AP Math
Graduation Proficiencies
(Performance Scoring Criteria)

Proficiency-Based Graduation Requirements (PBGRs) are the locally-delineated set of content knowledge and skills connected to state standards that, when supplemented with any additional locally-developed requirements, have been determined to qualify a student for earning a high school diploma. Vermont's Education Quality Standards (EQS) require that schools' graduation requirements be rooted in demonstrations of student proficiency, as opposed to time spent in classrooms. This requirement will take effect in Vermont beginning with the graduating class of 2020.

AP Calculus: Limits and Continuity

The idea of limits is essential for discovering and developing important ideas, definitions, formulas, and theorems in calculus. Students must have a solid, intuitive understanding of limits and be able to compute various limits, including one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to work with tables and graphs in order to estimate the limit of a functions at a point. Students should know the algebraic properties of limits and techniques for finding limits of indeterminate forms, they should be able to apply limits to understand the behavior of a function near a point and how to use to determine continuity.

Graduation Proficiency #1- **LIMITS AND CONTINUITY**

Find, describe, and compare limits and asymptotic/unbounded behavior of functions analytically, graphically, numerically, and verbally and define continuity in terms of limits and graphs(including Intermediate Value Theorem and Extreme Value Theorem).

Limits and Continuity

Performance Indicators:

C.LC.1: Understand the concept of limit and estimate limits from graphs and tables of values, including 1-sided limits.
C.LC.2: Find limits by substitution
C.LC.3: Find limits of sums, differences, products, and quotients.
C.LC.4: Find limits of rational functions that are undefined at a point.
C.LC.5: Find limits at infinity, decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior.
C.LC.6: Understand continuity in terms of limits and decide if a function is continuous at a point.
C.LC.7: Find the types of discontinuities of a function
C.LC.8: Understand and use the Intermediate Value Theorem on a function over a closed interval
C.LC.9: Understand and use the Extreme Value Theorem on a function over a closed interval

AP Calculus Scoring Criteria- Proficiency 1

	Getting Started	*Making Progress*	*Proficient*	*Going Beyond*
C.LC.1: Understand the concept of limit and estimate			C.LC.1: Understand the concept of limit and estimate limits from graphs and	

limits from graphs and tables of values, including 1-sided limits.			tables of values, including 1-sided limits.	
C.LC.2: Find limits by substitution	.		C.LC.2: Find limits by substitution	
C.LC.3: Find limits of sums, differences, products, and quotients.			C.LC.3: Find limits of sums, differences, products, and quotients.	
C.LC.4: Find limits of rational functions that are undefined at a point.			C.LC.4: Find limits of rational functions that are undefined at a point.	
C.LC.5: Find limits at infinity, decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior.			C.LC.5: Find limits at infinity, decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior.	
C.LC.6: Understand continuity in terms of limits and decide if a			C.LC.6: Understand continuity in terms of limits and decide if a function is	

function is continuous at a point.			continuous at a point.	
C.LC.7: Find the types of discontinuities of a function			C.LC.7: Find the types of discontinuities of a function	
C.LC.8: Understand and use the Intermediate Value Theorem on a function over a closed interval			C.LC.8: Understand and use the Intermediate Value Theorem on a function over a closed interval	
C.LC.9: Understand and use the Extreme Value Theorem on a function over a closed interval			C.LC.9: Understand and use the Extreme Value Theorem on a function over a closed interval	

AP Calculus: Differentiation and Derivatives: Students build the derivative using the concept of limits and use the derivative primarily to compute the instantaneous rate of change of a function. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the graph of a function and solving problems involving rectilinear motion. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. In addition, students should be

able to solve separable differential equations, apply the Mean Value Theorem and be familiar with a variety of real-world applications including related rates, optimization, and growth and decay models.

Graduation Proficiency #2: **Differentiation**

Differentiation

Performance Indicators:

C.D.1: Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivatives as a rate of change
C.D.2: State, understand and apply the definition of derivative
C.D.3: Find the derivative of functions, including algebraic, trigonometric, logarithmic, exponential and composite functions using the power rule, product and quotient rule and chain rule.
C.D.4: Find the derivatives of implicitly defined functions
C.D.5: Find derivatives of inverse functions
C.D.6: Find 2nd and higher order derivatives
C.D.7: Understand and apply the Mean Value Theorem

Performance Indicator	*Getting Started*	*Making Progress*	*Proficient*	*Going Beyond*
C.D.1: Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivatives as a rate of change			I can use the concept of derivative geometrically, numerically, and analytically, **and** interpret the derivatives as a rate of change	
C.D.2: State, understand and			I can state, understand and	

apply the definition of derivative			apply the definition of derivative	
C.D.3: Find the derivative of functions, including algebraic, trigonometric, logarithmic, exponential and composite functions using the power rule, product and quotient rule and chain rule.			I can find the derivative of functions, including algebraic, trigonometric, logarithmic, exponential and composite functions using the power rule, product and quotient rule and chain rule.	
C.D.4: Find the derivatives of implicitly defined functions			I can find the derivatives of implicitly defined functions	
C.D.5: Find derivatives of inverse functions			I can find derivatives of inverse functions	

C.D.6: Find 2nd and higher order derivatives			I can find 2nd and higher order derivatives	
C.D.7: Understand and apply the Mean Value Theorem			I can use and apply the Mean Value Theorem	

Graduation Proficiency #3: **Applications of Derivatives**

Applications of Derivatives

Performance Indicators:

C.AD. 1: Find the slope of a curve at a point, including points at which the are vertical tangents and no tangents and write the equation of the tangent line to a curve at a point and a local linear approximation.

C.AD.2: Decide where functions are decreasing and increasing and understand the relationship of the behavior of f and the sign of f'.

