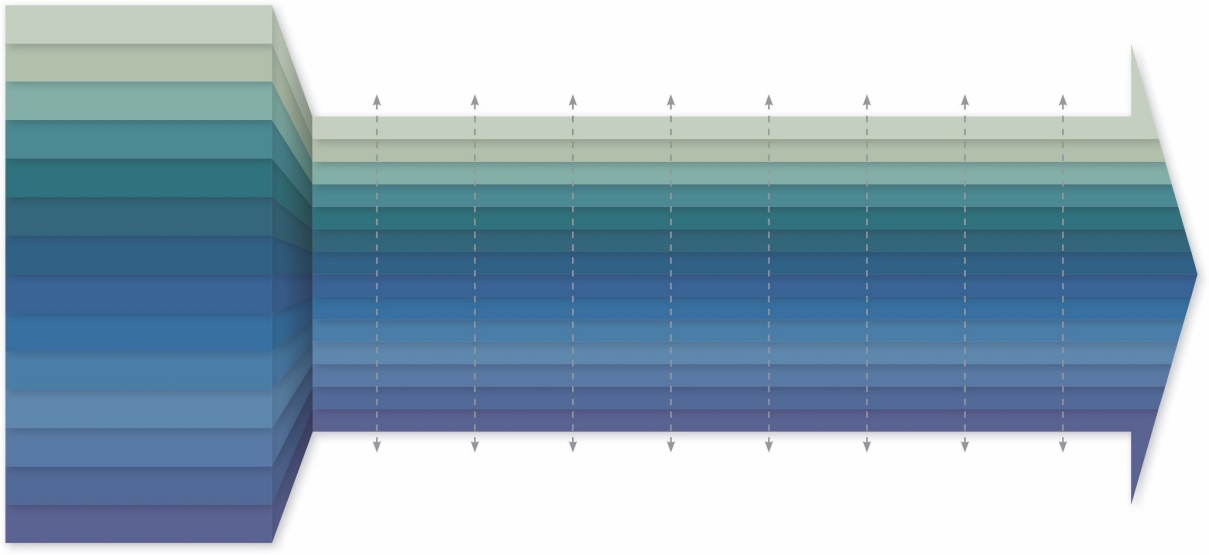
New York State Next Generation

Mathematics Learning Standards

**2017**



**Make   
sense of problems   
and   
persevere   
in solving them.**

**Reason  
abstractly**

**and**

**quantitatively.**

**Construct   
viable**

**arguments  
and critique**

**the reasoning of others.**

**Use appropriate**

**tools**

**strategically.**

**Look for**

**and make**

**use of**

**structure.**

**Model with**

**mathematics.**

**Attend to**

**precision.**

**Look for**

**and express**

**regularity   
in repeated reasoning.**

**Counting and Cardinality**

**Operations and Algebraic Thinking**

**Number and Operations in Base Ten**

**Number and Operations – Fractions**

**Ratios and Proportional Relationships**

**The Number System**

**Expressions and Equations**

**Functions**

**Measurement and Data**

**Geometry**

**Statistics and Probability**

**Number and Quantity**

**Algebra**

**Modeling**

# 

# Introduction

In 2015, New York State (NYS) began a process of review and revision of its current mathematics standards adopted in January of 2011. Through numerous phases of public comment, virtual and face-to-face meetings with committees consisting of NYS educators (Special Education, Bilingual Education and English as a New Language teachers), parents, curriculum specialists, school administrators, college professors, and experts in cognitive research, the *New York State Next Generation Mathematics Learning Standards (2017)* were developed. These revised standards reflect the collaborative efforts and expertise of all constituents involved.

The *New York State Next Generation Mathematics Learning Standards (2017)* reflect revisions, additions, vertical movement, and clarifications to the current mathematics standards. The Standards are defined as the knowledge, skills and understanding that individuals can and do habitually demonstrate over time because of instruction and learning experiences. These mathematics standards, collectively, are focused and cohesive—designed to support student access to the knowledge and understanding of the mathematical concepts that are necessary to function in a world very dependent upon the application of mathematics, while providing educators the opportunity to devise innovative programs to support this endeavor. As with any set of standards, they need to be rigorous; they need to demand a balance of conceptual understanding, procedural fluency and application and represent a significant level of achievement in mathematics that will enable students to successfully transition to post-secondary education and the workforce.

