New York State Next Generation

Mathematics Learning Standards

**2017**

**Make
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and
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**Reason
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**and**

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**Construct
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**Use appropriate**

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**Look for**

**and make**

**use of**

**structure.**

**Model with**

**mathematics.**

**Attend to**

**precision.**

**Look for**

**and express**

**regularity
in repeated reasoning.**

**Counting and Cardinality**

**Operations and Algebraic Thinking**

**Number and Operations in Base Ten**

**Number and Operations – Fractions**

**Ratios and Proportional Relationships**

**The Number System**

**Expressions and Equations**

**Functions**

**Measurement and Data**

**Geometry**

**Statistics and Probability**

**Number and Quantity**

**Algebra**

**Modeling**

#

# Introduction

In 2015, New York State (NYS) began a process of review and revision of its current mathematics standards adopted in January of 2011. Through numerous phases of public comment, virtual and face-to-face meetings with committees consisting of NYS educators (Special Education, Bilingual Education and English as a New Language teachers), parents, curriculum specialists, school administrators, college professors, and experts in cognitive research, the *New York State Next Generation Mathematics Learning Standards (2017)* were developed. These revised standards reflect the collaborative efforts and expertise of all constituents involved.

The *New York State Next Generation Mathematics Learning Standards (2017)* reflect revisions, additions, vertical movement, and clarifications to the current mathematics standards. The Standards are defined as the knowledge, skills and understanding that individuals can and do habitually demonstrate over time because of instruction and learning experiences. These mathematics standards, collectively, are focused and cohesive—designed to support student access to the knowledge and understanding of the mathematical concepts that are necessary to function in a world very dependent upon the application of mathematics, while providing educators the opportunity to devise innovative programs to support this endeavor. As with any set of standards, they need to be rigorous; they need to demand a balance of conceptual understanding, procedural fluency and application and represent a significant level of achievement in mathematics that will enable students to successfully transition to post-secondary education and the workforce.

## Context for Revision of the *NYS Next Generation Mathematics Learning Standards (2017)*

***Changing expectations for mathematics achievement***

Today’s children are growing up in a world very different from the one even 15 years ago. Seismic changes in the labor market mean that we are living and working in a knowledge-based economy—one that demands advanced literacy and Science, Technology, Engineering and Mathematics (STEM) skills, whether for application in the private or public sector. Today, information moves through media at lightning speeds and is accessible in ways that are unprecedented; technology has eliminated many jobs while changing and creating others, especially those involving mathematical and conceptual reasoning skills. One characteristic of these fast-growing segment of jobs is that the employee needs to be able to solve unstructured problems while working with others in teams. At the same time, migration and immigration rates around the world bring diversity to schools and neighborhoods. The exponential growth in interactions and information sharing from around the world means there is much to process, communicate, analyze and respond to in the everyday, across all settings. For a great majority of jobs, conceptual reasoning and technical writing skills are integral parts to the daily routine.

To prepare students for the changes in the way we live and work, and to be sure that our education system keeps pace with what it means to be mathematically literate and what it means to collaboratively problem solve, we need a different approach to daily teaching and learning. We need content-rich standards that will serve as a platform for advancing children’s 21st-century mathematical skills —their abstract reasoning, their collaboration skills, their ability to learn from peers and through technology, and their flexibility as a learner in a dynamic learning environment. Students need to be engaged in dialogue and learning experiences that allow complex topics and ideas to be explored from many angles and perspectives. They also need to learn how to think and solve problems for which there is no one solution—and learn mathematical skills along the way.

***Increasingly Diverse Learner Populations***

The need for a deeper, more innovative approach to mathematics teaching comes at a time when the system is already charged with building up language skills among the increasingly diverse population. Students who are English Language Learners (ELLs)/Multilingual Learners (MLLs) now comprise over 20% of the school-age population, which reflects significant growth in the past several decades. Between 1980 and 2009, this population increased from 4.7 to 11.2 million young people, or from 10 to 21% of the school-age population. This growth will likely continue in U.S. schools; by 2030, it is anticipated that 40% of the school-age population in the U.S. will speak a language other than English at home. [(1)](#WorksCited) Today, in schools and districts across the U.S., many students other than those classified as ELLs are learning English as an additional language, even if not in the initial stages of language development—these children are often described as “language minority learners.” Likewise, many students, large numbers of whom are growing up in poverty, speak a dialect of English that is different from the academic English found in school curriculum. [(2) (3) (4)](#WorksCited)