C.AD.3: Solve real world and other mathematical problems find local and absolute maximum and minimum points with and without technology.

C.AD.4: Find points of inflection and understand the relationship between the concavity of f and f''.

C.AD.5: Use the 1st and 2nd derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology.

C.AD.6: Solve optimization real-world problems with and without technology

C.AD.7: Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of average rate of change and interpret in the applications including distance, velocity and acceleration.

C.AD.8: Model rates of change, including related rates problems.

Performance Indicators	*Getting Started*	*Making Progress*	*Proficient*	*Going Beyond*
C.AD. 1: Find the slope of a curve at a point, including points at which the are vertical tangents and no tangents and write the equation of the tangent line to a curve at a point and a local linear approximation.			I can find the slope of a curve at a point, including points at which the are vertical tangents and no tangents and write the equation of the tangent line to a curve at a point and a local linear approximation.	
C.AD.2: Decide where functions are decreasing and increasing and understand the relationship of the behavior of f and the sign of f'.			I can decide where functions are decreasing and increasing and understand the relationship of the behavior of f and the sign of f'.	
C.AD.3: Solve real world and other mathematical problems find local			I can solve real world and other mathematical problems find local	

and absolute maximum and minimum points with and without technology.			and absolute maximum and minimum points with and without technology.	
C.AD.4: Find points of inflection and understand the relationship between the concavity of f and f''.			I can find points of inflection and understand the relationship between the concavity of f and f''.	
C.AD.5: Use the 1st and 2nd derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology.			I can use the 1st and 2nd derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology.	
C.AD.6: Solve optimization real-world problems with and without technology			I can solve optimization real-world problems with and without technology	

C.AD.7: Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of average rate of change and interpret in the applications including distance, velocity and acceleration.			I can find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of average rate of change and interpret in the applications including distance, velocity and acceleration.	
C.AD.8: Model rates of change, including related rates problems.			I can model rates of change, including related rates problems.	

AP Calculus: Integration and Integrals-- Students should understand the definition of a definite integral involving Riemann sum, able to approximate a definite integral using different methods, and be able to compute definite integrals using geometry. They should be familiar with basic techniques of integration and properties of integrals. The student should be familiar with area, volume and motion applications, as well as

with the use of the definite integral as an accumulation function. Students should be able to work with and analyze functions defined by an integral. Students also should have a grasp of the relationship between differentiation and integration as expressed by the Fundamental Theorem of Calculus.

Graduation Proficiency #4: **Integration**

Integration

Performance Indicators:
C.I.1: Use rectangle approximations(LRAM, RRAM, MRAM) to find approximate values of integrals
C.I.2: Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points
C.I.3: Interpret a definite integral as a limit of Riemann sums
C.I.4: Understand the Fundamental Theorem of Calculus: interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval $\int_a^b f(x)dx = F(b) - F(a)$

C.I.5: Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.
C.I.6: Understand and use the properties of definite integrals in evaluating functions.
C.I.7: Understand and use integration by substitution to find values of integrals
C.I.8: Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.

Performance Indicator	*Getting Started*	*Making Progress*	*Proficient*	*Going Beyond*
C.I.1: Use rectangle approximations(LRAM, RRAM, MRAM) to find approximate values of integrals			I can use rectangle approximations(LRAM, RRAM, MRAM) to find approximate values of integrals	

C.I.2: Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points			I can calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points	
C.I.3: Interpret a definite integral as a limit of Riemann sums			I can interpret a definite integral as a limit of Riemann sums	
C.I.4: Understand the Fundamental Theorem of Calculus: interpret a defineite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval $\int_a^b f(x)dx = F(b) - F(a$			I can understand the Fundamental Theorem of Calculus: interpret a defineite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval $\int_a^b f(x)dx = F(b) -$	
C.I.5: Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular			I can use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to	

antiderivatives. Perform analytical and graphical analysis of functions so defined.			represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.	
C.I.6: Understand and use the properties of definite integrals in evaluating functions.			I can understand and use the properties of definite integrals in evaluating functions.	
C.I.7: Understand and use integration by substitution to find values of integrals			I can understand and use integration by substitution to find values of integrals	
C.I.8: Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.			I can understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.	

Graduation Proficiency #5:**Application of Integrals**

High School 9-12

Performance Indicators:
C.AI.1: Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line.
C.AI.2: Solve separable differential equations and use them in modeling real-world problems with and without technology.
C.AI.3: Use definite integrals to find the area between a curve and the x-axis, or between two curves.
C.AI.4: Use definite integrals to find the average value of a function over a closed interval.
C.AI.5: Use definite integrals to find the volume of a solid with known cross-sectional area.
C.AI.6: Apply integration to model and solve(with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.

Performance Indicator	*Getting Started*	*Making Progress*	*Proficient*	*Going Beyond*
C.AI.1: Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity			I can find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions	

functions, and applications to motion along a line.			from velocity functions, and applications to motion along a line.	
C.AI.2: Solve separable differential equations and use them in modeling real-world problems with and without technology.			I can solve separable differential equations and use them in modeling real-world problems with and without technology.	
C.AI.3: Use definite integrals to find the area between a curve and the x-axis, or between two curves.			I can use definite integrals to find the area between a curve and the x-axis, or between two curves.	
C.AI.4: Use definite integrals to find the average value of a function over a closed interval.			I can use definite integrals to find the average value of a function over a closed interval.	
C.AI.5: Use definite integrals to find the volume of a solid with			I can use definite integrals to find the volume of a solid with known	

known cross-sectional area.			cross-sectional area.	
C.AI.6: Apply integration to model and solve(with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.			I can apply integration to model and solve(with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.	