## Context for Revision of the *NYS Next Generation Mathematics Learning Standards (2017)*

***Changing expectations for mathematics achievement***

Today’s children are growing up in a world very different from the one even 15 years ago. Seismic changes in the labor market mean that we are living and working in a knowledge-based economy—one that demands advanced literacy and Science, Technology, Engineering and Mathematics (STEM) skills, whether for application in the private or public sector. Today, information moves through media at lightning speeds and is accessible in ways that are unprecedented; technology has eliminated many jobs while changing and creating others, especially those involving mathematical and conceptual reasoning skills. One characteristic of these fast-growing segment of jobs is that the employee needs to be able to solve unstructured problems while working with others in teams. At the same time, migration and immigration rates around the world bring diversity to schools and neighborhoods. The exponential growth in interactions and information sharing from around the world means there is much to process, communicate, analyze and respond to in the everyday, across all settings. For a great majority of jobs, conceptual reasoning and technical writing skills are integral parts to the daily routine.

To prepare students for the changes in the way we live and work, and to be sure that our education system keeps pace with what it means to be mathematically literate and what it means to collaboratively problem solve, we need a different approach to daily teaching and learning. We need content-rich standards that will serve as a platform for advancing children’s 21st-century mathematical skills —their abstract reasoning, their collaboration skills, their ability to learn from peers and through technology, and their flexibility as a learner in a dynamic learning environment. Students need to be engaged in dialogue and learning experiences that allow complex topics and ideas to be explored from many angles and perspectives. They also need to learn how to think and solve problems for which there is no one solution—and learn mathematical skills along the way.

***Increasingly Diverse Learner Populations***

The need for a deeper, more innovative approach to mathematics teaching comes at a time when the system is already charged with building up language skills among the increasingly diverse population. Students who are English Language Learners (ELLs)/Multilingual Learners (MLLs) now comprise over 20% of the school-age population, which reflects significant growth in the past several decades. Between 1980 and 2009, this population increased from 4.7 to 11.2 million young people, or from 10 to 21% of the school-age population. This growth will likely continue in U.S. schools; by 2030, it is anticipated that 40% of the school-age population in the U.S. will speak a language other than English at home. [(1)](#WorksCited) Today, in schools and districts across the U.S., many students other than those classified as ELLs are learning English as an additional language, even if not in the initial stages of language development—these children are often described as “language minority learners.” Likewise, many students, large numbers of whom are growing up in poverty, speak a dialect of English that is different from the academic English found in school curriculum. [(2) (3) (4)](#WorksCited)

Each of these groups—ELLs/MLLs, language minority learners, and students acquiring academic English—often struggle to access the language, and therefore the knowledge that fills the pages of academic texts, despite their linguistic assets. Therefore, the context for this new set of Mathematics Standards is that there is a pressing need to provide instruction that not only meets, but exceeds standards, as part of system-wide initiative to promote equal access to math skills for all learners while capitalizing on linguistic and cultural diversity.

All academic work does, to some degree, involve the academic language needed for success in school. For many students, including ELLs/MLLs, underdeveloped academic language affects their ability to comprehend and analyze texts, limits their ability to write and express their mathematical reasoning effectively, and can hinder their acquisition of academic content in all academic areas in which learning is demonstrated and assessed through oral and written language. If there isn’t sufficient attention paid to building academic language across all content areas, students, including ELLs/MLLs, will not reach their potential and we will continue to perpetuate achievement gaps. The challenge is to design instruction that acknowledges the role of language; because language and knowledge are so inextricable.

In summary, today’s children live in a society where many of their peers are from diverse backgrounds and speak different languages; one where technology is ubiquitous and central to daily life. They will enter a workforce and economy that demands critical thinking skills, and strong communication and social skills for full participation in society. This new society and economy has implications for today’s education system—especially our instruction to foster a deeper and different set of communication and critical thinking skills, with significant attention to STEM.

***Students with Disabilities and the Standards***

One of the fundamental tenets guiding educational legislation (the *No Child Left Behind Act*, and *Every Student Succeeds Act*), and related policies over the past 15-years, is that all students, including students with disabilities, can achieve high standards of academic performance. A related trend is the increasing knowledge and skill expectations for PreK-Grade 12 students, especially in the area of reading and language arts, required for success in postsecondary education and 21st Century careers. Indeed, underdeveloped literacy skills have profound academic, social, emotional, and economic consequences for students, families, and society.

At the same time, the most recently available federal data [(5)](#WorksCited) presents a portrait of the field reflecting both challenges and opportunities.