Each of these groups—ELLs/MLLs, language minority learners, and students acquiring academic English—often struggle to access the language, and therefore the knowledge that fills the pages of academic texts, despite their linguistic assets. Therefore, the context for this new set of Mathematics Standards is that there is a pressing need to provide instruction that not only meets, but exceeds standards, as part of system-wide initiative to promote equal access to math skills for all learners while capitalizing on linguistic and cultural diversity.

All academic work does, to some degree, involve the academic language needed for success in school. For many students, including ELLs/MLLs, underdeveloped academic language affects their ability to comprehend and analyze texts, limits their ability to write and express their mathematical reasoning effectively, and can hinder their acquisition of academic content in all academic areas in which learning is demonstrated and assessed through oral and written language. If there isn’t sufficient attention paid to building academic language across all content areas, students, including ELLs/MLLs, will not reach their potential and we will continue to perpetuate achievement gaps. The challenge is to design instruction that acknowledges the role of language; because language and knowledge are so inextricable.

In summary, today’s children live in a society where many of their peers are from diverse backgrounds and speak different languages; one where technology is ubiquitous and central to daily life. They will enter a workforce and economy that demands critical thinking skills, and strong communication and social skills for full participation in society. This new society and economy has implications for today’s education system—especially our instruction to foster a deeper and different set of communication and critical thinking skills, with significant attention to STEM.

***Students with Disabilities and the Standards***

One of the fundamental tenets guiding educational legislation (the *No Child Left Behind Act*, and *Every Student Succeeds Act*), and related policies over the past 15-years, is that all students, including students with disabilities, can achieve high standards of academic performance. A related trend is the increasing knowledge and skill expectations for PreK-Grade 12 students, especially in the area of reading and language arts, required for success in postsecondary education and 21st Century careers. Indeed, underdeveloped literacy skills have profound academic, social, emotional, and economic consequences for students, families, and society.

At the same time, the most recently available federal data [(5)](#WorksCited) presents a portrait of the field reflecting both challenges and opportunities.

* *Students served under IDEA, Part B*: During the 2012-13 school year, there was a total of 5.83 million students with disabilities, ages 6-21; an increase from 5.67 million in 2010-11.
* *Access to the general education program*: More than 60 percent (62.1%) of students, ages 6 through 21 served under IDEA, Part B, were educated in the regular classroom 80% or more of the day, up from 60.5% in 2010-11.
* *Participation in state assessments*: Between 68.1 and 84.1 percent of students with disabilities in each of grades 3 through 8 and high school participated in the regular state assessment in reading based on grade-level academic achievement standards with or without accommodations.
* *English language arts proficiency*: The median percentages of students with disabilities in grades 3 through 8 and high school who were administered the 2012-13 state assessment in reading based on grade-level academic achievement standards who were proficient ranged from 25.4 to 37.3 percent.
* *Graduation*: Over sixty percent (65.1%) of students with disabilities graduated with a regular high school diploma.

Overall, the number of students with disabilities is increasing nationwide, as is their access to the general education curriculum, and participation in the state ELA and mathematics assessments. Attaining proficiency and graduating with a regular high school diploma are areas where significant improvements are needed.

Therefore, each student’s individualized education program (IEP) must be developed in consideration of the State learning standards and should include information for teachers to effectively provide supports and services to address the individual learning needs of the student as they impact the student’s ability to participate and progress in the general education curriculum. In addition to supports and services, special education must include specially designed instruction, which means adapting, as appropriate, the content, methodology or delivery of instruction to address the unique needs that result from the student’s disability. By so doing, the teacher ensures each student’s access to the general education curriculum so that he or she can meet the learning standards that apply to all students. The [*Blueprint for Improved Results for Students with Disabilities*](http://www.p12.nysed.gov/specialed/publications/2015-memos/documents/blueprint-students-disabilities-special-education.pdf) focuses on seven core evidence-based principles for students with disabilities to ensure they have the opportunity to benefit from high quality instruction and to reach the same academic standards as all students. For additional information, please see the Office of Special Education’s field advisory: [*Blueprint for Improved Results for Students with Disabilities*](http://www.p12.nysed.gov/specialed/publications/2015-memos/documents/blueprint-students-disabilities-special-education.pdf)*.*

