Understanding the Relationship between the Circumference and Area of a Circle
Mr. Wysong

CONTENT AND TASK DECISIONS

Grade Level(s): 7

Description of the Task:

Indiana Mathematics Content Standards: 7.GM.5 Understand the formulas for area and circumference of a circle and use them to solve real-world and other mathematical problems; give an informal derivation of the relationship between circumference and area of a circle.

Indiana Mathematics Process Standards:
PS.1: Make sense of problems and persevere in solving them.
- Students will have to think hard about connecting the formulas and be willing to persevere in trying to understand a new connection (between circles and rectangles).

PS.3: Construct viable arguments and critique the reasoning of others.
- Students will try to convince one another why the sides of the rectangles are related to the circle.

PS.4: Model with mathematics.
- Students will color code lengths to better visualize the connect between the formula for circumference and area.

PS.5: Use appropriate tools strategically.
- Students will have circle cut outs, string, and scissors.

PS.6: Attend to precision.
- The formula for area is more accurate the narrower the pieces of the circle are.

Mathematics Content Goals: Students will understand where the formula for circumference comes from and how it can be used to find the formula for area of a circle.

Language Objectives: Students will explain the relationship between the formulas for circumference and area of a circle using a picture and written explanation.

Materials:
Circles to cut out, scissors, string/yarn, tape, rulers, and pencils.

THE LESSON

Before:
- **Activate prior knowledge:** students will do a pre-write about circles. They will have a to draw a picture, label parts, write formulas, and anything else they know about circles.
- **Be sure the problem is understood:** It will be necessary to explain that we are not trying to memorize the formulas, but rather understand where they come from and be able to explain them.
- **Establish clear expectations:** Materials need to be used appropriately. Every student needs to record their thinking and include a picture to further their explanation.

During (PART 1):
- **Let go,** all students will be given a circle with a diameter printed on card stock. Circles will come in two sizes (each partner getting a different size). The task is to develop the relationship between the circumference and the diameter given a piece of string.
- **Listen actively,** I hope to hear students discussing the relationship (pi) and recognize it is the same for both circles.
- **Provide appropriate support**: Students might not naturally compare the diameter to the circumference. I will walk around the room and ask students how they might use the string to measure lengths (if they are stuck). I can also ask what similarities and what differences there are between the two circles.
- **Provide worthwhile extensions**: Students will be encouraged to write a formula and draw a related picture.

During (PART 2):
- **Let go,** students will next be given tape and scissors. The circles given in the second part will be divided into 8 parts and 20 parts. Students will explore how to find the area of the circle by cutting and taping the parts to form a rectangle (roughly). The students will work in partners and apply the formula for area of a rectangle to try to derive the formula for area of a circle.
- **Listen actively,** I hope to hear students discussing the base of the rectangle as it relates to the circumference and the height of the rectangle (being the radius).
- **Provide appropriate support**: Students might not figure out how they are to form the rectangle with the pieces of the circle. I would tell the students to start with two pieces and try to form the closest shape possible to a rectangle. Then, the students can continue in the same pattern with the other 6 or 18 pieces. Another potential issue is not seeing that the height is the radius. I will ask them where they would see the height in the original circle. The hardest part will be for students to recognize that the base is half the circumference. Again, I will ask them to think about where all of the round parts of a circle are. What do all of the round parts placed next to each other form? I will try to allow all students to make the connection of the base being half the circumference.
- **Provide worthwhile extensions**: I will challenge students to come up with a way to find the radius if they are given the area.

After:
- **Promote a mathematical community of learners** Most of the discussion will happen within the pairs. I will have a couple students (volunteers) bring their work up to the document camera and explain their thinking (for both parts).
- **Listen actively without evaluation** I will ask if any student did the task a different way. I will affirm a positive from each student's explanation.
- **Make connections** I can ask if any student made any connections that have not been presented. I will then ask if they think the perimeter and area of any other shapes are related. We can also discuss finding the formulas for various other shapes. How are triangles, trapezoids, and parallelograms related?
- **Summarize main ideas** The area of a circle can be derived from the formula for circumference.

