

Math Distance Learning Packet

Grade 7

Student Version

Understand

Addition of Positive and Negative Integers

Name: _____

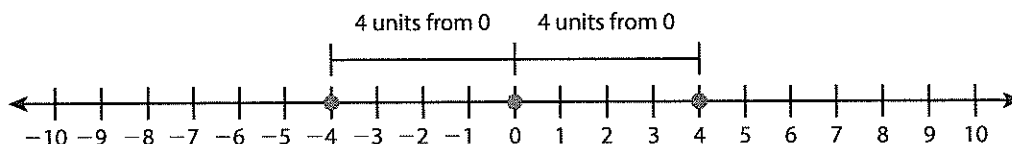
Prerequisite: What is the absolute value of a number?



Study the example showing how to find the absolute value of a number. Then solve problems 1–10.

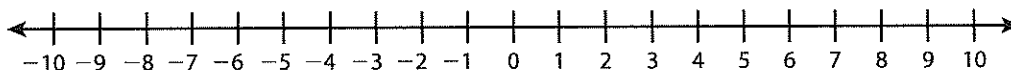
Example

The *absolute value* of a number is the distance from the number to 0 on a number line. Both -4 and 4 are 4 units from 0, so $|-4| = 4$ and $|4| = 4$.



- 1 What is the absolute value of 0? _____

Use the number line to solve problems 2–4.



- 2 Graph the numbers 6, 0, -10 , 9, and -6 on the number line.
- 3 Which number that you graphed has the greatest absolute value? What is the absolute value of that number? _____
- 4 Which two numbers that you graphed have the same absolute value? Explain.

- 5 Write $<$, $=$, or $>$ to compare the numbers.

a. -7 -1

d. $|0|$ $|-1|$

b. $|-7|$ $|-1|$

e. 12 $|12|$

c. 0 -1

f. -12 $-|12|$

Vocabulary

absolute value the distance a number is from 0 on the number line.

$|2| = 2$ $|-3| = 3$

Solve.

The table shows information about each of the Great Lakes. It shows the elevation of the surface of each lake and the elevation of the deepest point of each lake, both relative to sea level.

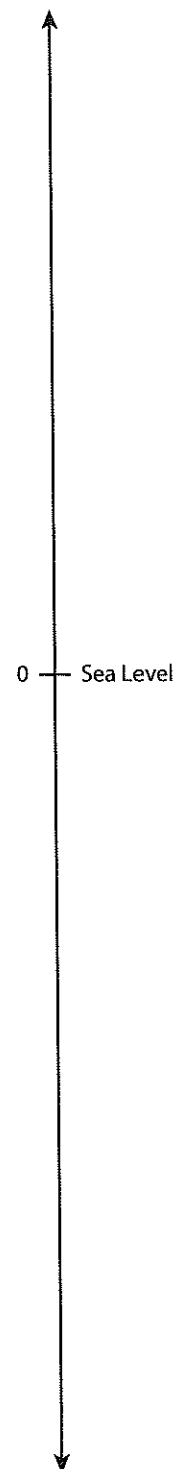
	Surface Level (ft)	Deepest Point (ft)
Lake Superior	601	-732
Lake Michigan	577	-346
Lake Ontario	243	-559
Lake Huron	577	-173
Lake Erie	569	210

- 6 Show the surface levels and deepest points from the table on the number line.
- 7 Which number in the table has the greatest absolute value? What is the absolute value of that number?

- 8 Which of the Great Lakes, if any, is entirely above sea level? Explain.

- 9 How can you use absolute value to find the distance from the surface of Lake Michigan to its deepest point? What is the distance?

- 10 The deepest point of Lake Titicaca in South America is -922 feet relative to its surface. The deepest point is 11,542 feet above sea level. What is the elevation of the surface of the lake? Use absolute value to explain.



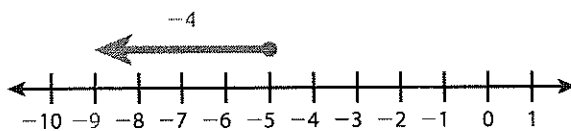
Add Positive and Negative Integers

Study the example problem showing how to add positive and negative integers. Then solve problems 1–8.

Example

Graph each situation on a number line. Then, model each situation with an equation.

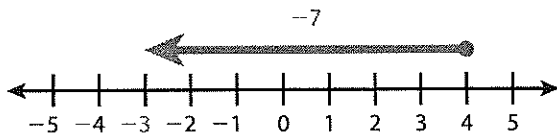
- Jordan already owes \$5 to Kiara and borrows \$4 from Don. How much money does Jordan have?



$$-5 + (-4) = ?$$

$$-5 + (-4) = -9$$

- Micah has \$4 and owes Ben \$7.

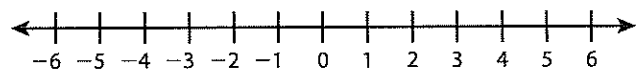


$$4 + (-7) = ?$$

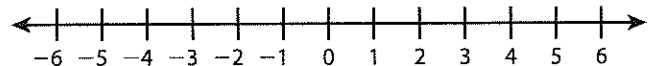
$$4 + (-7) = -3$$

1 Complete the equation and model each sum on a number line.

a. $\square + 4 = 0$



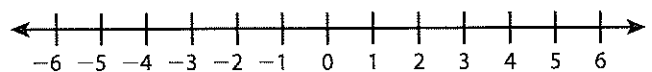
b. $-5 + \square = 0$



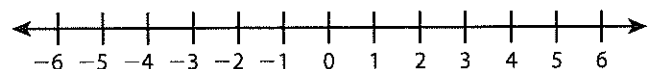
c. When will the sum of two numbers be zero?

2 In the first box of each equation, write an example of an integer that will result in the sum described. Then write the sum. Model each sum on a number line.

a. positive sum: $-5 + \square = \square$



b. negative sum: $-4 + \square = \square$



Solve.

- 3 Explain how you can use absolute value to tell whether the sum of two integers is positive or negative.

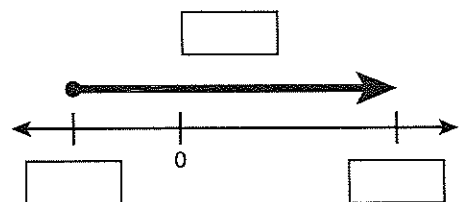
- 4 An elevator is two floors below ground level and goes up 5 floors. Write an addition equation that models the location of the elevator relative to ground level. What integer represents the new location?

- 5 One morning, the temperature was -5°F . By noon, the temperature had increased 12 degrees. What was the temperature at noon? Use a model to explain your answer.

- 6 A lobster fisherman moves a lobster trap from 20 feet below sea level to a location that is 15 feet deeper. Draw a number line and write an addition equation that models this situation. What integer represents the new location relative to sea level?

- 7 The sum of integers p and q is modeled on a number line, where $|p| < |q|$. In each box, write p , q , or $p + q$. Then write an addition equation using integers that could represent p , q , and $p + q$.

- 8 Show how the model in problem 7 would change if $|p| > |q|$. Draw the model, labeling p , q , and $p + q$. Then write an addition equation using integers that could represent p , q , and $p + q$.



Reason and Write

Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

You are designing a hike that starts at the bottom of a small mountain, below sea level. You plan to stop at three locations:

- a scenic location, which is below sea level
- a picnic area for lunch, which is at sea level
- the top of the mountain, which is 800 feet above sea level

Design the hike. Choose the location of each stop. Make a table that shows each location, the change in elevation from the previous location, and the elevation of each location relative to sea level. Show how you found the elevation of each location using integer addition. Graph your locations on a number line. Describe your hike, including the total change in elevation from the start of the hike to the mountaintop.

Show your work. Use integers, tables, models, equations, and words to explain your answer.

Elevation (feet)

Location	Change in Elevation	New Elevation
Start		-200
Scenic location	150	-50
Lunch	50	0
Mountaintop	800	800

Addition of integers also shows the elevation at each location and the total change in elevation.

$$-200 + 150 = -50$$

$$-50 + 50 = 0$$

$$0 + 800 = 800$$

$$\text{Total change: } 800 + 50 + 150 = 1,000$$

Starting at -200 feet and climbing 150 feet will bring me to an elevation of -50 feet. After climbing another 50 feet, I will be at sea level, or an elevation of 0 feet. Finally, climbing the final 800 feet will bring me to the mountaintop at an elevation of 800 feet. The total change in elevation will be 1,000 feet.

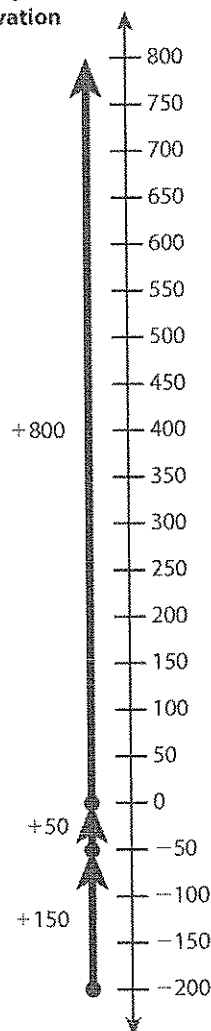
Where does the example...

- use integers?
- use a table and a number line?
- use an equation to model?
- use words to explain?
- answer the question?



Change in Elevation

Elevation



Solve the problem. Use what you learned from the model.

You are designing a hike that starts at the top of a canyon, above sea level. You plan to stop at three locations:

- a scenic landmark, which is at sea level
- a famous boulder, which is below sea level
- the bottom of the canyon, which is 600 feet below sea level

Design the hike. Choose the location of each stop. Make a table that shows each location, the change in elevation from the previous location, and the elevation of each location relative to sea level. Show how you found the elevation of each location using integer addition. Graph your locations on a number line. Describe your hike, including the total change in elevation from the start of the hike to the bottom of the canyon.

Did you ...

- use integers?
- use a table and a number line?
- use an equation to model?
- use words to explain?
- answer the question?

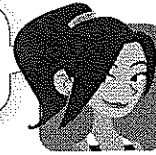


Show your work. Use integers, tables, models, equations, and words to explain your answer.

Understand Subtraction of Positive and Negative Integers

Name: _____

Prerequisite: How do you add integers using a number line?



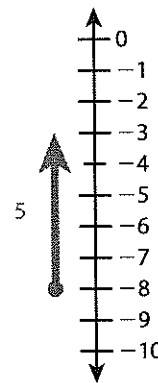
Study the example problem showing how to add positive and negative integers. Then solve problems 1–7.

Example

When Leo woke up he saw that the temperature was -8°F . By noon the temperature had increased 5°F . What was the temperature at noon?

You can use a number line to add $-8 + 5$.

$-8 + 5 = -3$, so the temperature at noon was -3°F .



- 1** By 3:00 PM, the temperature had increased by another 5°F . Was the temperature at 3:00 PM positive or negative? How do you know? What was the temperature at 3:00 PM?

- 2** By 11:00 PM, the temperature had dropped 8°F . Was the temperature at 11:00 PM positive or negative? Explain.

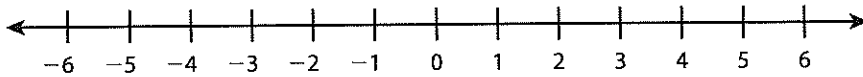
- 3** A swimmer dives 10 feet below the surface of a lake. How far must she swim before she reaches the surface? Use an addition equation to explain, and tell what each part of the equation means.



Solve.

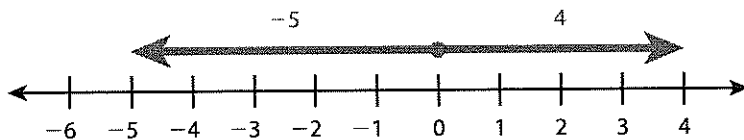
- 4 Use a number line to show $4 + (-7)$. Then complete the equation that shows the sum.

$4 + (-7) = \underline{\hspace{2cm}}$



- 5 In a game that you are playing, your friend says that she has -6 points “give or take” 4 points. You currently have -3 points in the game. Can you say who is winning? Why or why not? Use a number line to explain.

- 6 Leon used the number line below to show $-5 + 4$. Explain what is wrong with his model.



