**Mathematics**

**Numeracy strategies across the content areas.** The following pages present the best practices in mathematics instruction that are endorsed by the Public Schools of Robeson County as having the greatest impact on student achievement. The first column lists the best practice, the second column lists strategies to implement the practice, and the third column lists the effect on student learning and achievement. **Additional resources provided through links.**

**Suggested Teaching Strategies**

* **Deliberate and detailed planning**
* Number sense - daily
* Formative assessment - daily
* Daily cumulative review - daily
* Math fact fluency - daily
* Multiple methods - daily
* Multiple representations - daily
* Tasks - as often as possible but at least weekly
* Literacy/language rich mathematics classrooms - daily use of math vocabulary; use of several vocabulary strategies during the week; teach students to read math texts
* Mathematics embedded in real-world contexts - as often as possible but at least weekly

**Effect on Student Learning and Achievement**

* When teachers plan deliberate instruction and collaborate among their teaching peers, lessons become more engaging and rigorous for all students. Well planned instructions keeps students engaged and involved in classroom activities and less likely to become bored.
* When students see multiple representation (graphs, manipulatives, tables, and charts) of content, it is stored in different parts of the brain making recall much easier. Students begin to make connections to real world applications.
* Giving students words in context in the classroom allows them to understand the use and meaning of the word.

|  |  |  |
| --- | --- | --- |
| **Best Practice** | **Description/Strategies** | **Effect on Student Learning & Achievement** |
| **Deliberate and Detailed Planning** | **Effective mathematics instruction requires careful planning.**  Planning with other teachers helps focus lessons and increase [rigor](https://drive.google.com/open?id=1BVKm-QNHZi9IMe4yd9w6ajE2r5zM5ek-), as well as creating equivalent classroom experiences across grade levels and courses. The following items should be regularly included in lesson plans:   * Vertical alignment * Gradual release of instruction * Plan for teaching academic vocabulary * Careful selection and pre-planning of meaningful problems * Plan for misconceptions * Plan to help [struggling students](https://drive.google.com/drive/folders/1EmChEY_Kf7m1pBsMNClZ4V0cPnuo4AtE?usp=sharing) * [Plan for higher-level questions](https://drive.google.com/open?id=1CrzU0u-a5OS2Z2paG1sICuz-iI3qUGFM) * Manipulatives with a purpose * [Centers/Stations/Math Workshop](https://drive.google.com/drive/folders/1Qs4RVvHBRGjdoYV9E6nxMQJdiFoybMm3?usp=sharing) * Connect concepts from previous units/courses to current unit * Use of technology * iReady | * Provides avenues to address student’s ability to think, reason, and problem-solve * Allows for addressing conceptual nuances of lessons to be addressed by teacher * Facilitates differentiation * Improves proficiency in mathematics, across grade levels and diverse student populations * Allows for consideration of likely errors and misconceptions and for planning of strategies to address them * Encourages active engagement to activate the brain and increase retention |
| **Number Sense** | [Number sense](https://drive.google.com/drive/folders/18PdZxJa4IZsxzxxfbgN0R65e6Usa7U9Z?usp=sharing) is the ability to understand what numbers mean and their relationship to one another. It is the ability to use mental math, understand symbols, and use numbers to solve problems. Simply put, number sense is flexible and fluid thinking about numbers. Number sense establishes a comfort with numbers, including estimation, mental math, numerical equivalents, a sense of order and magnitude and a well-developed understanding of place value. Number sense is taught and reinforced in every math problem.   * Mental math * Estimation * Number magnitude * Place value (e.g., tenths versus tens) * Sense of order (e.g., numbers getting smaller and larger) * Recognizing part-whole relationships * Equivalence (e.g., 5+2=3+4, ½ =50%) * Manipulatives * Models * Number lines * Derived math facts (e.g., 7 X 8 can be thought of as two times 7 X 4) * Problem-solving | * Promotes flexible thinking and reasoning * Facilitates problem solving * Enables recognition of unreasonable answers * Allows for composing and decomposing numbers in different ways * Highlights connections among operations * Makes mental math easier * Enables students to make reasonable estimations * Improves recall and provides feedback mechanisms for students * Research has shown that high-achieving students use number sense but low-achieving students do not (Gray & Tall, 1994) |