* *Students served under IDEA, Part B*: During the 2012-13 school year, there was a total of 5.83 million students with disabilities, ages 6-21; an increase from 5.67 million in 2010-11.
* *Access to the general education program*: More than 60 percent (62.1%) of students, ages 6 through 21 served under IDEA, Part B, were educated in the regular classroom 80% or more of the day, up from 60.5% in 2010-11.
* *Participation in state assessments*: Between 68.1 and 84.1 percent of students with disabilities in each of grades 3 through 8 and high school participated in the regular state assessment in reading based on grade-level academic achievement standards with or without accommodations.
* *English language arts proficiency*: The median percentages of students with disabilities in grades 3 through 8 and high school who were administered the 2012-13 state assessment in reading based on grade-level academic achievement standards who were proficient ranged from 25.4 to 37.3 percent.
* *Graduation*: Over sixty percent (65.1%) of students with disabilities graduated with a regular high school diploma.

Overall, the number of students with disabilities is increasing nationwide, as is their access to the general education curriculum, and participation in the state ELA and mathematics assessments. Attaining proficiency and graduating with a regular high school diploma are areas where significant improvements are needed.

Therefore, each student’s individualized education program (IEP) must be developed in consideration of the State learning standards and should include information for teachers to effectively provide supports and services to address the individual learning needs of the student as they impact the student’s ability to participate and progress in the general education curriculum. In addition to supports and services, special education must include specially designed instruction, which means adapting, as appropriate, the content, methodology or delivery of instruction to address the unique needs that result from the student’s disability. By so doing, the teacher ensures each student’s access to the general education curriculum so that he or she can meet the learning standards that apply to all students. The [*Blueprint for Improved Results for Students with Disabilities*](http://www.p12.nysed.gov/specialed/publications/2015-memos/documents/blueprint-students-disabilities-special-education.pdf) focuses on seven core evidence-based principles for students with disabilities to ensure they have the opportunity to benefit from high quality instruction and to reach the same academic standards as all students. For additional information, please see the Office of Special Education’s field advisory: [*Blueprint for Improved Results for Students with Disabilities*](http://www.p12.nysed.gov/specialed/publications/2015-memos/documents/blueprint-students-disabilities-special-education.pdf)*.*

## Understanding the *NYS Next Generation Mathematics Learning Standards (2017)*

The *NYS Next Generation Mathematics Learning Standards (2017)* define what students should understand and be able to do as a result of their study of mathematics. To assess progress on the Standards, a teacher must assess whether the student has understood what has been taught and provide opportunities where a student can independently use and apply this knowledge to solve mathematical problems in similar or new contexts. While procedural skills are relatively straightforward to assess, teachers often ask: what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is accurate or where a mathematical rule comes from. Correctly using language to articulate mathematical understanding plays a part in this justification. Making the distinction between mathematical understanding and procedural skill is critical when designing curriculum and assessment; both are important for the mastery of these standards. That is, there is a world of difference between a student who can summon a mnemonic device to expand a product such as (*a* + *b*)(*x* + *y*) and a student who can explain what the mnemonic represents as a process for systematically approaching algebraic problems. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task, such as expanding (*a* + *b* + *c*)(*x* + *y*).

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English Language Learners (ELLs)/Multilingual Learners (MLLs) and for Students with Disabilities. However, the department ensured that teachers of English Language Learners (ELLs)/Multilingual Learners (MLLs) and Students with Disabilities participated in the revision of the standards. The New York State Education Department (NYSED) has created two statewide frameworks, the [*Blueprint for Improved Results for Students with Disabilities*](http://www.p12.nysed.gov/specialed/publications/2015-memos/documents/blueprint-students-disabilities-special-education.pdf) and the [*Blueprint for English Language Learner Success*](http://www.nysed.gov/common/nysed/files/programs/bilingual-ed/nysblueprintforellsuccess.2016.pdf), aimed to clarify expectations and to provide guidance for administrators, policymakers, and practitioners to prepare ELLs/MLLs and Students with Disabilities for success. These principles therein the frameworks are intended to enhance programming and improve instruction that would allow for students within these populations to reach the same standards as all students and leave school prepared to successfully transition to post school learning, living and working.

No set of grade-specific standards can fully reflect the variation in learning profiles, rates, and needs, linguistic backgrounds, and achievement levels of students in any given classroom. When designing and delivering mathematics instruction, educators must consider the cultural context and prior academic experiences of all students while bridging prior knowledge to new knowledge and ensuring that content is meaningful and comprehensible. In addition, as discussed above, educators must consider the relationship of language and content, and the vital role that language plays in obtaining and expressing mathematics content knowledge. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate adaptations to ensure equitable access and maximum participation of all students.

# High School – Introduction

## Organization of the High School Standards for Mathematical Content

The standards are organized by course at the high school level. The high school courses include Algebra I, Geometry, and Algebra II. The Plus (+) Standards do not make up a course. They are additional standards that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics. The Plus (+) Standards may be dispersed throughout any high school course, including Algebra I, Geometry, and Algebra II, but are not expected to be addressed on state assessments.