## Understanding the *NYS Next Generation Mathematics Learning Standards (2017)*

The *NYS Next Generation Mathematics Learning Standards (2017)* define what students should understand and be able to do as a result of their study of mathematics. To assess progress on the Standards, a teacher must assess whether the student has understood what has been taught and provide opportunities where a student can independently use and apply this knowledge to solve mathematical problems in similar or new contexts. While procedural skills are relatively straightforward to assess, teachers often ask: what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is accurate or where a mathematical rule comes from. Correctly using language to articulate mathematical understanding plays a part in this justification. Making the distinction between mathematical understanding and procedural skill is critical when designing curriculum and assessment; both are important for the mastery of these standards. That is, there is a world of difference between a student who can summon a mnemonic device to expand a product such as (*a* + *b*)(*x* + *y*) and a student who can explain what the mnemonic represents as a process for systematically approaching algebraic problems. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task, such as expanding (*a* + *b* + *c*)(*x* + *y*).

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English Language Learners (ELLs)/Multilingual Learners (MLLs) and for Students with Disabilities. However, the department ensured that teachers of English Language Learners (ELLs)/Multilingual Learners (MLLs) and Students with Disabilities participated in the revision of the standards. The New York State Education Department (NYSED) has created two statewide frameworks, the [*Blueprint for Improved Results for Students with Disabilities*](http://www.p12.nysed.gov/specialed/publications/2015-memos/documents/blueprint-students-disabilities-special-education.pdf) and the [*Blueprint for English Language Learner Success*](http://www.nysed.gov/common/nysed/files/programs/bilingual-ed/nysblueprintforellsuccess.2016.pdf), aimed to clarify expectations and to provide guidance for administrators, policymakers, and practitioners to prepare ELLs/MLLs and Students with Disabilities for success. These principles therein the frameworks are intended to enhance programming and improve instruction that would allow for students within these populations to reach the same standards as all students and leave school prepared to successfully transition to post school learning, living and working.

No set of grade-specific standards can fully reflect the variation in learning profiles, rates, and needs, linguistic backgrounds, and achievement levels of students in any given classroom. When designing and delivering mathematics instruction, educators must consider the cultural context and prior academic experiences of all students while bridging prior knowledge to new knowledge and ensuring that content is meaningful and comprehensible. In addition, as discussed above, educators must consider the relationship of language and content, and the vital role that language plays in obtaining and expressing mathematics content knowledge. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate adaptations to ensure equitable access and maximum participation of all students.

## How to Read the P-8 Standards for Mathematical Content

*\*See* [*High School – Introduction*](#HS_Intro) *for how to read the High School Standards for Mathematical Content.*

The standards are organized by grade level from Prekindergarten through grade eight.

**Standards** define what students should understand and be able to do.

**Clusters** summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

**Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.

**Coherence Linkages** connect standards one grade level forward and/or back when there are very direct linking standards in those grades. For a more thorough analysis of how standards link to one another, see <http://achievethecore.org/coherence-map/>.

**Citations** are indicated by a blue number when information was taken or adapted from another source. The number will match the source number in the *Works Cited* section at the end of this document. When viewing these standards electronically, the source information (including page number) will appear as hover-over text.

**Prekindergarten through Grade Eight**

**Domain**



**Citation**

**Notes to Clarify & Connect Standards**

**Coherence Linkages**

**Cluster Heading**

**Standards**

The order in which the standards are presented is not necessarily the order in which the standards need to be taught. Standards from various domains are connected, and educators will need to determine the best overall design and approach, as well as the instructional strategies needed to support their learners to attain grade-level expectations and the knowledge articulated in the standards. That is, the standards do not dictate curriculum or teaching methods; learning opportunities and pathways will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students, based on their pedagogical and professional impressions and information.