- 7 The sum of two integers is -4 .

a. Can the two integers both be positive? Explain.

b. Can the two integers both be negative? Explain.

Subtract Positive and Negative Integers

Study the example problem showing how to subtract two integers. Then solve problems 1–4.

Example

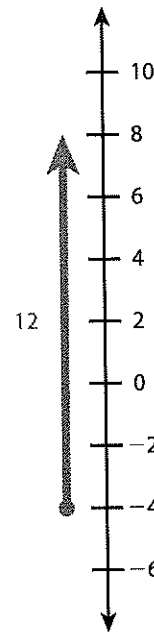
What is the difference between an elevation of -4 feet and an elevation of 8 feet?

Find the difference by subtracting: $8 - (-4)$.

Write the subtraction as an addition problem: $8 + 4$.

Model the addition problem on a number line.

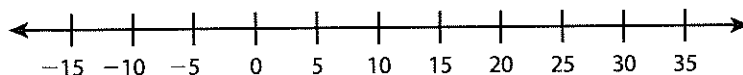
The difference in the elevations is 12 feet.



- 1** Marcie is playing a board game with a friend. She needs 20 points to win. She currently has -10 points. She wants to know the difference between the number of points she now has and the number of points she needs.

- a.** Write a subtraction problem to represent the situation. Then write the subtraction problem as an addition problem.

- b.** Model the addition problem on the number line.

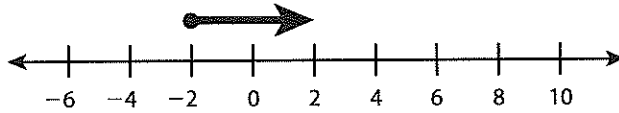


- c.** What is the difference between the number of points she needs to win and the number of points she now has?

Solve.

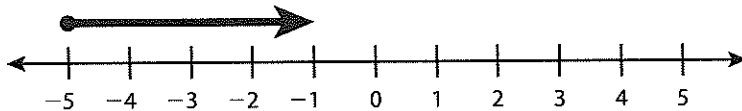
- 2** Jessie uses the number line below to help write $-2 + (-4)$ as a subtraction problem.

a. What is wrong with Jessie's number line?



b. Write the subtraction problem and the answer.

- 3** Use the number line below to solve the problems.



- a. What is the distance between -5 and -1 on the number line? _____
- b. What is $|-5 - (-1)|$? _____
- c. What do you notice about the absolute value of the difference between the two numbers?

- 4** What number must be subtracted from -5 for the difference to be -2 ? Explain your answer. Include a number line in your explanation.

Reason and Write

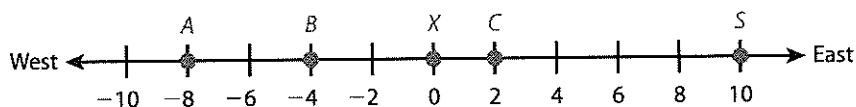
Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

You and your friends Aaron, Beth, and Craig all live on the same street that the school is on. Aaron lives farthest from the school. You and Aaron live the same distance from Beth's house. Craig lives closest to the school.

Place points *A* for Aaron's house, *B* for Beth's house, *C* for Craig's house, and *S* for the school on a number line. Let your house *X* be at 0. Describe each person's location and use absolute value to find the distance that each person has to walk to and from school. Then, list the locations in order from farthest from the school to closest to the school.

Show your work. Use the number line, words, absolute value, and equations to explain your answer.



Aaron's house is located at -8 on my number line, and the school is at 10 . So his distance from school is:
 $|10 - (-8)| = |18| = 18$ units. He has to walk
 $2(18) = 36$ units to and from school each day.

Beth's house is located at -4 on my number line, and the school is at 10 . So her distance from school is:
 $|10 - (-4)| = |14| = 14$ units. She has to walk
 $2(14) = 28$ units to and from school each day.

My house is located at 0 on my number line, and the school is at 10 . So my distance from school is:
 $|10 - 0| = |10| = 10$ units. I have to walk
 $2(10) = 20$ units to and from school each day.

Craig's house is located at 2 on my number line, and the school is at 10 . So his distance from school is:
 $|10 - 2| = |8| = 8$ units. He has to walk
 $2(8) = 16$ units to and from school each day.

In order from farthest to closest, the locations are
 Aaron's house, Beth's house, my house, Craig's house.

Where does the example...

- use the number line?
- use words?
- use absolute value?
- use equations?
- answer each part of the problem?

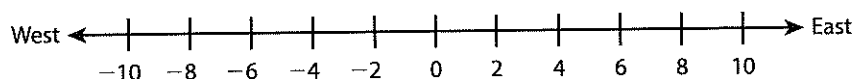


Solve the problem. Use what you learned from the model.

You and your friends Ari, Ben, and Carla all live on the same street that some tennis courts are on. Ari and Carla live the same distance from the tennis courts. Ben lives farthest from the courts. You live the same distance from Ben's house and the tennis courts.

Place points A for Ari's house, B for Ben's house, C for Carla's house, and T for the tennis courts on a number line. Let your house X be at 0. Describe each person's location and use absolute value to find the distance that each person has to walk to and back home from the tennis courts. Then, list the locations in order from farthest from the courts to closest to the courts.

Show your work. Use the number line, words, absolute value, and equations to explain your answer.



Did you...

- use the number line?
- use words?
- use absolute value?
- use equations?
- answer each part of the problem?



Add and Subtract Positive and Negative Integers

Name: _____

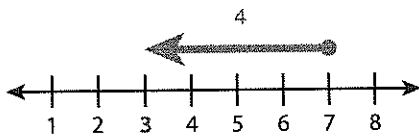
Prerequisite: Connect Addition and Subtraction

Study the example showing how to connect addition and subtraction. Then solve problems 1–9.

Example

Solve the subtraction problem: $7 - 4 = \square$.

To solve, you can represent $7 - 4 = \square$ using a number line. Start at 7 and move 4 units to the left to represent subtracting 4 from 7. You end at 3, so $7 - 4 = 3$.



Because addition and subtraction are inverse operations, you can also rewrite the subtraction problem as an addition problem. Think: "What number do I add to 4 to get 7?" Because $4 + 3 = 7$, you know $7 - 4 = 3$.

- 1 Draw a number line that represents $7 + (-4)$.
- 2 How does the number line you drew in problem 1 compare to the number line in the example?
- 3 Use your answers to the last two problems to complete this equation.

$$7 - 4 = 7 + \underline{\hspace{2cm}}$$

- 4 Complete each equation.

a. $-1 + (-4) = \underline{\hspace{2cm}}$

c. $-1 + (-4) = -1 - \underline{\hspace{2cm}}$

b. $-1 - \underline{\hspace{2cm}} = -5$

d. $1 - (-4) = 1 + 4 = \underline{\hspace{2cm}}$

Solve.

- 5 The average low temperature for one winter day is 10°F .
The low temperature on that day was actually -2°F .

a. Write a subtraction problem to represent the situation. Then write the subtraction problem as an addition problem.

b. Model the addition problem on a number line.

c. What is the difference in the temperatures?

- 6 Write an addition expression that is equivalent to $65 - 79$. Then evaluate the expression.

- 7 Explain how to write any subtraction problem as an addition problem. Why does it help to write a subtraction problem as an addition problem?

- 8 Write an absolute value expression to represent the distance between -1 and 6 on a number line. Then evaluate the expression.

- 9 The expression $x - 4$ represents a negative integer. What integers could x represent?

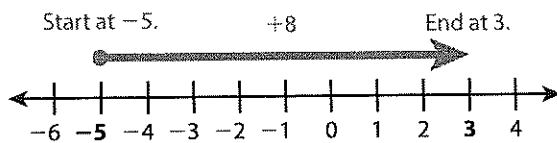
Addition Methods for Integers

Study the example problem showing how to add positive and negative integers. Then solve problems 1–7.

Example

At the end of the first round of a game, Luis has a score of -5 points. At the end of the second round, he has a score of 3 points. How many points did he score during the second round?

You can use a number line to help you understand the problem.



The number line shows that Luis scored 8 points during the second round.

- 1** Complete this model to represent the problem.

score at the end of first round $+$ $=$

- 2** Write an addition equation using numbers to represent the verbal equation in problem 1.

- 3** Amy said she would have solved this problem differently. She saw that she was looking for the difference between the score in the second round, 3, and the score in the first round, -5 , so she wrote the expression $3 - (-5)$. Does her method work? Explain.



Solve.

- 4 The temperature in Indianapolis was -4°F at 7:00 AM. The temperature rose 3°F by noon. What was the temperature at noon? Use a number line to find the answer.
-

- 5 Aiden had saved \$22 before he earned \$25 mowing a lawn. He then spent \$32 on a suitcase. How much money does he have now? Explain how you found your answer.
-

- 6 Omar has a score of 12 in a bean-bag toss game. On his next turn he gets -8 points. Use the bar model to write an addition sentence that shows Omar's score now.

12	
?	8

- 7 Gina works in a clothing store. At noon, she has \$125 in the cash register. James gives her \$60 for a sweater, and she gives him \$7 change. Hana then gives her \$40 for a blouse and receives \$3 change. Use a series of addition equations to find out how much money Gina has in her cash register at the end of these sales.

Show your work.

Solution: _____

Subtraction Methods for Integers

Study the example showing how to subtract positive and negative integers. Then solve problems 1–4.

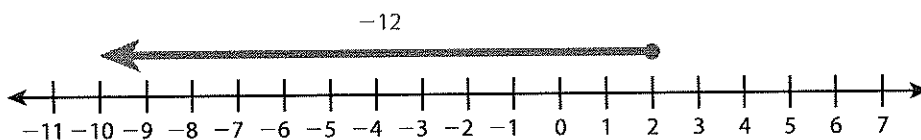
Example

Two friends are playing a game with a spinner that has positive and negative numbers on it. Each player takes turns spinning the spinner. The table shows the results of the first 4 rounds.

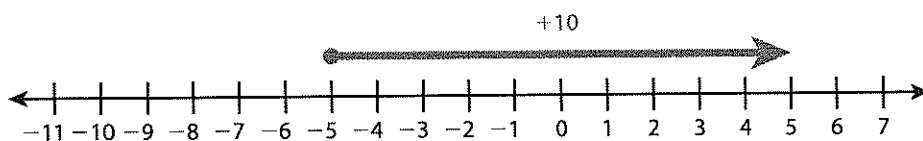
	Round 1	Round 2	Round 3	Round 4
Player 1	2	-5	-5	8
Player 2	12	-10	2	-4
Team Score				

The team score at the end of each round is found by subtracting Player 2's score from Player 1's score.

For Round 1: Subtracting 12 from 2 is the same as adding -12 . Start at 2 and move left 12 units to arrive at -10 .



For Round 2: Subtracting -10 is the same as adding 10. Start at -5 and move right 10 units to arrive at 5.



- 1** Use a number line to find the team score for Round 3.

Round 3 team score: _____

Solve.

- 2** Refer to the table from the previous page.

	Round 1	Round 2	Round 3	Round 4
Player 1	2	-5	-5	8
Player 2	12	-10	2	-4
Team Score				

On which round did the team get the highest score?
Explain your answer.

- 3** Anita recorded the daily high and low temperatures as 13°F and -3°F , respectively.

- a. Write the difference in temperatures, in $^{\circ}\text{F}$, as a subtraction equation. Then write the difference in temperatures as an addition equation.

- b. Give an example of a positive temperature and a negative temperature that have a difference of 5°F .

- 4** Consider the following problems.

- a. Write a subtraction equation that involves one negative integer but results in a positive difference. Does the other integer have to be positive? Explain your answer.

- b. Write a subtraction problem involving two positive integers with a negative difference. Explain the relationship between the two integers that must exist for the difference to be negative.

Add and Subtract Positive and Negative Integers

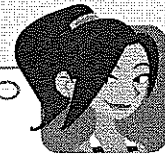
Solve the problems.

- 1 The element bromine turns into a liquid at -7°C , and it turns into a gas at 59°C . From the temperature at which bromine becomes a liquid, by how many degrees must the temperature change for it to turn into a gas?

A -66°C C 52°C
 B -52°C D 66°C

Johnathan chose C as his answer. How did he get that answer?

Should you add or subtract?