The high school courses and the Plus (+) Standards are grouped by conceptual categories. These categories include Number and Quantity, Algebra, Functions, Geometry, Statistics and Probability, and Modeling. Conceptual categories portray a coherent view of high school mathematics; a student’s work with functions, for example, crosses a number of courses, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

# High School – Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

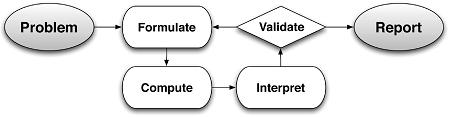
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

* Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
* Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
* Designing the layout of the stalls in a school fair so as to raise as much money as possible.
* Analyzing stopping distance for a car.
* Modeling savings account balance, bacterial colony growth, or investment growth.
* Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
* Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
* Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

****The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

## How to read the High School Standards for Mathematical Content

**Standards** define what students should understand and be able to do.

**Clusters** summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

**Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.

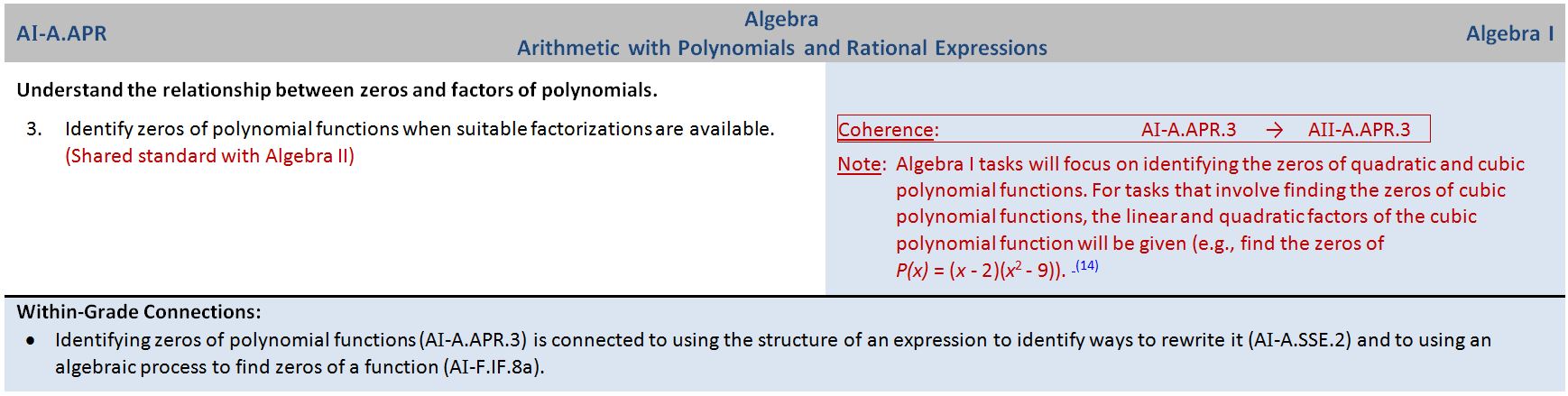
**Conceptual Categories** provide coherence across the courses.

**Coherence Linkages** connect standards one grade level forward and/or back when there are very direct linking standards in those grades. For a more thorough analysis of how standards link to one another, see <http://achievethecore.org/coherence-map/>.

**Citations** are indicated by a blue number when information was taken or adapted from another source. The number will match the source number in the *Works Cited* section at the end of this document. When viewing these standards electronically, the source information (including page number) will appear as hover-over text.

**High School Courses and Plus (+) Standards**

**Conceptual Category**



**Coherence Linkages**

**Citation**

**Notes to Clarify & Connect Standards**

**Standards**

**Cluster Heading**

**Domain**

The order in which the standards are presented is not necessarily the order in which the standards need to be taught. Standards from various domains are connected, and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain course expectations and the knowledge articulated in the standards. That is, the standards do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information.

**Geometry** **Overview**

Geometry is intended to be the second course in mathematics for high school students. During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions to establish the validity of geometric conjectures through deduction, proof, or mathematical arguments. Over the years, students develop an understanding of the attributes and relationships of two- and three-dimensional geometric shapes that can be applied in diverse contexts.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformations. Fundamental are the rigid motions: translations, rotations, reflections, and sequences of these, all of which are here assumed to preserve distance and angle measure reflections and rotations each explain a particular type of symmetry leading to insight into a figure’s attributes. Two geometric figures are defined to be congruent if there is a sequence of rigid motions that maps one figure onto the other. For triangles, congruence means that all corresponding pairs of sides and all corresponding pairs of angles are congruent. This leads to the triangle congruence criteria ASA, SAS, SSS, AAS and Hypotenuse-Leg (HL). Once these criteria are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations define similarity as a sequence of dilations and/or rigid motions that maps one figure onto another. Students formalize the similarity ideas of "same shape" and "scale factor" developed in the middle grades by establishing that similar triangles have all pairs of corresponding angles congruent and all corresponding pairs of sides proportional. These transformations lead to the criteria AA, SSS similarity, and SAS similarity for similar triangles.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, along with the Pythagorean Theorem and are fundamental in many mathematical situations. Radian measure will be introduced in Algebra II, along with the unit circle.