## The Standards for Mathematical Practice

The Standards for each grade level and course begin with eight Standards for Mathematical Practice. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. [(6)](#WorksCited) The second are the strands of mathematical proficiency specified in the National Research Council’s report [*Adding it Up*](https://www.nap.edu/read/9822/chapter/8#182): adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

1. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

1. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

1. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

1. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

1. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

1. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 ‑ 3(*x* ‑ *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

1. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (*y* – 2)/(*x* – 1) = 3. Noticing the regularity in the way terms cancel when expanding (*x* - 1)(*x* + 1), (*x* - 1)(*x*2 + *x* + 1), and (*x* - 1)(*x*3 + *x2* + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

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**Grade** **5 Overview**

In Grade 5, instructional time should focus on three areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimals into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume. Please note that while every standard/topic in the grade level has not been included in this overview, all standards should be included in instruction.

1. Through their learning in the ***Number and Operations – Fractions*** and ***Operations and Algebraic Thinking*** domains, students:
* apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators;
* develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them; and
* use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
1. Through their learning in the ***Operations and Algebraic Thinking*** and ***Number and Operations in Base Ten*** domains, students:
* develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations;
* apply understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths;
* develop fluency with decimal computations to hundredths, and make reasonable estimates of their results; and
* use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense.
1. Through their learning in the ***Measurement and Data*** and ***Geometry*** domains, students:
* recognize volume as an attribute of three-dimensional space;
* understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps;
* understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume;
* select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume;
* decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes; and
* measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

| **Mathematical Practices** |
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| 1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
 | 1. Use appropriate tools strategically.
2. Attend to precision.
3. Look for and make use of structure.
4. Look for and express regularity in repeated reasoning.
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| **NY-5.OA Operations and Algebraic Thinking**  |
| **Write and interpret numerical expressions.** |  |
| 1. Apply the order of operations to evaluate numerical expressions.
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| Coherence: |  |  | NY-5.OA.1 | → | NY-6.EE.2 |
| --- | --- | --- | --- | --- | --- |

e.g., * 6 + 8 ÷ 2
* (6 + 8) ÷ 2

Note:Exponents and nested grouping symbols are not included. |
| 1. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
 |

| Coherence: |  |  | NY-5.OA.2 | → | NY-6.EE.2NY-6.EE.3NY-6.EE.4 |
| --- | --- | --- | --- | --- | --- |

e.g., Express the calculation “add 8 and 7, then multiply by 2” as (8 + 7) × 2. Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product. |

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| **NY-5.OA Operations and Algebraic Thinking**  |
| **Analyze patterns and relationships.** |  |
| 1. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
 |

| Coherence: | NY-4.OA.5 | → | NY-5.OA.3 | → | NY-6.EE.9 |
| --- | --- | --- | --- | --- | --- |

e.g.,Given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. |

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| **NY-5.NBT Number and Operations in Base Ten**  |
| **Understand the place value system.** |  |
| 1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.
 |

| Coherence: | NY-4.NBT.1 | → | NY-5.NBT.1 |  |  |
| --- | --- | --- | --- | --- | --- |

 |
| 1. Use whole-number exponents to denote powers of 10. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
 |

| Coherence: |  |  | NY-5.NBT.2 | → | NY-6.EE.1 |
| --- | --- | --- | --- | --- | --- |

 |
| 1. Read, write, and compare decimals to thousandths.
 |  |
| a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.  |

| Coherence: | NY-4.NBT.2aNY-4.NF.6 | → | NY-5.NBT.3a |  |  |
| --- | --- | --- | --- | --- | --- |

e.g., * 47.392 = 4 × 10 + 7 × 1 + 3 × $\frac{1}{10}$ + 9 × $\frac{1}{100}$ + 2 × $\frac{1}{1000}$
* 47.392 = (4 × 10) + (7 × 1) + (3 × $\frac{1}{10}$ ) + (9 × $\frac{1}{100}$) + (2 ×$\frac{1}{1000}$)
* 47.392 = (4 × 10) + (7 × 1) + (3 × 0.1) + (9 × 0.01) + (2 × 0.001)
 |
| b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. |

| Coherence: | NY-4.NBT.2bNY-4.NF.7 | → | NY-5.NBT.3b | → | NY-6.NS.7 |
| --- | --- | --- | --- | --- | --- |

 |
| 1. Use place value understanding to round decimals to any place.
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| Coherence: | NY-4.NBT.3 | → | NY-5.NBT.4 |  |  |
| --- | --- | --- | --- | --- | --- |