- 2 Lamont keeps track of his math grades by recording them in a table. He wants to keep an average of 90, so he also lists the amount that each grade is above or below 90.

a. Complete the table.

Test	1	2	3	4	5	6
Grade	83	94	79		96	
Above/Below 90	-7	4		7		-3

- b. Use the numbers in the *Above/Below 90* row to find out whether Lamont's average is above or below 90.

Show your work.

Solution: _____

- c. What grade does Lamont need to get on the next test to have an average of exactly 90? Explain your answer.
- _____
- _____

You may want to group the positive numbers and the negative numbers.

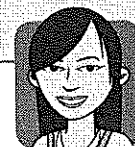


Solve.

- 3 Which expressions are equivalent to -9 ? Select all that are correct.

- A $8 - 8 + 9$
- B $3 - (-6) + (-18)$
- C $-1 + 7 - (-3)$
- D $4 - 5 - 8$

Recall how to write a subtraction problem as an addition problem.



- 4 Tell whether each equation is *True* or *False*.

- | | | |
|-------------------------|-------------------------------|--------------------------------|
| a. $-4 + (-7) = 11$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| b. $5 + (-4) = -5 + 4$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| c. $-10 + 7 = 7 - 10$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| d. $14 + (-3) = 10 + 1$ | <input type="checkbox"/> True | <input type="checkbox"/> False |

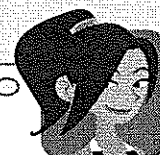
What should be your first step?



- 5 A duck is sitting on a ledge that is 11 feet above the surface of a pond. The duck dives 27 feet straight down to get food at the bottom of the pond. Which expression represents the position of the bottom of the pond, in feet, relative to its surface level?

- | | |
|----------------|----------------|
| A $27 + 11$ | C $11 - (-27)$ |
| B $27 + (-11)$ | D $11 + (-27)$ |

What does a negative value mean in this situation?



- 6 Which of the following are negative integers? Select all that are correct.

- A the sum of two positive integers
- B the sum of two negative integers
- C the difference of a positive integer and an integer that is greater than it
- D the difference of a negative integer and an integer that is greater than it but that is not its opposite

You may want to draw a number line and try sample numbers.



Multiply and Divide Positive and Negative Integers

Name: _____

Prerequisite: Add and Subtract Positive and Negative Integers

Study the example showing how to add and subtract positive and negative integers. Then solve problems 1–6.

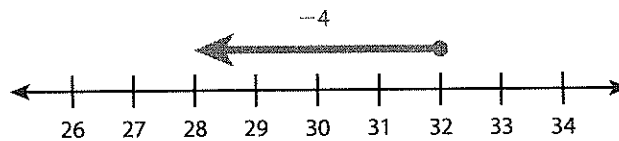
Example

Geneva and Juan are playing a game that involves a spinner. The table shows how the player's score changes according to the color the spinner lands on.

Color	Red	Yellow	Blue	Green
Number of Points	+2	−4	+3	−1

Juan has 32 points. Then his spin lands on yellow. What is his score now?

Yellow means Juan's score changes by −4 points.



$32 - 4 = 32 + (-4) = 28$. Juan's score is 28 points.

- 1** Geneva has 24 points. Then her spin lands on blue. What is her score now? Explain.

- 2** Geneva and Juan start a new game. Each player has 0 points.

- a.** Geneva's first two spins are yellow and red. What is her score now? Explain.

- b.** Juan's first two spins are red and green. What is his score now? Explain.

- c.** Which player has a greater score now? Explain.



Solve.

- 3 Describe a situation that the expression $-4 + (-1)$ could represent. Find the sum and tell what it represents in terms of the situation.

- 4 A water bird is flying 11 feet above the surface of a pond. It dives 15 feet down and then rises 3 feet to catch a fish. What is the new position of the bird relative to the surface of the water?

Show your work.

Solution: _____

- 5 Steve's average long jump distance is 13 ft. He uses a table to keep track of the distance he jumps in six long jumps during his next practice.

Jump Number	1	2	3	4	5	6
Distance (ft)	14	11	15	12	13	10
Difference from Average of 13 (ft)	1	-2				

Complete the table. Then use the results to explain whether Steve's overall long jump performance on these six jumps is better or worse than average.

- 6 Is the expression $|-3| + |4|$ equal to $|-3 + 4|$? Justify your answer.

Multiplying Integers

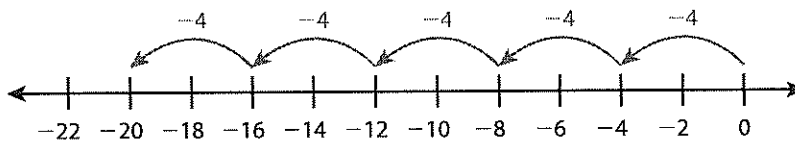
Study the example problem showing how to multiply positive and negative integers. Then solve problems 1–8.

Example

Aidan likes to dive on vacation. He stops to take a picture every 4 feet he dives. How has his elevation changed relative to the surface of the water after he takes 5 pictures?

You can think of the change in Aidan's elevation between pictures as -4 . Aidan takes 5 pictures, so there are 5 groups of -4 .

Use the model to show the total change in Aidan's elevation as jumps on a number line. Start at 0 and make jumps of -4 .



The number -20 represents the total change in Aidan's elevation.

$$(-4) + (-4) + (-4) + (-4) + (-4) = -20$$

- 1** Rewrite the addition of groups as a multiplication of groups.

$$(-4) + (-4) + (-4) + (-4) + (-4) = \underline{\hspace{2cm}}$$

- 2** Complete: Adding 5 groups of (-4) is the same as _____ 5 groups of (-4) .

- 3** What is the total distance he dives in feet? _____

- 4** Use a model to multiply $5 \cdot (-3)$.

Show your work.

Solve.

- 5 A swimming pool has 500 gallons of water in it and is leaking. The change in the amount of water in the pool is -3 gallons per hour. How many gallons of water will the pool contain after 6 hours?

Show your work.

Solution: _____

- 6 Write the first two products in the table below. Then use the pattern in the products to write the last two products in the table.

	$2 \cdot (-2)$	$1 \cdot (-2)$	$0 \cdot (-2)$	$(-1) \cdot (-2)$	$(-2) \cdot (-2)$
Product			0		

- 7 Complete each statement about multiplying positive and negative integers using the word *positive* or *negative*. For each statement, provide an example.

positive \cdot positive = positive _____

positive \cdot negative = _____

negative \cdot negative = _____

- 8 Mrs. Krin has \$300 deducted from her checking account every month for her car payment. She also has \$150 deducted every month for her insurance. After 1 year, by how much do these payments change her checking account balance? Explain how you found your answer.

Dividing Integers

Study the example problem showing how to divide positive and negative integers. Then solve problems 1–7.

Example

Chen is labeling a thermometer that is used to measure very cold temperatures from 0°F to -40°F . He divides it into 8 equal sections. How many degrees does each section represent?

Think: I need to divide -40 by 8,
so I can ask $8 \cdot ? = -40$.

$$8 \cdot (-5) = -40.$$

This means that $-40 \div 8 = -5$.

- 1** Draw Chen's thermometer. Be sure to label each section.

- 2** Chen labels another thermometer that uses 10 different sections. Complete the table for this thermometer.

Total number of degrees ($^{\circ}\text{F}$)	\div	Number of sections	$=$	Degrees in each section ($^{\circ}\text{F}$)
-40			$=$	

- 3** A scuba diver descends 48 feet in 4 minutes. What is the diver's average change in position per minute relative to where she started?
- _____



Solve.

- 4 Multiplication and division are related operations. Look at these examples.

$2 \cdot 4 = 8$	$(-2) \cdot (-4) = 8$	$-2 \cdot 4 = -8$	$2 \cdot (-4) = -8$
$8 \div 4 = 2$	$8 \div (-4) = -2$	$-8 \div 4 = -2$	$-8 \div (-4) = 2$

Think about the signs of the factors and the sign of the product in the multiplication problems. How do the signs of the numbers you are dividing relate to their quotients?

- 5 Describe a situation that the expression $-15 \div (-5)$ could represent. Find the quotient and tell what it represents in terms of the situation.

- 6 The low temperatures in degrees Fahrenheit in Marsh City during a week in January were:

$-3, -5, 3, 7, -2, -2, -5$

What was the average low temperature for that week?

Show your work.

Solution: _____

- 7 The quotient $x \div 5$ is a negative integer. Name two integers that x could represent and find the quotients.

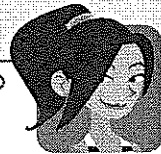
Multiply and Divide Positive and Negative Integers

Solve the problems.

- 1 A whale dives at a speed of 3 feet per second. What is the change in the position of the whale relative to where it started after 12 seconds?

A -36 feet C 4 feet
B -4 feet D 36 feet

Will your answer be positive or negative?



- 2 Tell whether each equation is *True* or *False*.

a. $-7 \cdot 8 = 7 \cdot (-8)$

☐

True

☐

False

b. $-7 \cdot (-8) = 7 \cdot 8$

☐

True

☐

False

c. $7 \cdot (-8) = 7 \cdot 8$

☐

True

☐

False

How can the signs of the factors in each multiplication equation help you solve this problem?



- 3 Myra withdraws the same amount of money from her checking account each week. In 4 weeks, she withdraws a total of \$200. Which equation represents the amount of money her account changes by each week?

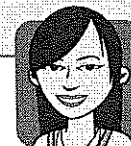
A $-200 \div (-4) = 50$

B $-200 \div 4 = -50$

C $-200 \div 4 = 50$

D $-200 \div (-4) = -50$

What can the signs of the numbers in a division problem tell you about the quotient?



Sam chose C as the correct answer. How did he get that answer?

Solve.

- 4** Kain made two number cubes to use in a game. The faces on each cube contain the numbers 1, -2 , 3, -4 , 5, and -6 . After Kain rolls the two cubes, he multiplies the two numbers.

- a. Give an example of two numbers that Kain could roll to get a positive product.

- b. Give an example of two numbers that Kain could roll to get a negative product.

What is true about the signs of two factors if their product is positive?



- 5** Savannah solves each of the following problems as shown below.

a. $-6 \cdot 12 \div (-4) = 18$

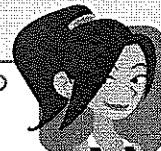
b. $8 \cdot (-3) \div 6 = -4$

c. $-40 \cdot (-2) \div (-10) = 8$

d. $-7 \cdot 5 \cdot (-2) \div 5 = 14$

Are the answers correct? Explain any incorrect answers.

Remember to pay careful attention to the signs of numbers as you find quotients and products.



Understand Proportional Relationships

Name: _____

Prerequisite: How do you find equivalent ratios?



Study the example problem showing how to find equivalent ratios. Then solve problems 1–7.

Example

There are 3 counselors assigned to a group of 24 campers. At this rate, how many counselors are needed for 40 campers?

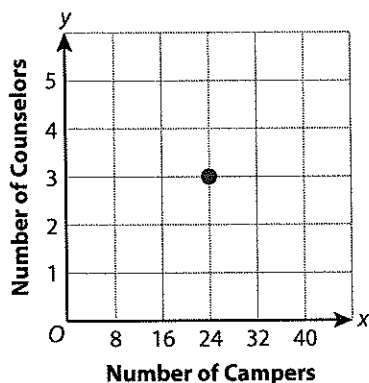
The given ratio is 24 campers to 3 counselors. That's a rate of 8 campers to 1 counselor. You can use this information to make a table of equivalent ratios.

Number of Campers	8	16	24	32	40
Number of Counselors	1	2	3	4	5

From the table you can see that 5 counselors are needed for 40 campers.

- 1** Explain how to use the unit rate to make the table in the example.

- 2** One ordered pair from the table is plotted on the coordinate plane. Finish plotting the ordered pairs.



- 3** Suppose you had 9 counselors available. How many campers could you have?

Vocabulary

equivalent ratios

two or more ratios that are equal to one another.



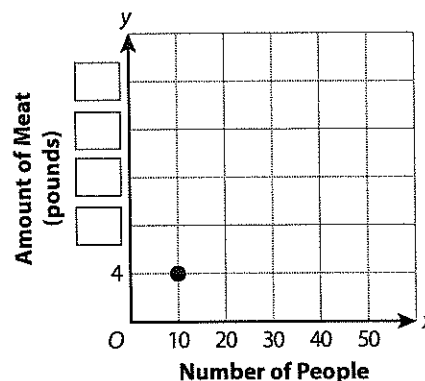
Solve.