Students’ experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. They reason abstractly and quantitatively to model problems using volume formulas.

Students prove and apply basic theorems about circles and study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students explain the correspondence between the definition of a circle and the equation of a circle written in terms of the distance formula, its radius, and coordinates of its center. Given an equation of a circle, students graph the circle in the coordinate plane and apply techniques for solving quadratic equations.

***Note:*** *For scaffolding purposes, the use of a variety of tools and methods for construction is encouraged. These scaffolds include compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena using visualization, reasoning, and geometric modeling to solve problems, in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. Students can create geometric models and ideas to solve not only problems in mathematics, but in other disciplines or everyday situations.*

| **Mathematical Practices** | |
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| 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. | 1. Use appropriate tools strategically. 2. Attend to precision. 3. Look for and make use of structure. 4. Look for and express regularity in repeated reasoning. |

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| **GEO-G.CO** | **Geometry**  **Congruence** | | **Geometry** |
| Experiment with transformations in the plane. | |  | |
| 1. Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc as these exist within a plane. | |  | |
| 1. Represent transformations as geometric functions that take points in the plane as inputs and give points as outputs. Compare transformations that preserve distance and angle measure to those that do not. | | | Coherence: | NY-8.F.1  NY-8.G.1 | → | GEO-G.CO.2 |  |  | | --- | --- | --- | --- | --- | --- |   Note: Instructional strategies may include drawing tools, graph paper, transparencies and software programs. | |
| 1. Given a regular or irregular polygon, describe the rotations and reflections (symmetries) that map the polygon onto itself. | | | Coherence: | NY-8.G.4 | → | GEO-G.CO.3 |  |  | | --- | --- | --- | --- | --- | --- |   Note: The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as “A quadrilateral with *at least* one pair of parallel sides.” | |
| 1. Develop definitions of rotations, reflections, and translations in terms of points, angles, circles, perpendicular lines, parallel lines, and line segments. | | | Coherence: | NY-8.G.1 | → | GEO-G.CO.4 |  |  | | --- | --- | --- | --- | --- | --- |   Notes:   * Include point reflections. * A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. * A rotation requires knowing the center/point and the measure/direction of the angle of rotation. * A line reflection requires a line and the knowledge of perpendicular bisectors. | |
| 1. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another. | | | Coherence: | NY-8.G.2 | → | GEO-G.CO.5 |  |  | | --- | --- | --- | --- | --- | --- |   Notes:   * Include point reflections. * A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. * A rotation requires knowing the center/point and the measure/direction of the angle of rotation. * A line reflection requires a line and the knowledge of perpendicular bisectors. * Instructional strategies may include graph paper, tracing paper, and geometry software. * Singular transformations that are equivalent to a sequence of transformations may be utilized, such as a glide reflection.  However, glide reflections are not an expectation of the course. | |

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| **GEO-G.CO** | **Geometry**  **Congruence** | | **Geometry** |
| Understand congruence in terms of rigid motions. | |  | |
| 1. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | | Coherence: | NY-8.G.2 | → | GEO-G.CO.6 |  |  | | --- | --- | --- | --- | --- | --- |   Notes:   * A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. * A rotation requires knowing the center/point and the measure/direction of the angle of rotation. * A line reflection requires a line and the knowledge of perpendicular bisectors. | |
| 1. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | | Coherence: | NY-8.G.2 | → | GEO-G.CO.7 |  |  | | --- | --- | --- | --- | --- | --- | | |
| 1. Explain how the criteria for triangle congruence (ASA, SAS, SSS, AAS and HL (Hypotenuse Leg)) follow from the definition of congruence in terms of rigid motions. | |  | |
| **Connecting the Standards for Mathematical Practice to Mathematical Content:**   * One place in geometry where precision (MP.6) is necessary and useful is in the refinement of conjectures so that initial conjectures that are not correct can be salvaged — two angle measures and a side length do not determine a triangle, but a certain configuration of these parts leads to the angle-side-angle theorem (GEO-G.CO.8). [(14)](#WorksCited) | | | |