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| **Within-Grade Connections:*** Conversions within the metric system (NY-5.MD.1) represent an important practical application of the place value system and an opportunity to develop understanding of it (NY-5.NBT.1 - 2). [(14)](#WorksCited)
* Understanding that in a multi-digit number, a digit in one place represents $\frac{1}{10}$ of what it represents in the place to its left (NY-5.NBT.1) is an example of multiplying a quantity by a fraction (NY-5.NF.4). [(14)](#WorksCited)

**Connecting the Standards for Mathematical Practice to Mathematical Content:*** When students explain patterns in the number of zeros of the product when multiplying a number by powers of 10 (NY-5.NBT.2), they have an opportunity to look for and express regularity in repeated reasoning (MP.8). When they use these patterns in division, they are making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2). [(14)](#WorksCited)
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| **NY-5.NBT Number and Operations in Base Ten**  |
| **Perform operations with multi-digit whole numbers and with decimals to hundredths.** |  |
| 1. Fluently multiply multi-digit whole numbers using a standard algorithm.
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| Coherence: | NY-4.NBT.5 | → | NY-5.NBT.5 | → | NY-6.NS.3 |
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 |
| 1. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
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| Coherence: | NY-4.NBT.6 | → | NY-5.NBT.6 | → | NY-6.NS.2 |
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Notes on and/or: * Students should be taught to use strategies based on place value, the properties of operations, *and* the relationship between multiplication and division; however, when solving any problem, students can choose any strategy.
* Students should be taught to use equations, rectangular arrays, *and* area models; however, when illustrating and explaining any calculation, students can choose any strategy.
 |
| 1. Using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations:
	* add and subtract decimals to hundredths;
	* multiply and divide decimals to hundredths.

Relate the strategy to a written method and explain the reasoning used. |

| Coherence: | NY-4.NF.5 | → | NY-5.NBT.7 | → | NY-6.NS.3 |
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Notes on and/or: Students should be taught to use concrete models and drawings; as well as strategies based on place value, properties of operations, *and* the relationship between operations. When solving any problem, students can choose to use a concrete model *or* a drawing. Their strategy must be based on place value, properties of operations, or the relationship between operations.Note: Division problems are limited to those that allow for the use of concrete models or drawings, strategies based on properties of operations, and/or the relationship between operations (e.g., 0.25 ÷ 0.05). Problems should not be so complex as to require the use of an algorithm (e.g., 0.37 ÷ 0.05). |
| **Note on *Fluency* with Procedures:*** *Fluency* with procedures *(procedural fluency)* means students are accurate, efficient, flexible, and know when and how to use them appropriately. Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations. Students also need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation of the mathematical basis for the procedure. [(12)](https://www.nap.edu/read/9822/chapter/6#121) [(13)](#WorksCited) For more on developing procedural fluency, see [*Adding it Up*](https://www.nap.edu/read/9822/chapter/6#121), pp. 121-124.

**Connecting the Standards for Mathematical Practice to Mathematical Content:*** When students break divisors and dividends into sums of multiples of base-ten units (NY-5.NBT.6), they are seeing and making use of structure (MP.7) and attending to precision (MP.6). Initially for most students, multi-digit division problems take time and effort, so they also require perseverance (MP.1) and looking for and expressing regularity in repeated reasoning (MP.8). [(14)](#WorksCited)
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| **NY-5.NF Number and Operations—Fractions**  |
| **Use equivalent fractions as a strategy to add and subtract fractions.** |  |
| 1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
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| Coherence: | NY-4.NF.1NY-4.NF.3c | → | NY-5.NF.1 |  |  |
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e.g., * $\frac{1}{3}$+ $\frac{2}{9}$ = $\frac{3}{9}$ + $\frac{2}{9}$ = $\frac{5}{9}$
* $\frac{2}{3}$ + $\frac{5}{4}$ = $\frac{8}{12}$ + $\frac{15}{12}$ = $\frac{23}{12}$
 |
| 1. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators.
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| Coherence: | NY-4.NF.3d | → | NY-5.NF.2 |  |  |
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e.g., using visual fraction models or equations to represent the problem |
| Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. | e.g., Recognize an incorrect result $\frac{2}{5}$ + $\frac{1}{2}$ = $\frac{3}{7}$ by observing that $\frac{3}{7}$ < $\frac{1}{2}$.  |
| **Connecting the Standards for Mathematical Practice to Mathematical Content:*** When students use benchmark fractions and number sense to estimate mentally (NY-5.NF.2) they are reasoning abstractly and quantitatively (MP.2).  When students assess the reasonableness of answers they are making sense of problems (MP.1).
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| **NY-5.NF Number and Operations—Fractions**  |
| **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** |  |
| 1. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}$= *a* ÷ *b*).
 |