ASSESSMENT

Observe: All students will fill out an exit ticket. On the exit ticket, students must explain (in their own words) how the circumference of a circle is related to the area of a circle. Students will then be given a problem in which they are given they circumference of a circle and are asked to give the area. Lastly, students will reflect on the task…sharing the hardest part and what they learned

PART 1
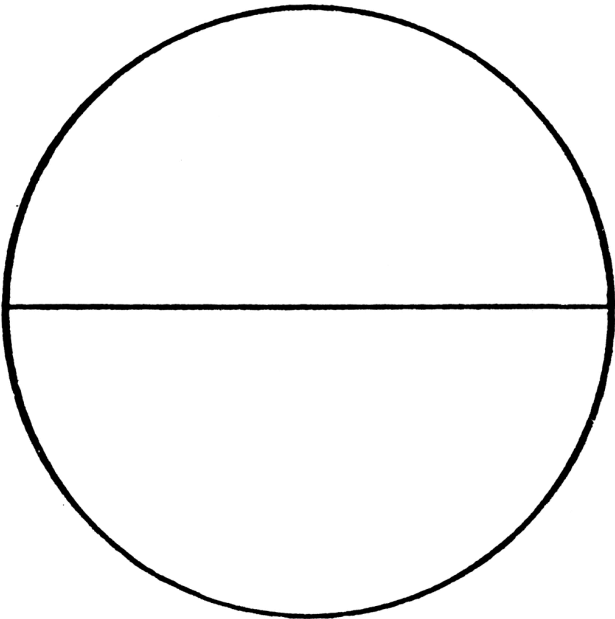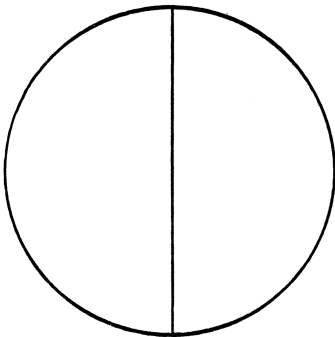
PART 2
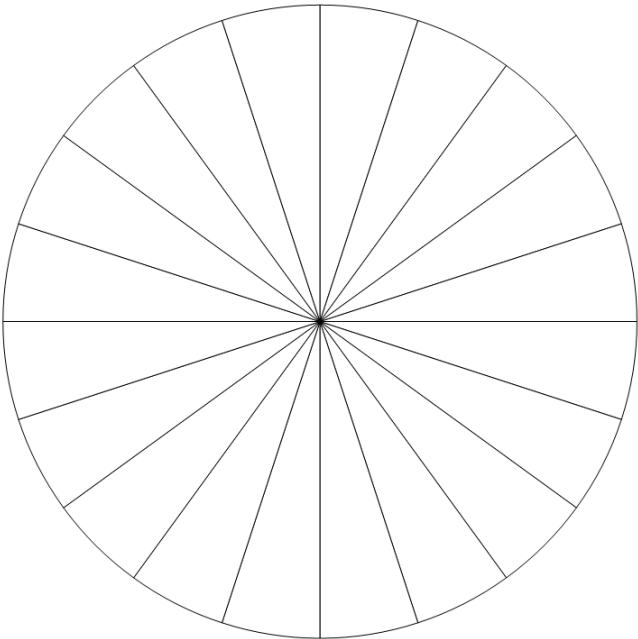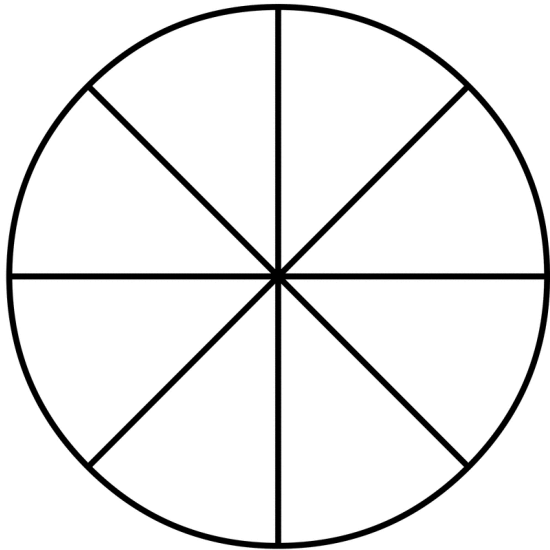
Write and draw everything you know about circles.

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_____

_____

Part 1

What is the formula for circumference of a circle? _____

Draw a picture (of a circle) that shows the formula.

Name: _____

Circle Exit Ticket

1) Explain (in your own words) how the circumference of a circle is related to the area of a circle.

_____

_____

2) Reflect on the task…what did you learn?

_____

_____

3) Reflect on the task…what was the hardest part?

_____

_____

4) What is the area of a circle if the circumference is 50.24cm?