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| **GEO-G.CO** | **Geometry**  **Congruence** | | **Geometry** |
| Prove geometric theorems. | |  | |
| 1. Prove and apply theorems about lines and angles. | | | Coherence: | NY-8.G.5 | → | GEO-G.CO.9 |  |  | | --- | --- | --- | --- | --- | --- |   Notes:   * Include multi-step proofs and algebraic problems built upon these concepts. * Examples of theorems include but are not limited to:   + Vertical angles are congruent.   + If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.   + The points on a perpendicular bisector are equidistant from the endpoints of the line segment. | |
| 1. Prove and apply theorems about triangles. | | | Coherence: | NY-8.G.5 | → | GEO-G.CO.10 |  |  | | --- | --- | --- | --- | --- | --- |   Notes:   * Include multi-step proofs and algebraic problems built upon these concepts. * Examples of theorems include but are not limited to:   + Angle Relationships:     - The sum of the interior angles of a triangle is 180 degrees.     - The measure of an exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles of the triangle.   + Side Relationships:     - The length of one side of a triangle is less than the sum of the lengths of the other two sides.     - In a triangle, the segment joining the midpoints of any two sides will be parallel to the third side and half its length.   + Isosceles Triangles:     - Base angles of an isosceles triangle are congruent. | |
| 1. Prove and apply theorems about parallelograms. | | Notes:   * Include multi-step proofs and algebraic problems built upon these concepts. * The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as “A quadrilateral with *at least* one pair of parallel sides.” * Examples of theorems include but are not limited to:   + A diagonal divides a parallelogram into two congruent triangles.   + Opposite sides/angles of a parallelogram are congruent.   + The diagonals of parallelogram bisect each other.   + If the diagonals of quadrilateral bisect each other, then quadrilateral is a parallelogram.   + If the diagonals of a parallelogram are congruent then the parallelogram is a rectangle. * Additional theorems covered allow for proving that a given quadrilateral is a particular parallelogram (rhombus, rectangle, square) based on given properties. | |
| **Connecting the Standards for Mathematical Practice to Mathematical Content:**   * Abstraction (MP.2) is used in geometry when, for example, students use a diagram of a specific isosceles triangle as an aid to reason about *all* isosceles triangles (GEO‑G.CO.9). [(14)](#WorksCited) * When students develop the skill of creating and presenting proofs (GEO-G.CO.9 & 10), they are constructing viable arguments and have opportunities to critique the reasoning of others (MP.3). [(14)](#WorksCited) * Seeing structure in geometric configurations (MP.7) can lead to insights and proofs. This often involves the creation of auxiliary lines not originally part of a given figure. Two classic examples are the construction of a line through a vertex of a triangle parallel to the opposite side as a way to see that the angle measures of a triangle add to 180 degrees and the introduction of a symmetry line in an isosceles triangle to see that the base angles are congruent (GEO-G.CO.9 & 10). [(14)](#WorksCited) | | | |

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| **GEO-G.CO** | **Geometry**  **Congruence** | | **Geometry** |
| Make geometric constructions. | |  | |
| 1. Make, justify, and apply formal geometric constructions. | | Notes:   * Examples of constructions include but are not limited to:   + Copy segments and angles.   + Bisect segments and angles.   + Construct perpendicular lines including through a point on or off a given line.   + Construct a line parallel to a given line through a point not on the line.   + Construct a triangle with given lengths.   + Construct points of concurrency of a triangle (centroid, circumcenter, incenter, and orthocenter).   + Construct the inscribed circle of a triangle.   + Construct the circumscribed circle of a triangle.   + Constructions of transformations. (see G.CO.A.5) * This standard is a fluency recommendation for Geometry. Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs. [(14)](#WorksCited) | |
| 1. Make and justify the constructions for inscribing an equilateral triangle, a square and a regular hexagon in a circle. | |  | |
| **Note on *Fluency* with Procedures:**   * *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure. [(12)](https://www.nap.edu/read/9822/chapter/6#121) [(13)](#WorksCited)   **Connecting the Standards for Mathematical Practice to Mathematical Content:**   * Dynamic geometry environments can help students look for invariants in a whole class of geometric constructions (GEO-G.CO.12 & 13), and the constructions in such environments can sometimes lead to an idea behind a proof of a conjecture. This is an example of using appropriate tools strategically (MP.5). [(14)](#WorksCited) | | | |