| Coherence: |  |  | NY-5.NF.3 | → | NY-6.RP.2 |
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e.g., Interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$.  |
| Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. | e.g., using visual fraction models or equations to represent the probleme.g., If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? |
| 1. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number or a fraction.
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| Coherence: | NY-4.NF.4 | → | NY-5.NF.4 | → | NY-6.G.1 |
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| 1. Interpret the product $\frac{a}{b} $× *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*.
 | e.g., Use a visual fraction model to show $\frac{2}{3} $× 4 = $\frac{8}{3}$, and create a story context for this equation. Do the same with$ \frac{2}{3} $× $\frac{4}{5} $ = $\frac{8}{15} $. |
| 1. Find the area of a rectangle with fractional side lengths by tiling it with rectangles of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
 | e.g.,  | The right side of the illustration shows a 1 by 1 unit square. It is subdivided into 3 equal rows and 4 equal colums. Each resulting sub-portion, then, has dimensions 1/3 by 1/4.  The left side of the illustration shows the same 1 by 1 unit square subdivided into 3 equal rows and 4 equal columns. This one is used to show the rectangle with dimensions 2/3 by 3/4 as an illustration of the area model for multiplying fractions. |
| 1. Interpret multiplication as scaling (resizing).
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| Coherence: | NY-4.OA.1 | → | NY-5.NF.5 | → | NY-6.RP.1 |
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| 1. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
 | e.g., In the case of 10 × $\frac{1}{2}$ = 5, 5 is half of 10 and 5 is 10 times larger than $\frac{1}{2}$ . |
| 1. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case). Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. Relate the principle of fraction equivalence

$\frac{a}{b}$= $\frac{a}{b}$ × $\frac{n}{n}$ to the effect of multiplying $\frac{a}{b}$by 1. | e.g.,* Explain why 4 × $\frac{3}{2}$ is greater than 4.
* Explain why 4 × $\frac{1}{2}$ is less than 4.
* $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ because $\frac{1}{3}$ × $\frac{2}{2}$ = $\frac{2}{6}$.
 |
| 1. Solve real world problems involving multiplication of fractions and mixed numbers.
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| Coherence: | NY-4.NF.4c | → | NY-5.NF.6 |  |  |
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e.g., using visual fraction models or equations to represent the problem |
| 1. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

Note: Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement until grade 6 (NY-6.NS.1). |

| Coherence: |  |  | NY-5.NF.7 | → | NY-6.NS.1 |
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| 1. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
 | e.g., Create a story context for $\frac{1}{3}$ ÷ 4 and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3}$ ÷ 4 = $\frac{1}{12}$ because $\frac{1}{12}$ × 4 = $\frac{1}{3}$. |
| 1. Interpret division of a whole number by a unit fraction, and compute such quotients.
 | e.g., Create a story context for 4 ÷ $\frac{1}{5}$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ $\frac{1}{5}$ = 20 because 20 × $\frac{1}{5}$ = 4. |
| 1. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.
 | e.g., using visual fraction models and equations to represent the probleme.g., How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb. of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins? |
| **Within-Grade Connections:*** Understanding that in a multi-digit number, a digit in one place represents $\frac{1}{10}$ of what it represents in the place to its left (NY-5.NBT.1) is an example of multiplying a quantity by a fraction (NY-5.NF.4). [(14)](#WorksCited)