- 4 In 10 seconds, Jamal travels 550 feet on his bicycle. At this speed, how many feet can he travel in 1 minute? Explain.

- 5 Cala uses 2 pounds of Feed-All fertilizer for a 100-foot row of vegetables at her farm. At this rate, how many pounds of fertilizer would she use for a 450-foot row of vegetables? Explain.

- 6 The directions for Grow Better fertilizer say that the 25-pound bag covers a 1,000-foot row of vegetables. Which brand would Cala need more of to fertilize her vegetables? Explain.

- 7 Giselle has a catering business. There is a proportional relationship between the number of people and the amount of meat she uses for an event. The graph shows the amount of meat Giselle uses for 10 people. Find the amount of meat needed for 20, 30, 40, and 50 people. Finish labeling the graph and plotting ordered pairs. Finally, explain how to calculate the amount of meat needed for a party of 75 people.



Identify Proportional Relationships

Study the example showing one way to test whether a relationship is proportional. Then solve problems 1–7.

Example

The tables show the prices for ordering photo mugs from two different companies. You can use the information to write ratios showing the relationship between the cost and the corresponding number of mugs.

Company A

Number of Mugs	5	10	25	50
Cost (\$)	15	30	75	150

$$\frac{15}{5} = 3 \quad \frac{30}{10} = 3 \quad \frac{75}{25} = 3 \quad \frac{150}{50} = 3$$

These ratios are all equivalent.
This relationship is proportional.

Company B

Number of Mugs	5	10	25	50
Cost (\$)	20	35	80	155

$$\frac{20}{5} = 4 \quad \frac{35}{10} = 3.5 \quad \frac{80}{25} = 3.2 \quad \frac{155}{50} = 3.1$$

These ratios are not all equivalent.
This relationship is not proportional.

- 1 Explain how you can tell whether a group of ratios represents a proportional relationship.

- 2 Look at the ratios for Company A in the example. What is the constant of proportionality and what does it mean?

- 3 Write an equation to represent the relationship between the number of mugs and cost for Company A. Use c for cost and m for the number of mugs.

- 4 Complete the table to show a proportional relationship. Write the constant of proportionality.

Number of Yoga Classes	2	4	6	8
Cost (\$)		60		

constant of proportionality: _____

Vocabulary

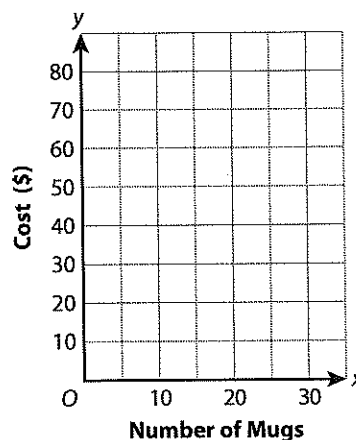
constant of proportionality

another term for the unit rate in a proportional relationship.

Solve.

- 5 Plot a point for some of the ordered pairs from the example problem shown at the right. Model the relationships by drawing a line from the y -axis through each point. Explain how the graphs show which relationship is proportional and which is not proportional.

Company A: (5, 15), (10, 30), (25, 75)
Company B: (5, 20), (10, 35), (25, 80)



- 6 Determine whether each equation below does or does not represent a proportional relationship. Support your answer using either a table or a graph.

Equation A: $y = x$

Equation B: $y = x + 2$

- 7 Zahra has paper rectangles of different sizes. Every rectangle is 5 centimeters longer than it is wide. Is there a proportional relationship between the lengths and widths of these rectangles? Explain.

Reason and Write

Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

Describe a relationship involving some product or service and its cost that is NOT proportional. Explain how you know that it is not a proportional relationship.

Show your work. Use tables, graphs, words, and numbers to explain your answer.

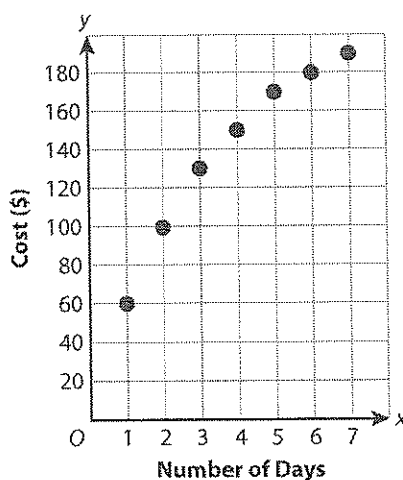
Best Bike Rentals rents bikes by the day. The longer you rent the bike, the better their rate is. The table shows the cost of renting a bike for up to 7 days. The ratios of cost to days in this table are not equivalent because the relationship is not proportional.

Number of Days	1	2	3	4	5	6	7
Cost (\$)	60	100	130	150	170	180	190

For example, $\frac{60}{1} = 60$, $\frac{100}{2} = 50$, and $\frac{130}{3} = 43\frac{1}{3}$. These three ratios are not equivalent. The rate of dollars per day is less the longer you keep the bike.

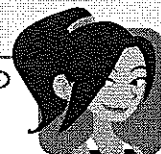
I can also plot the ordered pairs from the table on a coordinate grid.

The points cannot be connected with a straight line that goes through the origin. This is another way to show that the relationship is not proportional.



Where does the example...

- answer both parts of the problem?
- use a table or graph to explain?
- use numbers to explain?
- use words to explain?
- give details?



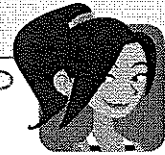
Solve the problem. Use what you learned from the model.

Describe a relationship involving some product or service and its cost that IS proportional. Explain why it is a proportional relationship, and identify the constant of proportionality.

Show your work. Use tables, graphs, words, and numbers to explain your answer.

Did you...

- answer both parts of the problem?
- use a table or graph to explain?
- use numbers to explain?
- use words to explain?
- give details?



Equations for Proportional Relationships

Name: _____

Prerequisite: Understand Proportional Relationships

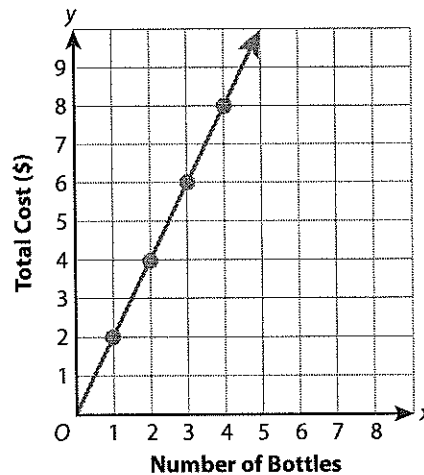
Study the example showing proportional relationships in a table and a graph. Then solve problems 1–5.

Example

Bottles of water cost \$2 each. Is the relationship of the cost to the number of bottles a proportional relationship?

You can use a table or a graph to see whether the relationship is proportional.

Number of Bottles	1	2	3	4
Total Cost (\$)	2	4	6	8
Total Cost Number of Bottles	$\frac{2}{1}$	$\frac{4}{2} = \frac{2}{1}$	$\frac{6}{3} = \frac{2}{1}$	$\frac{8}{4} = \frac{2}{1}$



All of the ratios in the table are equal to $\frac{2}{1}$.

This is the unit rate and means that it costs \$2 for one bottle of water. The graph is a straight line through the origin (0, 0).

So this is a proportional relationship.

- What is the cost of 12 bottles? Explain.

- Find the ratio of the cost of 12 bottles to the number of bottles. Is the ratio equivalent to the unit rate?

- How much will the total cost increase for each additional bottle of water purchased? Compare this value to the constant of proportionality.

Vocabulary

proportional relationship the relationship among a group of ratios that are equivalent.

constant of proportionality the unit rate in a proportional relationship.

Solve.

- 4 The table shows the number of grams of fiber in different amounts of winter squash.

Number of Cups	2	4	5	8
Grams of Fiber	20	40	50	80

- a. What is the ratio of the number of grams of fiber to the number of cups?

For 2 cups _____ For 4 cups _____

For 5 cups _____ For 8 cups _____

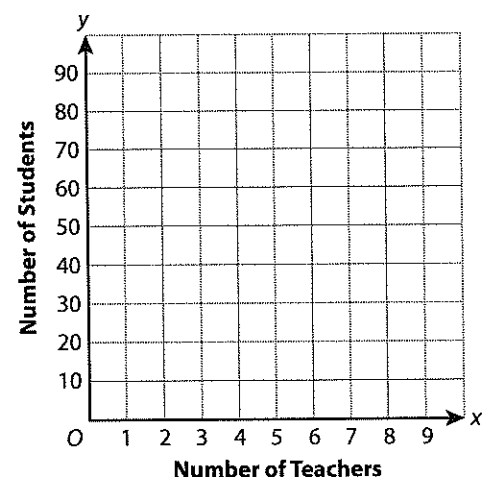
- b. Are the data in the table in a proportional relationship? If so, what is the constant of proportionality? Explain.

- c. How could you use a graph to show whether the data are in a proportional relationship?

- 5 There are currently 2 teachers and 22 students signed up for a field trip. For every additional 10 students, the school will assign 1 more teacher. Mel says that this is a proportional relationship. Do you agree? Complete the graph and the table to explain.

Teachers	2				
Students	22				

Show your work.



Solution: _____

Write Equations for Proportional Relationships

Study the example showing how to identify a proportional relationship. Then solve problems 1–9.

Example

The table shows the relationship between the money that Leo earns and the number of lawns that he mows. Is the relationship proportional?

Number of Lawns	2	4	5	8
Money Earned (\$)	10	20	25	40

The ratios of the money earned to the number of lawns all simplify to $\frac{5}{1}$, or 5, so the relationship is proportional.

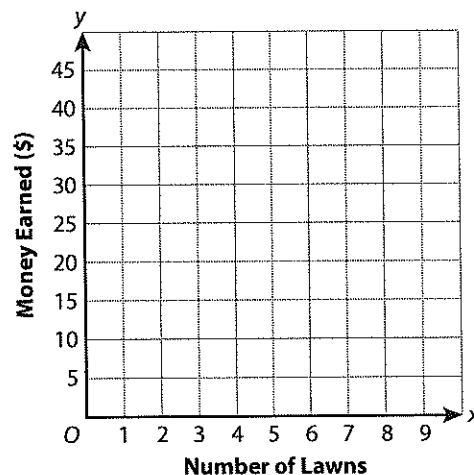
- 1 Graph the relationship between the money earned and the number of lawns and connect the points. How does the graph tell you that the relationship is proportional?

- 2 What does the ratio $\frac{5}{1}$ represent in terms of the example?

- 3 How can you use the graph to find the constant of proportionality? What is the constant of proportionality?

- 4 Use the constant of proportionality to write an equation that represents the amount of money earned, y , for mowing x lawns. _____

- 5 If you know the constant of proportionality, m , for two proportional quantities, x and y , what equation can you write to describe the relationship?



Solve.

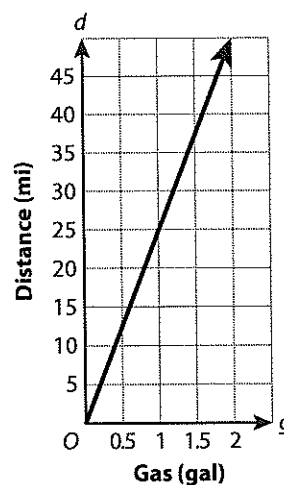
- 6 Nila uses the equation $c = 8h$ to figure out the total amount c she should charge a customer if she babysits for h hours. Find the constant of proportionality and explain what it means.
-
- 7 Use the information in problem 6 to solve this problem. Nila decides to increase the rate she charges customers by \$2 per hour. What equation should she now use to determine how much to charge her customers? Explain.
-
- 8 The table shows the cost of several bunches of bananas. What equation can be used to represent the cost c of a bunch that weighs p pounds?

Number of Pounds	2.5	3.5	4	4.5
Cost (\$)	1.05	1.47	1.68	1.89

Show your work.

Solution: _____

- 9 The graph shows the relationship between the distance that Dustin can drive his car and the amount of gas needed for that distance. Explain how Dustin can use the graph to predict the number of gallons of gas he will need for a trip of 120 miles. Then find the amount of gas he will need.