| **Formative Assessment** | Formative assessment is an embedded, on-going process that provides evidence of student achievement to inform instructional planning and to adapt what happens in classrooms to meet student needs. It is a means of eliciting and gathering evidence of student understanding at strategic points during instruction.   * [Running records](https://drive.google.com/drive/folders/1Z72TyrgW_Y61ECtpQ9GG7Lf73xJUCECb?usp=sharing) * Quick writes * [Entrance/exit tickets](https://drive.google.com/open?id=1wDnJS9L37rDbOQMz_w8gv3XGU9G_8CKo) * Summary writing * Mid-chapter check points * Self assessments * Think-Ink-Pair-Share * Example/non-example * [Graphic Organizers](https://drive.google.com/open?id=1Gisx--URaOznsm85KkdDrKvOWnUy9urS) * [Shaping up review](https://drive.google.com/open?id=1R-dsvkbjySKn_qltng5xRvyUlECE0K6B) * [Checking for understanding](https://drive.google.com/open?id=1r-swiPrdXCDFeO8qlQWgBQYciJJn9pU_)   + White boards   + Manipulatives   + Response Options     - Cards     - Student Response Systems   + Choral Responses   + Brainstorming   + Hand signals   + Green-Red-Yellow     - Cups     - Table tents     - Chips | * Allows students time to process the information, and provide teachers with valuable information about re-teaching, regrouping or moving forward * Provides teachers and students with information that identifies student’s achievement of intended learning goals * Provides opportunities for descriptive feedback * Helps students identify weaknesses and strengths |
| **Daily cumulative review** | [IDEA - Intentional, Daily, Engaging review for All students](https://mrdolan.wordpress.com/2012/03/16/death-to-review-day-intentional-daily-engaging-review-for-all-students/). **Daily cumulative review at some point is essential in every lesson and has proven to be one of the most effective strategies for fostering mastery as well as retention**. Research has proven that students need at least 5 to 7 exposures to a new concept before that concept will move from short-term memory to long-term memory. Daily cumulative review includes a combination of vocabulary, estimation, geometry, measurement, probability, number sense, computation, problem solving, logic, probability, and measurement on a regular basis. This can be accomplished through:   * Bell work * Spiral review problems  * [Exit tickets](https://drive.google.com/open?id=1wDnJS9L37rDbOQMz_w8gv3XGU9G_8CKo) * Written summary  * [Brain breaks](https://drive.google.com/open?id=1uu6JvJUUzpw1UNf4y9VQgziYrZszrRWQ) * Daily routines * Instructional resources through activity * Distributed practice   + Strategically placing review problems within the daily work | * Activates prior knowledge * Moves knowledge from short-term to long-term memory * Informs students and teachers whether or not there is mastery of key concepts * Keeps skills and understanding current * Reinforces previously taught material * Gives students the opportunity to clarify understandings and misunderstandings * Provides formative information necessary to adjust instruction to improve performance * Gives additional time to process the concept * Helps with recognition of the connections between various mathematical ideas. |
| **Math Fact Fluency** | [Fluency](https://drive.google.com/drive/folders/1xdOnr0chNwmoaKrbPQj8XIxHBGKS2wrR?usp=sharing) is the effortless, error-free recall of basic math facts by carrying out procedures flexibly, accurately, efficiently, and appropriately. It is the ability to not only efficiently recall a fact but to also understand how to solve the fact. Fluency includes, but is not limited to, automatic recall of addition, subtraction, multiplication and division facts. It is knowing which procedure is appropriate and most effective in a given situation.   * Fluent students   + Understand the math they are doing   + Use a variety of strategies to compute   + Think flexibly by:     - Putting together and taking apart numbers     - Knowing the relationship among numbers * [Automaticity](http://www.mathcoachscorner.com/2017/08/developing-automaticity-basic-math-facts/) (student can recall math facts without having to think about them) is achieved through targeted meaningful practice that:   + Is frequent and short in duration   + Includes interactive activities   + Requires that students talk about how numbers relate to one another and participate in discussions of alternative approaches and students use * Fluency does not focus on speed or memorization | * The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense (Boaler, 2015). * As students work on meaningful number activities, they will commit math facts to heart at the same time understanding numbers in math. They will enjoy and learn important mathematics rather than memorize, dread and fear mathematics (Boaler, 2015). * Improves students’ mathematical abilities (Seeley, 2009). * To be a successful problem solver, students must be able to accurately compute answers, but more than that, they must be able to figure out how to build equations that correspond to problem situations (O’Connell & SanGiovanni). * Promotes mathematical proficiency (Seely, 2009). * Allows a student to solve problems more effectively. * Speeds up math tasks. * Math timed tests can cause math stress and math anxiety (Boaler, 2012). * Math anxiety robs people of working memory, which is important to solving problems (Boaler, 2012). * If fluency is not developed, the development of higher order mathematical abilities may be impaired. |
| **Multiple Methods** | Using multiple methods or strategies teaches students that mathematics is a sense-making process for understanding ”why?” This teaches students that there is not just one way to get a correct answer which allows students the opportunities to use various approaches to solving a problem. Effective instruction incorporates deliberate attention to both multiple representations and to alternative approaches to solutions which accommodates diverse learning styles within every class.   * [Productive struggle](https://drive.google.com/drive/folders/1A915BlOgG5j6H3cuYvunh9g3xYcvN2xQ?usp=sharing) is a student-based process that allows for discovering ways to solve problems when an approach/method/solution is not given, it is a necessary component of learning math with understanding (Hiebert & Grouws, 2007) * Compare/contrast methods show more than one approach will give a correct answer for a problem * Argumentative discourse requires students to defend, justify, and explain their method for solving a problem when their method differs from the methods of other students * [Think-alouds](https://drive.google.com/open?id=1WxU70O6nUopDFD3tSyxRq10YVmOQC5SL) model for students how they may solve problems in a multitude of ways * [Math talk](https://drive.google.com/drive/folders/1X5jS44xJSrlmhFZwsOkHiNGdmivfDTiY?usp=sharing) gives students the opportunity to explain their mathematical thinking * Multiple methods allow for student choice | * Allows students to make sense in their own way * Enriches instruction and provides new levels of access to mathematical understanding * Gives students multiple methods to solve problems * Allows extra time to process the concept * Allows students to revise their collection of methods and retain those that are most appropriate for each situation * Increases competency when using mathematical language * Allows students to productively struggle with new problem situations |
| **Multiple representation** | Multiple representations allow access to the mathematics of a problem for all types of learners. These representations, such as models, drawings, number lines, tables, graphs, support the visualization and deeper understanding of skills and concepts. Using multiple representations and allowing students time to practice concepts with those representations is the bridge between concrete and abstract in the [concrete-representational-abstract (CRA)](https://drive.google.com/drive/folders/1ZpVgYlZhm5ClGTKnibghbtk5ttaoShme?usp=sharing) sequence of instruction.   * Number lines * Tables * Graphs * Pictorial representations * Manipulatives * [Word wall](https://drive.google.com/open?id=1wMpOMShIjJCSKQKcJt1uVjv4BdCW55ss) (with representation) * Kinesthetic activities * [Graphic organizers](https://drive.google.com/drive/folders/1Gisx--URaOznsm85KkdDrKvOWnUy9urS?usp=sharing) * Area models * Tape diagrams | * Allows for understanding through at least one method * Provides different ways to examine a problem * Assists students in making sense of abstract concepts * Helps students see there are many ways to interpret * Assists with the recall of information when it is stored in multiple parts of the brain: nonlinguistic content is stored in multiple parts of the brain than linguistic content, thus giving the brain two ways of remembering content, and enhancing the brain’s ability to recall the information * Gives extra time to process the concept * Allows learners to discover misconceptions and correct them through the use of concrete experiences |