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| **GEO-G.SRT** | **Geometry**  **Similarity, Right Triangles, and Trigonometry** | | **Geometry** |
| Understand similarity in terms of similarity transformations. | |  | |
| 1. Verify experimentally the properties of dilations given by a center and a scale factor. | | | Coherence: | NY-8.G.3 | → | GEO-G.SRT.1 |  |  | | --- | --- | --- | --- | --- | --- | | |
| * 1. Verify experimentally that dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | |  | |
| * 1. Verify experimentally that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. | |  | |
| 1. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar. Explain using similarity transformations that similar triangles have equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | | Coherence: | NY-8.G.4 | → | GEO-G.SRT.2 |  |  | | --- | --- | --- | --- | --- | --- |   Notes:   * The center and scale factor of the dilation must always be specified with dilation. * A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector. * A rotation requires knowing the center/point and the measure/direction of the angle of rotation. * A line reflection requires a line and the knowledge of perpendicular bisectors. | |
| 1. Use the properties of similarity transformations to establish the AA, SSS, and SAS criterion for two triangles to be similar. | | | Coherence: | NY-8.G.5 | → | GEO-G.SRT.3 |  |  | | --- | --- | --- | --- | --- | --- | | |

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| **GEO-G.SRT** | **Geometry**  **Similarity, Right Triangles, and Trigonometry** | | **Geometry** |
| Prove theorems involving similarity. | |  | |
| 1. Prove and apply similarity theorems about triangles. | | Notes:   * Include multi-step proofs and algebraic problems built upon these concepts. * Examples of theorems include but are not limited to:   + If a line parallel to one side of a triangle intersects the other two sides of the triangle, then the line divides these two sides proportionally (and conversely) .   + The length of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the lengths of the two segments of the hypotenuse.   + The centroid of the triangle divides each median in the ratio 2:1. | |
| 1. Use congruence and similarity criteria for triangles to:    1. Solve problems algebraically and geometrically.    2. Prove relationships in geometric figures. | | Notes:   * ASA, SAS, SSS, AAS, and Hypotenuse‐Leg (HL) theorems are valid criteria for triangle congruence. AA, SAS, and SSS are valid criteria for triangle similarity. * This standard is a fluency recommendation for Geometry. Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks. [(14)](#WorksCited) | |
| **Note on *Fluency* with Procedures:**   * *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure. [(12)](https://www.nap.edu/read/9822/chapter/6#121) [(13)](#WorksCited)   **Connecting the Standards for Mathematical Practice to Mathematical Content:**   * When students use area as a device to establish results about proportions, such as the important theorem (and its converse) that a line parallel to one side of a triangle divides the other two sides proportionally (GEO-G.SRT.4), they are making use of structure (MP.7). [(14)](#WorksCited) | | | |

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| **GEO-G.SRT** | **Geometry**  **Similarity, Right Triangles, and Trigonometry** | | **Geometry** |
| Define trigonometric ratios and solve problems involving right triangles. | |  | |
| 1. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of sine, cosine and tangent ratios for acute angles. | | | Coherence: |  |  | GEO-G.SRT.6 | → | AII-F.TF.2  (+)-G.SRT.10 | | --- | --- | --- | --- | --- | --- | | |
| 1. Explain and use the relationship between the sine and cosine of complementary angles. | | | Coherence: |  |  | GEO-G.SRT.7 | → | AII-F.TF.4 | | --- | --- | --- | --- | --- | --- | | |
| 1. Use sine, cosine, tangent, the Pythagorean Theorem and properties of special right triangles to solve right triangles in applied problems. ★ | | | Coherence: | NY-8.G.7 | → | GEO-G.SRT.8 | → | (+)-G.SRT.10 | | --- | --- | --- | --- | --- | --- |   Note: Special right triangles refer to the 30-60-90 and 45-45-90 triangles. | |

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| **GEO-G.SRT** | **Geometry**  **Similarity, Right Triangles, and Trigonometry** | | **Geometry** |
| Apply Trigonometry to general triangles. | |  | |
| 1. Justify and apply the formula A= ab sin (C) to find the area of any triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | |  | |

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| **GEO-G.C** | **Geometry**  **Circles** | | **Geometry** |
| Understand and apply theorems about circles. | |  | |
| 1. Prove that all circles are similar. | |  | |
| 1. Identify, describe and apply relationships between the angles and their intercepted arcs of a circle. | | Note: These relationships that pertain to the circle may be utilized to prove other relationships in geometric figures, e.g., the opposite angles in any quadrilateral inscribed in a circle are supplements of each other. | |
| 1. Identify, describe and apply relationships among radii, chords, tangents, and secants of a circle. | | | Coherence: |  |  | GEO-G.C.2b | → | (+)-G.C.4 | | --- | --- | --- | --- | --- | --- |   Note: Include algebraic problems built upon these concepts. | |
| **Connecting the Standards for Mathematical Practice to Mathematical Content:**   * In geometry, students conjecture about geometric phenomena that pertain to infinitely many cases at once (ex., *every* angle inscribed in a semicircle is a right angle) because it is impossible to check every case (GEO-G.C.2a & b). When they do this, they are constructing viable arguments and have opportunities to critique the reasoning of others (MP.3). [(14)](#WorksCited) | | | |