Show your work.

Solution: _____

Equations for Proportional Relationships

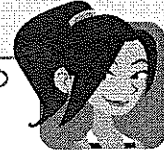
Solve the problems.

- 1 Micah paints birdhouses to sell at a fair. The table shows the amount of paint he uses. Is this a proportional relationship? If so, find the constant of proportionality and write an equation for the relationship.

Cans of Paint (p)	$\frac{1}{4}$	0.75	$1\frac{1}{2}$	2.5
Number of Birdhouses (b)	3	9	18	30

Show your work.

How can you simplify the ratios?

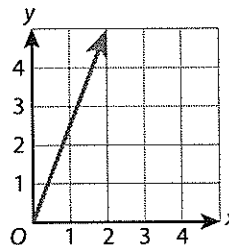


Solution: _____

- 2 Consider the table, equation, and graph. Which of them represents a proportional relationship?

x	3	5	8
y	3.6	6	7.2

$y = 2x + 5$

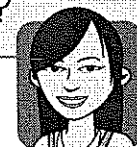


How can you identify a proportional relationship?



- 3 Cayley says that the equations $p = 1.5q$ and $\frac{2}{3}p = q$ both represent the same proportional relationship. Mariah says that can't be true because the constants of proportionality are different. With which student do you agree? Explain.

How can you identify the equation for a proportional relationship?



Solve.

- 4** Jason runs the same distance each day. In one 7-day period he ran $40\frac{1}{4}$ miles. He knows that there is a proportional relationship between n , the number of days, and t , the total distance he runs. Tell whether each statement is *True* or *False*.

- a. The relationship can be expressed as $n = 5.25t$. ☐ True ☐ False
- b. The graph of the equation is a straight line through $(0, 0)$. ☐ True ☐ False
- c. The unit rate is 5.75. ☐ True ☐ False

How can finding the unit rate help you?



- 5** A farmer charges \$6 for 4 pounds of tomatoes. Which equation can the farmer use to find how many dollars d he should charge for p pounds of tomatoes?

- A $d = \frac{2}{3}p$ C $d = 1.5p$
- B $d = 6p$ D $d = 4p$

Rosa chose **A** as her answer. Explain her error.

What is the form of an equation for a proportional relationship?



- 6** When Chef Alice makes rice pilaf for 30 people, she uses 15 cups of chicken broth and 10 cups of rice. Dan wants to make the same recipe for 9 people. Write and use equations to find how much broth and how much rice Dan should use.

Show your work.

Finding unit rates could be helpful.



Solution: _____

Problem Solving with Proportional Relationships

Name: _____

Prerequisite: Write Equations for Proportional Relationships

Study the example showing how to write an equation for a proportional relationship. Then solve problems 1–6.

Example

Jamie is making bracelets using black and red beads. She uses 6 black beads for every 4 red beads. Represent the number of black beads for any given number of red beads using a table, a graph, and an equation. Identify the constant of proportionality.

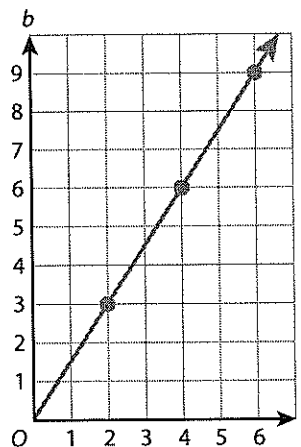
The ratio of black beads to red beads will be the same for all quantities.

$$\frac{\text{black beads}}{\text{red beads}} = \frac{6}{4} = \frac{3}{2}$$

You can use this ratio to make a table.

Red Beads, r	2	4	6
Black Beads, b	3	6	9

You can use the table to make a graph by plotting and connecting the ordered pairs. The constant of proportionality is $\frac{3}{2}$. This situation can be represented by the equation $b = \frac{3}{2}r$.



- 1 If Jaime uses 12 red beads, how many black beads does she use?

- 2 If Jamie wants to maintain the relationship of black beads to red beads, could she make a bracelet with 5 red beads? Explain your answer.

Vocabulary

constant of proportionality the unit rate in a proportional relationship.

Solve.

- 3 Ana hikes at a constant speed. She travels 6 miles in 2 hours.
- a. Find her speed in miles per hour. Use it to write an equation for the distance, d , that Ana travels in h hours.

- b. Use the equation to find the distance Ana travels in 1.5 hours.

- 4 Mana is saving to buy a new bicycle. The equation $d = 12w$ represents the amount in dollars, d , that Mana saves in w weeks. What is the constant of proportionality? What does it represent in this situation?

- 5 The table shows the number of gallons of water, g , that a water pump transfers in s seconds. How many gallons of water are pumped per second? What is an equation for the gallons of water, g , that the station can pump in s seconds?

Seconds, s	12	16	20
Gallons Pumped, g	9	12	15

Show your work.

Solution: _____

- 6 Joleen and Pablo want to fertilize a rectangular garden with an area of A square feet. They know that 5 cups of fertilizer will cover an area of 240 square feet. They each write an equation to represent the relationship between the area, A , and the number of cups of fertilizer, c .

Joleen's equation: $A = 48c$

Pablo's equation: $c = \frac{1}{48}A$

Which of the equations is correct? Explain how you know.



Proportional Relationships with Simple Interest

Study the example showing how to find simple interest. Then solve problems 1–8.

Example

Nora borrows \$500 to buy a computer. She agrees to pay back the total amount of the computer plus 6% simple interest in 1 year. Write an expression for the total amount that Nora will have to pay back.

You can use a bar model to help you write an expression.

Amount Borrowed	Amount of Interest
\$500	6% of \$500 for 1 year
Total Amount to Pay Back	
t	

$$\begin{array}{ccccccc}
 \text{Amount Borrowed} & + & \text{Amount of Interest} & = & \text{Total to Pay Back} \\
 \downarrow & & \downarrow & & \downarrow \\
 500 & + & (0.06 \times 500) & = & t
 \end{array}$$

- What part of the bar model refers to the amount that Nora has to pay back in addition to the \$500 that she borrowed?

- What does each number in the expression 0.06×500 represent?

- Show how to find the amount of interest and the total amount that Nora will have to pay after 1 year.

- Describe how to find the total amount to pay on a 1-year loan when paying simple interest.

Vocabulary

simple interest a percent of an amount borrowed (or invested) that is paid to the lender (or investor) in addition to the original amount.

Solve.

- 5 Petra borrows \$200 for 1 year with a simple interest rate of 4.5%. Complete the equation that represents the total amount that Petra has to pay after 1 year.

$$\begin{array}{ccccc} \text{Amount Borrowed} & + & \text{Amount of Interest} & = & \text{Total to Pay Back} \\ \downarrow & & \downarrow & & \downarrow \\ \underline{\hspace{2cm}} & + & (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) & = & t \end{array}$$

- 6 Franco borrows \$400 and will pay 4% simple interest. Write an equation to answer each question.
- What will be the amount of interest if Franco pays back the loan in 1 year? _____
 - What will be the amount of interest if he pays back the loan in 2 years? _____
 - If Franco pays off the loan in 2 years, what is the total amount he will pay the lender? _____
- 7 Miguel deposits \$680 in an account that pays 3.5% simple interest. If he neither adds more money nor withdraws any money, what amount will be in the account after 6 years?

Show your work.

Solution: _____

- 8 Dan borrows money to buy a new trumpet for \$400 at a simple interest rate of 5%. He writes the equation $t = 400 + (0.5 \times 400)$ to represent the amount of money he will need to pay back after one year.

Is Dan's equation correct? Explain your answer and determine how much money Dan will need to pay back after one year.

Problem Solving with Percents

Study the example showing how to solve problems involving percents. Then solve problems 1–8.

Example

Tara buys a tablet that costs only \$320 because it is on sale for 20% off the original price. Write an equation that you can use to find the original price, p , of the tablet.

You can use a bar model to help you understand the problem.

Original Price p	
Discounted Price \$320 or 80% of Original Price	Discount 20% of Original Price

Tara pays $100\% - 20\%$, or 80% , of the original price.

The discounted price is 80% of p , so $0.80p = 320$.

- 1** In the equation $0.80p = 320$, what do 0.80, p , and 320 represent?

- 2** Solve the equation $0.80p = 320$ to find the original price.

Show your work.

Solution: _____

- 3** The original price of the tablet that you found in problem 2 increases by 20%. Write an expression for finding the new price. What is the new price?



Solve.

- 4 This year, a softball coach raised \$1,200 for new equipment. That is 4% less than he raised last year. How much did he raise last year? Explain.

- 5 You buy a calculator for \$65. A 6% sales tax is added. Write and solve an equation to find the total price, t .

- 6 A store is having a sale with 10% off everything.
- a. Write an equation to show the sale price s of any item given its regular price r .

- b. Does your equation represent a proportional relationship? Explain.

- 7 A store owner buys cell phones for \$40 and marks up the price by 25%. Explain how to find the price at which she sells the cell phones.

- 8 A video game that usually costs \$50 is on sale for \$32.50. What percent of the regular price is the discount?

Show your work.

Solution: _____

Vocabulary

tax a percent of a purchase that is added to the purchase and paid to a government.

markup a percent of the cost of an item that is added to the cost to determine the item's selling price.

Problem Solving with Proportional Relationships

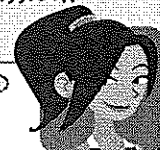
Solve the problems.

- 1 Cara earns a base pay of \$1,800 per month at a car dealership plus a commission of 6% of her sales. What are Cara's total earnings in a month in which she sells \$40,000 worth of merchandise?

Show your work.

Solution: _____

Which number should you multiply the percent by to find the commission?



- 2 Roy buys a coat for \$86.40, which includes an 8% sales tax. Which equation could you use to find the cost of the coat, c , without the sales tax? Select all that apply.

- A $0.92 \times 86.40 = c$ C $0.92c = 86.40$
 B $c + 0.08c = 86.40$ D $1.08c = 86.40$

How can you think of the price of the coat without the tax as a percent?



- 3 Which items have the same percent discount?

Item	Original Price	Sale Price
Sweater	\$40.00	\$32.00
Shorts	\$30.00	\$24.00
Jeans	\$50.00	\$40.00
Shirt	\$45.00	\$35.00

- A sweater and shorts ONLY
 B sweater, shorts, and shirt ONLY
 C jeans and shirt ONLY
 D sweater, shorts, and jeans ONLY

Victor chose C. How did he get that answer?

How can you find the amount of discount for each item?



Solve.

- 4 Is the expression a valid way to calculate a 15% tip on a cab fare of d dollars? Answer Yes or No.

- a. $0.10d + 0.05d$ ☐ Yes ☐ No
 b. $0.85d$ ☐ Yes ☐ No
 c. $1.015d$ ☐ Yes ☐ No
 d. $0.15d$ ☐ Yes ☐ No

What are some equivalent ways to express the amount of the tip?



- 5 Which situation can be modeled by Diagram A and which can be modeled by Diagram B?

Diagram A:

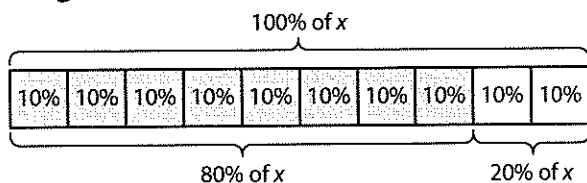
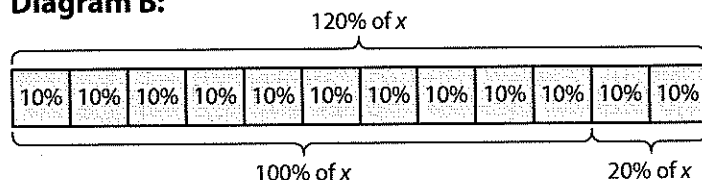


Diagram B:



- a. Your bill at a restaurant is \$68 and you want to leave a 20% tip. What is the total amount you will leave?
 b. You buy a sweater that is on sale for 20% off of the original price. The sweater cost \$28. What was the original price?



Which model shows addition? Which model shows subtraction?