**Connecting the Standards for Mathematical Practice to Mathematical Content:*** The understanding of multiplication as scaling (NY-5.NF.5) is an important opportunity for students to reason abstractly (MP2). [(15)](#WorksCited)
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| **NY-5.MD Measurement and Data**  |
| **Convert like measurement units within a given measurement system.** |  |
| 1. Convert among different-sized standard measurement units within a given measurement system when the conversion factor is given. Use these conversions in solving multi-step, real world problems.
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| Coherence: | NY-4.MD.1 | → | NY-5.MD.1 | → | NY-6.RP.3d |
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Notes: * All conversion factors will be given.
* Grade 5 expectations for decimal operations are limited to work with decimals to hundredths.
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| **Within-Grade Connections:*** Work on this standard supports computation with decimals to hundredths (NY-5.NBT.7). For example, converting 5 cm to 0.05 m involves computation with decimals to hundredths. [(14)](#WorksCited)
* Conversions within the metric system (NY-5.MD.1) are an important practical application of the place value system and an opportunity to develop understanding of it (NY-5.NBT.1 - 2).
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| **NY-5.MD Measurement and Data**  |
| **Represent and interpret data.** |  |
| 1. Make a line plot to display a data set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Use operations on fractions for this grade to solve problems involving information presented in line plots.
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| Coherence: | NY-4.MD.4 | → | NY-5.MD.2 | → | NY-6.SP.2NY-6.SP.4 |
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e.g., Given different measurements of liquid in identical beakers, make a line plot to display the data and find the total amount of liquid in all of the beakers. |
| **Within-Grade Connections:*** Displaying data of measurements in fractions of a unit and solving problems involving information presented in line plots (NY-5.MD.2), provides an opportunity for solving real-world problems with operations on fractions (NY-5.NF). [(14)](#WorksCited)
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| **NY-5.MD Measurement and Data**  |
| **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** |  |
| 1. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
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| Coherence: | NY-3.MD.5 | → | NY-5.MD.3 |  |  |
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| 1. Recognize that a cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
 |  |
| 1. Recognize that a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
 |  |
| 1. Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.
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| Coherence: | NY-3.MD.6 | → | NY-5.MD.4 |  |  |
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| 1. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
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| Coherence: | NY-3.MD.7 | → | NY-5.MD.5 | → | NY-6.G.2NY-6.G.5 |
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| 1. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base.
 |  |
| 1. Apply the formulas *V* = *l* × *w* × *h* and *V* = *B* × *h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
 |  |
| 1. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
 | e.g., | An illustration of a 3-dimensional "L" shape with the dimensions labeled. The "L" shape can be decomposed into 2 or 3 non-overlapping right rectangular prisms. The volume of the whole shape can be found by adding the volumes of these non-overlapping rectangular prisms. |
| **Connecting the Standards for Mathematical Practice to Mathematical Content:*** When students show that the volume of a right rectangular prism is the same as would be found by multiplying the side lengths (NY-5.MD.5), they also have an opportunity to look for and express regularity in repeated reasoning (MP.8). They attend to precision (MP.6) as they use correct length or volume units, and they use appropriate tools strategically (MP.5) as they understand or make drawings to show these units. [(14)](#WorksCited)
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| **NY-5.G Geometry**  |
| **Graph points on the coordinate plane to solve real-world and mathematical problems.** |  |
| 1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.
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| Coherence: |  |  | NY-5.G.1 | → | NY-6.NS.6 |
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| Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond. | e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate |
| 1. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
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| Coherence: |  |  | NY-5.G.2 | → | NY-6.NS.8NY-6.G.3 |
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| **NY-5.G Geometry**  |
| **Classify two-dimensional figures into categories based on their properties.** |  |
| 1. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
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| Coherence: | NY-4.G.2 | → | NY-5.G.3 |  |  |
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e.g., All rectangles have four right angles and squares are rectangles, so all squares have four right angles.Note: The inclusive definition of a trapezoid will be utilized, which defines a trapezoid as “A quadrilateral with *at least* one pair of parallel sides.” |
| 1. Classify two-dimensional figures in a hierarchy based on properties.
 |  |