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| **GEO-G.C** | **Geometry**  **Circles** | | **Geometry** |
| Find arc lengths and area of sectors of circles. | |  | |
| 1. Using proportionality, find one of the following given two others; the central angle, arc length, radius or area of sector. | | Note: Angle measure is in degrees. | |

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| **GEO-G.GPE** | **Geometry**  **Expressing Geometric Properties with Equations** | | **Geometry** |
| Translate between the geometric description and the equation of a conic section. | |  | |
| 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. Find the center and radius of a circle, given the equation of the circle. | | | Coherence: | NY-8.G.8 | → | GEO-G.GPE.1a | → | (+)-G.GPE.2 | | --- | --- | --- | --- | --- | --- |   Notes:   * Finding the center and radius may involve completing the square. The completing the square expectation for Geometry follows Algebra I:  leading coefficients will be 1 (after possible removal of GCF) and the coefficients of the linear terms will be even. * Completing the square may yield a fractional radius. | |
| 1. Graph circles given their equation. | | Note: For circles being graphed, the center will be an ordered pair of integers and the radius will be a positive integer. | |

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| **GEO-G.GPE** | **Geometry**  **Expressing Geometric Properties with Equations** | | **Geometry** |
| Use coordinates to prove simple geometric theorems algebraically. | |  | |
| 1. On the coordinate plane, algebraically prove geometric theorems and properties. | | | Coherence: | NY-8.G.8 | → | GEO-G.GPE.4 |  |  | | --- | --- | --- | --- | --- | --- |   Notes:   * Examples include but not limited to:   + Given points and/or characteristics, prove or disprove a polygon is a specified quadrilateral or triangle based on its properties.   + Given a point that lies on a circle with a given center, prove or disprove that a specified point lies on the same circle. * This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields. [(14)](#WorksCited) | |
| 1. On the coordinate plane:    1. Explore the proof for the relationship between slopes of parallel and perpendicular lines;    2. Determine if lines are parallel, perpendicular, or neither, based on their slopes; and    3. Apply properties of parallel and perpendicular lines to solve geometric problems. | | | Coherence: | NY-8.EE.6 | → | GEO-G.GPE.5 |  |  | | --- | --- | --- | --- | --- | --- |   Note: This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields. [(14)](#WorksCited) | |
| 1. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | | Note: Midpoint formula is a derivative of this standard. | |
| 1. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. ★ | | Note: This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields. [(14)](#WorksCited) | |
| **Note on *Fluency* with Procedures:**   * *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure. [(12)](https://www.nap.edu/read/9822/chapter/6#121) [(13)](#WorksCited)   **Note on the Word *Explore*:**   * *Explore* indicates that the topic is an important concept that builds the foundation for progression toward mastery in later grades. Repeated experiences with these concepts, with immersion in the concrete, are vital. | | | |

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| **GEO-G.GMD** | **Geometry**  **Geometric Measurement and Dimension** | | **Geometry** |
| Explain volume formulas and use them to solve problems. | |  | |
| 1. Provide informal arguments for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | | Coherence: | NY-8.G.9 | → | GEO-G.GMD.1 |  |  | | --- | --- | --- | --- | --- | --- | | |
| 1. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★ | | | Coherence: |  |  | GEO-G.GMD.3 | → | (+)-G.GMD.2 | | --- | --- | --- | --- | --- | --- | | |

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| **GEO-G.GMD** | **Geometry**  **Geometric Measurement and Dimension** | | **Geometry** |
| Visualize relationships between two-dimensional and three-dimensional objects. | |  | |
| 1. Identify the shapes of plane sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | Note: Plane sections are not limited to being parallel or perpendicular to the base. | |

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| **GEO-G.MG** | **Geometry**  **Modeling with Geometry** | | **Geometry** |
| Apply geometric concepts in modeling situations. | |  | |
| 1. Use geometric shapes, their measures, and their properties to describe objects. | | | Coherence: | NY-8.G.9 | → | GEO-G.MG.1 |  |  | | --- | --- | --- | --- | --- | --- | | |
| 1. Apply concepts of density based on area and volume of geometric figures in modeling situations. | |  | |
| 1. Apply geometric methods to solve design problems. | | Note: Applications may include designing an object or structure to satisfy constraints such as area, volume, mass, and cost. | |