- 6 A store advertises "Buy one item at full price and take $\frac{1}{2}$ off the cost of a second item with a lesser price." Steve buys one item with a price tag of \$40 and a second item with a price tag of \$60. What percent discount did he receive on his total purchase?

- A 80% C 40%
 B 50% D 20%

How can you find the amount of the discount?



Proportional Relationships

Name: _____

Prerequisite: Solving Problems with Percents

Study the example problem showing how to solve a problem with percents. Then solve problems 1–7.

Example

Chumani has a coupon for 15% off the total bill at a new restaurant. Her original bill is \$32.00. After the discount, what amount is her bill?

First, find the amount of the discount.

$$15\% = 0.15 \text{ and } 0.15 \times 32 = \$4.80$$

Subtract the discount from the original amount.

$$\$32.00 - \$4.80 = \$27.20$$

So Chumani's bill is \$27.20.

- 1 A bar model can also be used to represent the problem. Complete the bar model.

Original Bill <div style="border: 1px solid black; width: 100px; height: 20px; margin: 0 auto;"></div>	
Discounted Bill <i>b</i>	Amount of Discount <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; width: 40px; height: 20px; margin-right: 5px;"></div> % of \$ <div style="border: 1px solid black; width: 40px; height: 20px; margin-left: 5px;"></div> </div>

- 2 Write and solve an equation to represent the relationship shown in the bar model.
- _____
- 3 The percent of the discount in the problem above is 15%. What percent of the original bill is Chumani's bill? How could you solve the problem using this percent?
- _____
- _____

Solve.

- 4 Chander earns a base pay of \$2,200 per month. He also earns a commission of 4% of his total sales. One month Chander earned \$2,400. Explain how to find Chander's total sales for that month. How much were his total sales?

- 5 Marian borrowed money to buy a sound system that costs \$450. She is charged 5% simple interest for one year. What is the total amount that she pays for the sound system if she pays the full amount in one year?

Show your work.

Solution: _____

- 6 Dan paid \$34.30 for a sweater. The price included a 40% markup. Find the cost of the sweater before the markup was added.

Show your work.

Solution: _____

- 7 A jacket that originally sold for \$60.00 was on sale for 10% off. When it didn't sell after several weeks, the sale price was discounted another 40% off the discounted price. What was the final price of the jacket? Is the total discount equal to 50%? Explain.

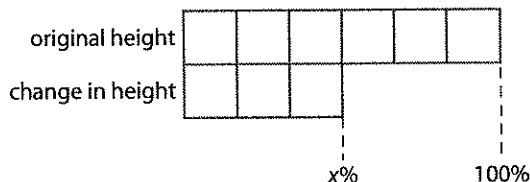
Finding Percent Change

Study the example problem showing how to find percent change. Then solve problems 1–7.

Example

The plants that Loma grew for her science project averaged 6 inches in height. Two weeks later, the plants averaged 9 inches in height. What was the percent increase in the average height of the plants?

You can use a bar model to compare the original height to the change in height.



You can also use the proportion below to compare the change to the original amount.

$$\frac{\text{amount of change}}{\text{original amount}} = \text{percent change}$$

$$\frac{9 - 6}{6} = \frac{x}{100}$$

- 1 Use either the bar model or the proportion to solve for x .
What was the percent increase in the average height of the plants?

- 2 Would the percent change be *greater than* or *less than* 50% if the plants had grown to 8 inches instead of 9?

- 3 After 4 weeks, the height of the plants had grown from 6 inches to 12 inches. Write and solve a proportion to find the percent increase in the height of the plants.

Solve.

- 4 Students donated 2,500 cans of food to the local food pantry last year. They donated 4,000 cans this year. What is the percent increase in the number of cans donated?

Show your work.

Solution: _____

- 5 Mike plays basketball. He attempted 32 free throws in January and 28 free throws in February.
- Is the percent change a *percent increase* or a *percent decrease*? _____
 - Write and solve a proportion to find the percent change in the number of free throws.

Show your work.

Solution: _____

- 6 Find the percent of increase or decrease.
- x to $5x$ _____
 - $2.5y$ to $1.5y$ _____
 - n to $\frac{4}{5}n$ _____
 - $3.2t$ to $5.2t$ _____

- 7 A store manager pays \$40 for a shirt and adds a markup of 20%. During a store sale, the manager marks the cost of the shirt down by 20%. What is the percent of change from the original cost, \$40, to the sale price?

Percent Error

Study the example problem showing how to find percent error. Then solve problems 1–8.

Example

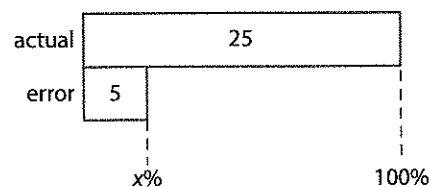
A thermometer manufacturer compares the reading on one of its thermometers to a thermometer that they know is accurate. The accurate thermometer reads 25°C. Their thermometer reads 30°C. What is the percent error in their thermometer's reading?

You can use a bar model to help you understand the problem.

You can also use a proportion.

$$\frac{\text{amount of error}}{\text{actual amount}} = \frac{x}{100}$$

$$\frac{30 - 25}{25} = \frac{x}{100}$$



- 1** Use either the bar model or the proportion to solve for x . What is the percent error in the thermometer's reading?

- 2** Explain the relationship between the x in the bar model and the x in the proportion.

- 3** The reading on a different thermometer is 23°C. Do you think the percent error of this thermometer is more or less than for the first thermometer? Explain your answer.

- 4** Find the percent error of a thermometer reading of 23°C. _____



Solve.

- 5 At the school carnival, students are asked to guess the number of marbles in a jar to win a prize. There are 240 marbles in the jar. The closest guess is 280 marbles. What is the percent error of the guess? Explain. Round your answer to the nearest percent.
-
-

- 6 Kai needed to cut 25 inches from a long board. He accidentally cut 24 inches from the board. What is his percent error? Explain.
-

- 7 Bev weighs a bag of apples labeled 5 pounds and finds that the weight is actually 72 ounces. To the nearest percent, what is the percent error in the weight? (1 pound = 16 ounces)

Show your work.

Solution: _____

- 8 Semira did a physics activity during which she rolled toy cars down a ramp and measured the distance each car traveled. On one trial, the actual distance that the car traveled was 75 cm. Semira's measurement was too short and had a 12% error. What was her distance measurement?

Show your work.

Solution: _____



Proportional Relationships

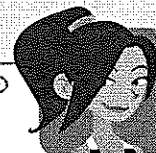
Solve the problems.

- 1** For each of the following situations, is the percent of change between 20% and 30%?

Select Yes or No for each situation.

- a. A \$12 cost increases to \$15. ☐ Yes ☐ No
- b. A boy's height increases from 52 inches to 61 inches. ☐ Yes ☐ No
- c. The temperature falls from 3°F to 2°F . ☐ Yes ☐ No

Which value goes in the denominator when you calculate a percent of change?



- 2** A customer pays \$18 for a DVD that originally cost \$20. What is the percent decrease in the cost of the DVD?

- A 2%
- B 10%
- C 11%
- D 18%

What is the change in the cost of the DVD?



Barbara chose **C** as the correct answer. How did she get that answer?

- 3** Brady needs to cut a piece of scrapbook paper 12 centimeters long. He cuts pieces of the following lengths. Which cut results in a percent error of 15%? Select all that apply.

- A 10.2 centimeters
- B 11.2 centimeters
- C 13.8 centimeters
- D 18.0 centimeters

A diagram might help you solve this problem.



Solve.

- 4 Jasmine sells beaded jewelry. She calculates the price at which she sells the jewelry by adding a percent markup to the amount it costs her to make the jewelry. Complete the following table. Record money amounts to the nearest cent and markups to the nearest whole percent.

Show your work.

Type of Jewelry	Cost to Make (\$)	Percent Markup (%)	Selling Price (\$)
Bracelet	7.69	40	
Necklace	8.66		12.56

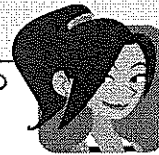
How does a markup affect the original cost?



- 5 In the late 1980s, there were only 22 California condors, which are large predatory birds, living in the wild. In 2012, the population had increased to 405 condors. To the nearest whole percent, what is the percent increase in the California condor population?

Show your work.

What is the change in the number of California condors?



Solution: _____

- 6 The manufacturer of an oven states that the temperatures displayed while cooking are within a 5% error. Rachel is using the oven to cook a chocolate cake that must be cooked at a temperature below 325°C . The oven display shows a temperature of 310°C . Can Rachel be certain that the temperature is suitable for cooking her cake? Explain your answer.

What is the ratio of the amount of error to the highest temperature?



Volume of Solids

Name: _____

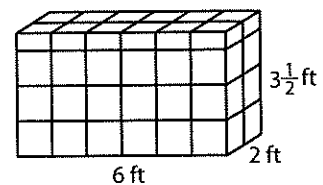
Prerequisite: Volume of Rectangular Prisms

Study the example showing how to find the volume of a rectangular prism. Then solve problems 1–7.

Example

Alex is constructing a box in which to grow vegetables on his patio. The box will be 6 feet long, 2 feet wide, and $3\frac{1}{2}$ feet deep. What is the volume of soil needed to fill the box?

You can model the volume using 1-foot unit cubes. Notice that the first three layers are whole cubes, and the top layer is made up of half-cubes.



Cubes in one of the bottom 3 layers: $6 \cdot 2 = 12$

Total cubes in the bottom 3 layers: $3 \cdot 12 = 36$

Cubes in the top layer: $\frac{1}{2}(6 \cdot 2) = 6$

Total cubes needed to fill the box: $36 + 6 = 42$

Alex needs 42 cubic feet of soil to fill the box.

- 1** Why do you multiply 12 by 3 to find the total number of cubes in the bottom 3 layers?

- 2** Why do you multiply $(6 \cdot 2)$ by $\frac{1}{2}$ to find the number of cubes in the top layer?

- 3** Use the formula $V = lwh$ to find the volume of the box. Compare the volume found using the formula with the volume computed above.



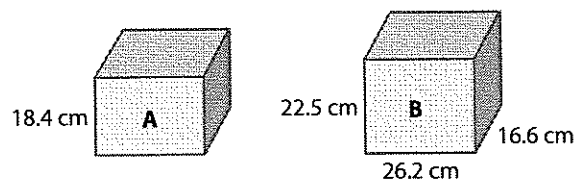
Solve.

- 4 A rectangular box of pasta is 10 inches long, 8 inches wide, and $2\frac{1}{4}$ inches deep. What is the volume of the box?
-

- 5 Meghan says that the formula $V = Bh$, where B is the area of the base, can be used to find the volume of any rectangular prism. Do you agree? Explain.
-

- 6 Lao is thinking of buying a fish tank. Tank A has a base area of 530 square centimeters. Which fish tank has the greater volume? How much greater?

Show your work.



Solution: _____

- 7 If you double the length, the width, and the height of a rectangular prism, how does the volume of the prism change? Use algebra to justify your answer. Then give a numerical example.

Show your work.

Solution: _____

Volume of Prisms

Study the example problem showing how to find the volume of a prism. Then solve problems 1–7.

Example

A triangular prism is shown at the right. What is the volume of the prism?

The bases of this prism are the right triangles at either end of the prism. So, first find the area of one of the bases.

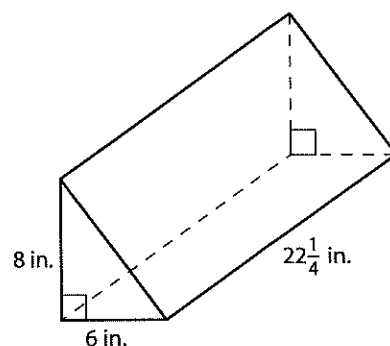
$$\frac{1}{2}bh = \frac{1}{2}(6)(8) = 24$$

The area of a triangular base is 24 square inches.

Next, use the formula $V = Bh$, where B is 24 and h is $22\frac{1}{4}$.

$$V = Bh = 24(22\frac{1}{4}) = 534$$

The volume of the prism is 534 cubic inches.



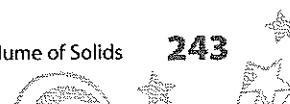
- 1** How do you know that the right triangles are the bases of the prism?

- 2** Describe the faces of the prism that are not bases. How are they alike? How are they different?

- 3** A second prism has dimensions that are $\frac{1}{2}$ of the dimensions of the prism in the example. Kathy says that the volume of the smaller prism is $\frac{1}{2}$ of the volume of the prism in the example. Do you agree? Explain.

Show your work.

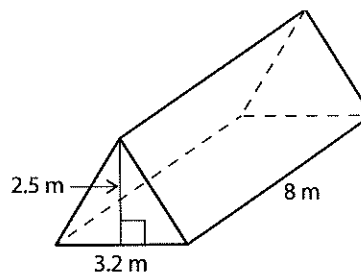
Solution: _____



Solve.

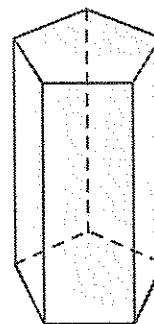
- 4 What is the volume of the prism shown?

Show your work.



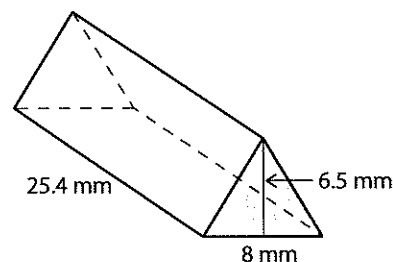
Solution: _____

- 5 A pentagonal prism is shown. The volume of the prism is 91.8 cubic inches. If the height of the prism is 10.8 inches, what is the area of each base? Explain.



- 6 A store sells two types of tiny crystals. One of the crystals is a triangular prism whose dimensions are shown at the right. The other crystal is shaped like a rectangular prism with a length of 26 millimeters, a width of 8.5 millimeters, and a height of 7 millimeters. Alice says that the volume of the rectangular crystal is greater than two times the volume of the triangular crystal. Find the volumes to prove whether or not Alice is correct.

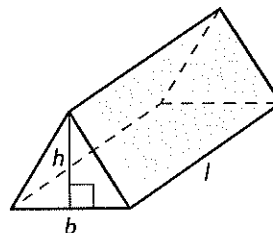
Show your work.



Solution: _____

- 7 Use the diagram at the right to write a formula in terms of b , h , and l for the volume of a triangular prism.

Show your work.



Solution: _____

Volume of Composite Solids

Study the example showing how to find the volume of a composite solid. Then solve problems 1–7.

Example

Lázaro is designing a set of blocks for young children. He needs to know the volume of the block shown.

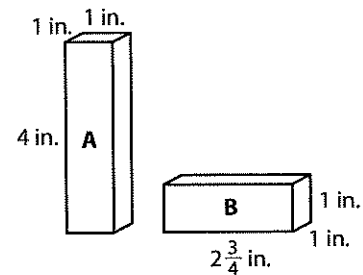
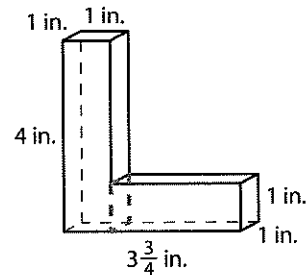
First he draws lines to divide the block into two rectangular prisms. Then he draws the two prisms and labels the dimensions of each.

$$\text{Volume of prism A} = 1 \times 1 \times 4 = 4$$

$$\text{Volume of prism B} = 2\frac{3}{4} \times 1 \times 1 = 2\frac{3}{4}$$

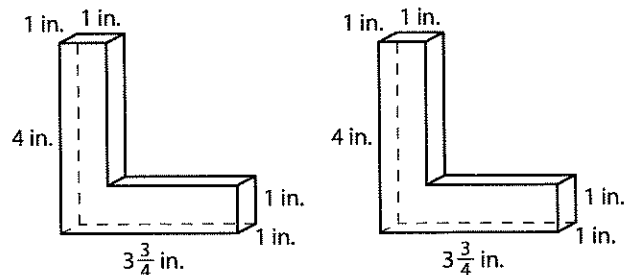
$$\text{Total volume of block} = 4 + 2\frac{3}{4} = 6\frac{3}{4}$$

The block has a volume of $6\frac{3}{4}$ cubic inches.



- 1 How did Lázaro find the length of prism B?

- 2 Draw lines to show two other ways that Lázaro could divide the block into other prisms.



- 3 Choose one of the ways in which you divided the block in problem 2. Sketch the prisms, label the dimensions of each one, and find the total volume.

Show your work.

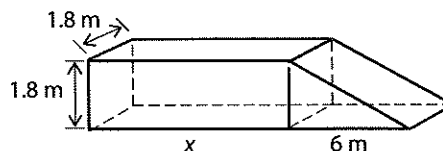
Solution: _____

Solve.

- 4 How is the process for finding the volume of a complex three-dimensional figure like the process for finding the area of a complex two-dimensional figure?

Use the figure at the right for problems 5–7.

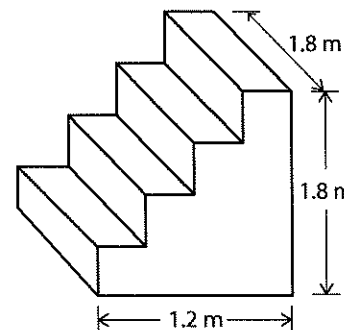
- 5 Sierra is designing a stage platform with a ramp. Draw lines on the figure at the right to divide the figure into solid figures whose volume you know how to find.
- 6 The total volume of the stage and ramp is 42.12 square cubic meters. Find the width of the stage, x .



Solution: _____

- 7 Sierra decides to add a set of stairs to the stage on the side opposite the ramp. Each stair has the same width and height. By how much does the volume of the structure increase?

Show your work.



Solution: _____

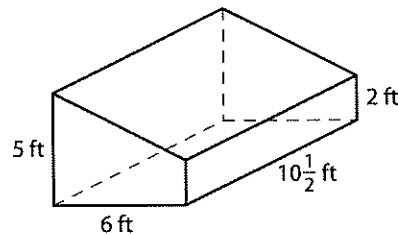
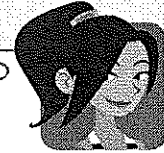
Volume of Solids

Solve the problems.

- 1 What is the volume of the figure shown?

Show your work.

What shapes make up this solid?

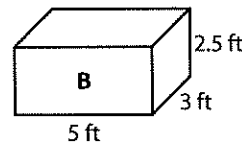
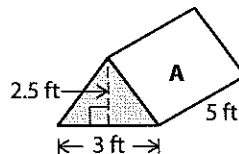


Solution: _____

- 2 How do the volumes of the two figures compare?
Select all that are correct.

- A Volume A = Volume B
B Volume A = $\frac{1}{2}$ Volume B
C Volume A > Volume B
D Volume B > Volume A

How can you answer without actually computing the volumes?



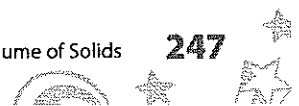
- 3 Jayden needs to store boxes that are 4 feet long, 3 feet wide, and 2 feet high. The boxes must remain upright with one of the 4-foot by 3-foot sides on top. Jayden's storage locker is 12 feet long, 6 feet wide, and 9 feet high. What is the greatest number of boxes that he can store in the locker?

- A 24 B 27 C 30 D 32

Be sure to satisfy all conditions of the problem.

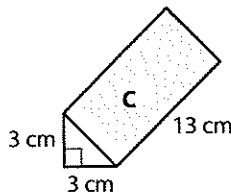
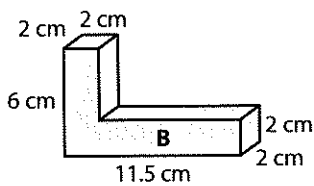
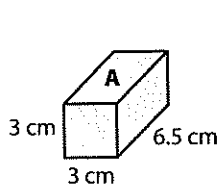


June chose B as her answer. How did she get that answer?



Solve.

- 4 Find the volume of each figure. Tell whether each statement is *True* or *False*.

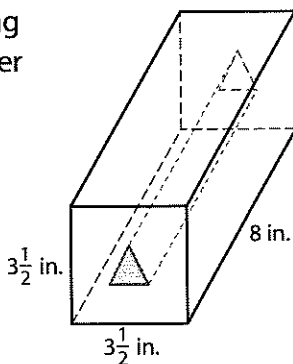


Do you have to compute the volumes of figures A and C to know if they are the same?



- | | | |
|---|-------------------------------|--------------------------------|
| a. Figure B has the greatest volume. | <input type="checkbox"/> True | <input type="checkbox"/> False |
| b. Figures A, B, and C have equal volumes. | <input type="checkbox"/> True | <input type="checkbox"/> False |
| c. No two figures have equal volumes. | <input type="checkbox"/> True | <input type="checkbox"/> False |
| d. Only figures A and B have equal volumes. | <input type="checkbox"/> True | <input type="checkbox"/> False |

- 5 A child's toy is made by removing a triangular prism from the center of a wooden rectangular prism. The triangular base of the triangular prism has a base length of 1 inch and a height of 1 inch. Write and solve an equation to find the volume of the toy.



Should you add or subtract to find the volume?



Show your work.

Solution: _____

Surface Area of Solids

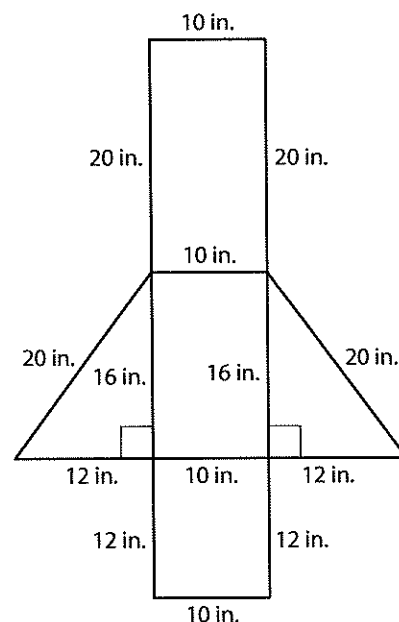
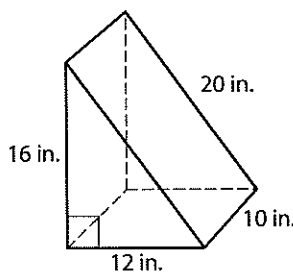
Name: _____

Prerequisite: Use a Net to Find Surface Area

Study the example showing how to use a net to find the surface area of a prism. Then solve problems 1–7.

Example

Kioshi needs to find the surface area of a triangular prism with the dimensions shown. He begins by drawing a net of the triangular prism and labeling the dimensions of each face. Find the area of each face of the prism.



Face	Base (in.)	Height (in.)	Area (sq in.)
Triangle	12	16	96
Triangle	12	16	96
Rectangle	10	12	120
Rectangle	10	16	160
Rectangle	10	20	200

- 1** Why do the two triangles have the same base length and height? How did you find the area of each triangle?

- 2** Do all the rectangles have equal areas? Explain.

- 3** What is the surface area of the prism? Explain how you found the surface area.

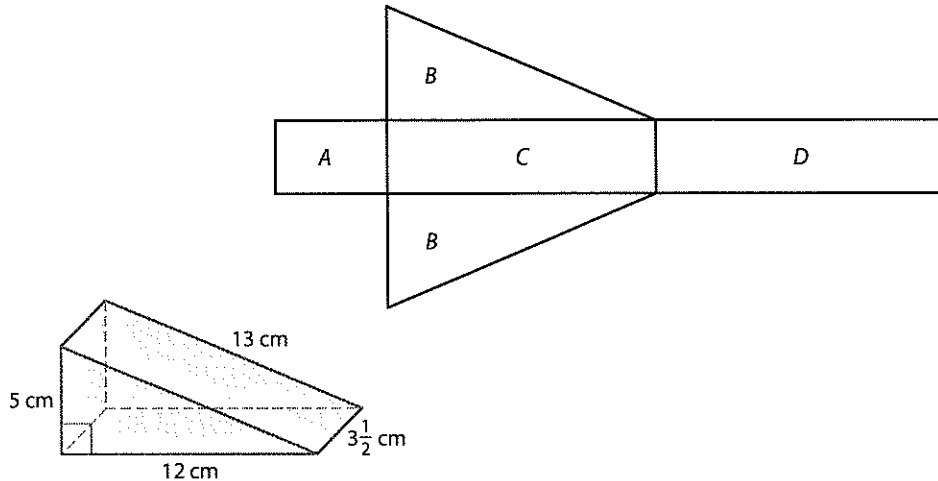
Vocabulary

net a flat representation of a solid when it is "unfolded."

surface area the sum of the areas of all of the faces of a three-dimensional figure.

Solve.

- 4 A doorstop is a triangular prism with the dimensions shown. Label the side lengths (in centimeters) of the net for the doorstop.



- 5 Using the letters in each section of the net in problem 4, write and solve an equation to find the total surface area of the prism, S .

- 6 Write an expression that represents the surface area of any rectangular prism with length l , width w , and height h . Explain.

- 7 The height of a rectangular prism is 20 centimeters. It has a surface area of 2,400 square centimeters. What are two possible sets of lengths and widths? Find one set of dimensions with l and w equal in length as well as a set of dimensions that are not equal.

Show your work.

Solution: _____

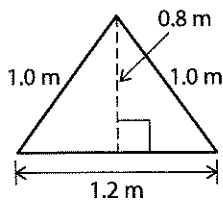
Surface Area of a Triangular Prism

Study the example problem showing how to find the surface area of a triangular prism. Then solve problems 1–7.

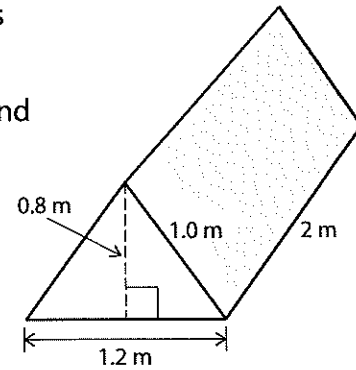
Example

The bases of the prism shown are isosceles triangles. What is the surface area of the prism?

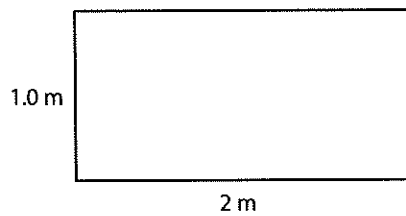
You can draw the faces of the prism, label the dimensions, and find the area of each face. There are two identical triangular faces. Draw and label one of them and find its area.



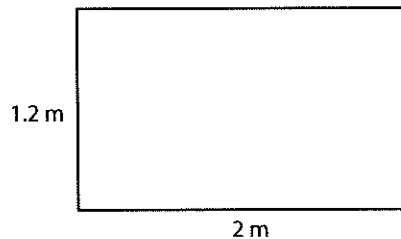
$$A = \frac{1}{2} (1.2)(0.8) = 0.48$$



Draw and label one of the identical rectangular faces. Then draw the third rectangular face. Find the areas.



$$A = (1.0)(2) = 2$$



$$A = (1.2)(2) = 2.4$$

$$\text{Surface area} = 2(0.48) + 2(2) + 2.4 = 7.36 \text{ square meters}$$

- 1** What formula was used to find the area of the triangular faces?

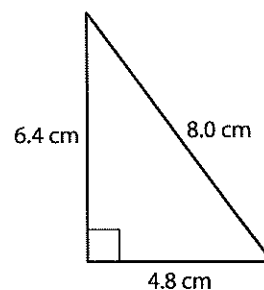
- 2** What formula was used to find the area of the rectangular faces?

- 3** Why is the area of one of the rectangles different from the areas of the other two rectangles?



Solve.

- 4 A manufacturer packages a product in a box shaped like a triangular prism with a height of 5.0 centimeters. The dimensions of the bases of the prism are shown. Draw each rectangular face of the box. Label the dimensions. Write the area of each rectangle inside it.



- 5 Write and solve an equation to find the surface area of the prism in problem 4.
Show your work.

Solution: _____

- 6 If the height of the box in problem 4 is doubled from 5 centimeters to 10 centimeters, does the surface area double? Explain your result.
Show your work.

Solution: _____

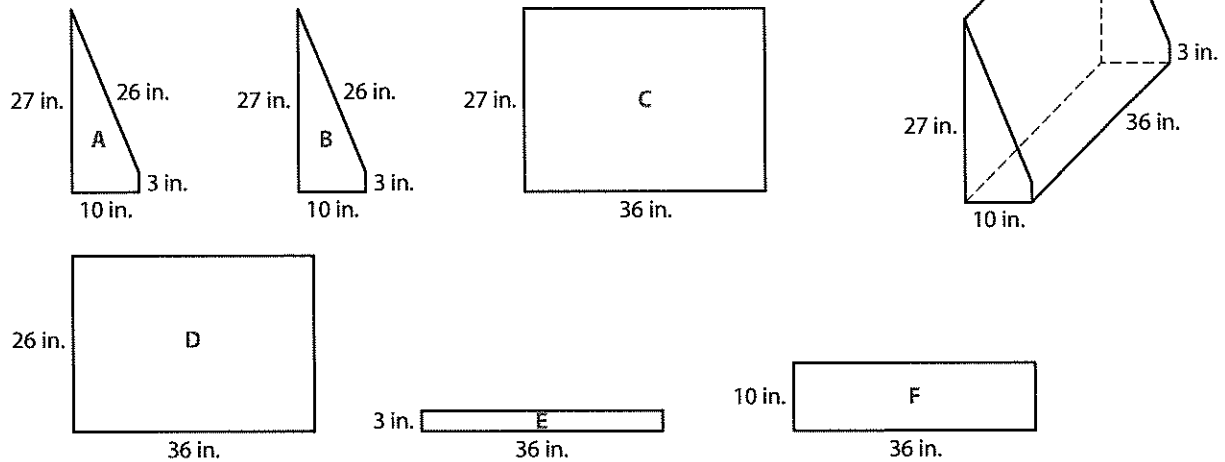
- 7 Find an expression for the surface area of a cube with edges of length s . Justify your answer.

Surface Area of a Complex Solid

Study the example showing how to find the surface area of a complex solid. Then solve problems 1–5.

Example

Tiana has a storage box with the dimensions shown. To find the surface area, she draws and labels the faces of the prism as shown. Make a table with the area of each face.



You can use the formulas you know to find the area of each figure that Tiana drew. Notice that figures A and B are composed of a triangle and a rectangle.

Face	Area (sq in.)
A	$A = (3)(10) + \frac{1}{2}(10)(24) = 150$
B	$A = (3)(10) + \frac{1}{2}(10)(24) = 150$
C	$A = (36)(27) = 972$

Face	Area (sq in.)
D	$A = (36)(26) = 936$
E	$A = (36)(3) = 108$
F	$A = (36)(10) = 360$

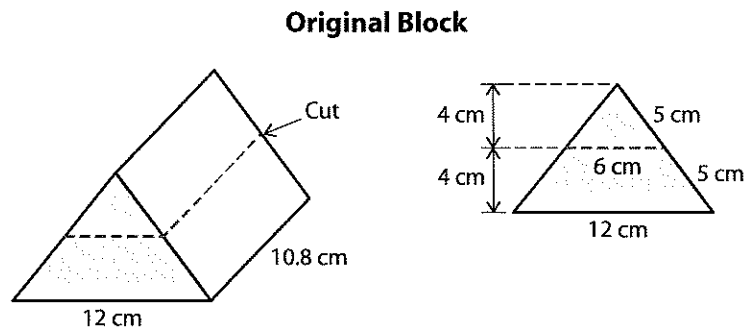
- 1** What is the surface area of the box?

- 2** Chris says that he can find the surface area faster by thinking of the box as a triangular prism on top of a rectangular prism. He finds the surface area of each one and adds them together. Does his method make sense?

Solve.

- 3 Milo is making a set of blocks for his child-care business. He has a block that is a triangular prism. He cuts the top off of this block to make a smaller triangular prism, which he paints red. A diagram of the original triangular face is shown. What is the surface area that Milo paints red?

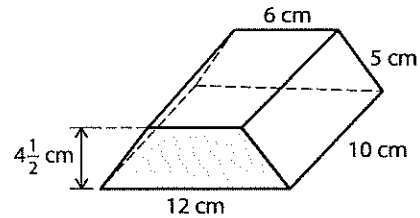
Show your work.



Solution: _____

- 4 Refer to problem 3. Milo paints the lower piece green. Draw one of the trapezoidal faces and label its dimensions. What is the surface area of the block that Milo paints green?

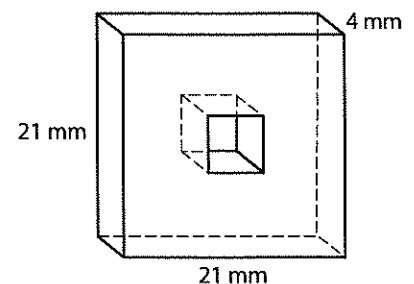
Show your work.



Solution: _____

- 5 A metal machine part is 21 millimeters long, 21 millimeters wide, and 4 millimeters thick. A square that is 6 millimeters on each side is punched from the middle of the metal solid. What is the surface area of the resulting figure?

Show your work.



Solution: _____

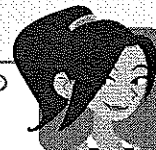
Surface Area of Solids

Solve the problems.

- 1 Viola says that she can find the surface area of a rectangular prism by finding the area of one face and multiplying by 6. When is this true? When is this *not* true? Use diagrams to explain your answer.

Show your work.

Think about the dimensions of a rectangular prism.

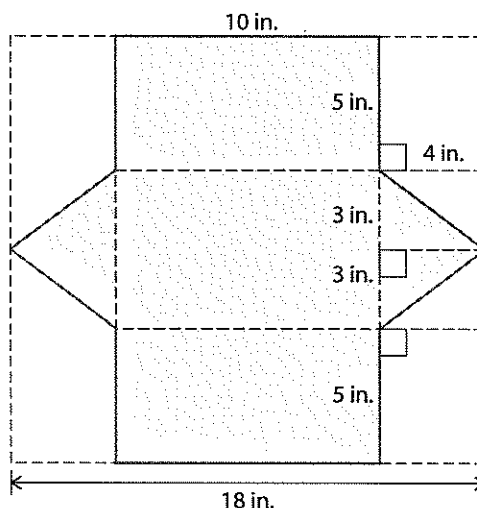


Solution: _____

- 2 Which expression can be used to find the surface area of the triangular prism represented by the net? Select all that apply.

- A $3(10 \cdot 5) + 2\left(\frac{1}{2}\right)(6 \cdot 4)$
 B $2(10 \cdot 5) + (10 \cdot 6) + 2\left(\frac{1}{2}\right)(6 \cdot 4)$
 C $(18 \cdot 16) - 4(5 \cdot 4) - 4\left(\frac{1}{2}\right)(3 \cdot 4)$
 D $(18 \cdot 16) - 2(5 \cdot 4) - 2\left(\frac{1}{2}\right)(3 \cdot 4)$

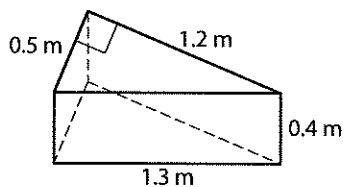
Which parts have equal areas?



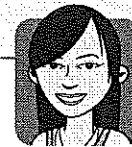
Solve.

- 3** What is the surface area of the prism shown?

- A** 1.5 square meters
- B** 1.8 square meters
- C** 2.16 square meters
- D** 2.4 square meters



How many faces does this prism have?

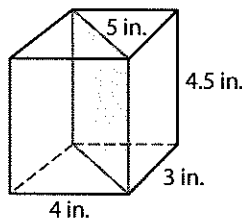


- 4** A storage box without a top is $18\frac{1}{4}$ inches long, 15 inches wide, and 10 inches high. The outside of the box is painted yellow. What is the total area that is painted yellow? Explain.

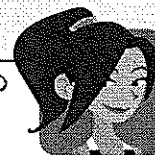
Would drawing a diagram help?



- 5** Lyle sliced a block of cheese along a diagonal into two triangular prisms, as shown. Lyle says that the surface area of the block is 2 times the surface area of one of the triangular prisms. Do you agree? Explain your answer.



How can you compare the surface areas?



- 6** Refer to problem 5. Find the difference between the surface area of the two triangular prisms and the surface area of the rectangular prism. Show your work.

What operation does the word *difference* indicate?