| **Rich Tasks** | [Rich math tasks](https://drive.google.com/open?id=1hzJkIgrwofJ870UG0eMjrRiCbFvNuALs) are high cognitive demand tasks in which all students are able to engage in the math of the problem. The tasks have multiple entry points for students and a variety of solution paths. Rich math tasks typically have the following six characteristics (Audet, L. 2016):   * Accessibility to all learners (low floor-high ceiling- wide walls)   + Low floor- anyone can access the task   + High ceiling - lots of possibilities for taking the task further through discussions and questioning   + Wide walls - students can explore multiple paths to a solution * Real-life tasks   + The tasks have some basis in real life   + The topic is something with which students have some real-life experience * Multiple approaches and representations   + The task lends itself to different approaches and representations which promotes student confidence in their mathematical abilities and enables them to learn from their peers’ different perspectives * Collaboration and discussion   + Every task should include some collaboration and discussion during the process of completing the task even if the task requires students to work independently first * Engagement, curiosity, and creativity   + The task should have a problem that captures the students’ interest so they will work harder to complete it (perseverance, SMP#1) * Opportunities for extension   + A rich task offers challenges/extensions for students who finish quickly while others continue to work   For a task to be effective, the teacher must launch, or introduce, the task in such a way as to pique the students’ curiosity without giving any details of the task or giving the students any course of action to complete the task. A good task launch has four important characteristics (Jackson et al, 2013) (NC2ML, 2018):   * Discuss the key features of the task   + The teacher must be certain that students understand the context or scenario of the task * Discuss the key mathematical ideas of the task   + The teacher must convey the mathematical ideas that are presented in the task through questioning and discussions * Develop common language to describe the key features   + Discussion with students is needed to develop terminology that refers to the key features of the task that are vital to student success on the task * Maintain the cognitive demand of the task   + The teacher must be careful to not suggest methods to solve the task because doing so will rob the students of their opportunity to develop the understanding of the mathematics by themselves   It is important that students can access the math, understand the mathematical goal they are working towards and can get started on the task. An effective launch supports students’ [productive struggle](https://drive.google.com/open?id=1A915BlOgG5j6H3cuYvunh9g3xYcvN2xQ) and persistence in completing the task (SMP#1). | * Improves students’ conceptual understanding (Boston & Wolf, 2006) * Improves students’ ability to communicate, problem solve and make mathematical connections (Boston & Wolf, 2006) * Improves performance on achievement tests (Boston & Wolf, 2006) * Improves engagement * Promotes mathematical discourse and learning through the discourse * Promotes development of the Standards for Mathematical Practice (SMP) * Allows students to show what they can do * Automatic differentiation |
| **Literacy/language-**  **rich mathematics classrooms** | Mathematics is a language, and as such must be encountered by reading, writing, and speaking while emphasizing academic vocabulary, terminology, symbols, explanations, and justifying solutions.   * Speaking   + Types of math talk/discourse     - [Think alouds](https://drive.google.com/drive/folders/1WxU70O6nUopDFD3tSyxRq10YVmOQC5SL?usp=sharing) by teachers and students     - [Math talk moves](https://drive.google.com/drive/folders/1w_xhFu1w5EmsFTVbAckEbvXzWR17czT0?usp=sharing)       * Revoicing: The teacher repeats what a student has said then asks the student to verify whether the revoicing is correct (e.g. “So what you are saying is...Am I correct?”       * Repeating: Students repeat or rephrase what another student has said (e.g. “Can you repeat what he just said in your own words?”)       * Reasoning - Agree/Disagree: Students make their own reasoning explicit by applying thinking to someone else’s contribution (“Do you agree or disagree? Why?” or “I agree/disagree because…”)       * Adding On: The teacher asks other students to contribute to the discussion (“Who can add something more?”)       * Waiting: The teacher waits at least ten seconds for students to think before calling on someone for an answer (“Take your time...we’ll wait.”) * Writing   + Types of mathematical writing:     - Affective Writing: Writing that explains the students’ attitudes and feeling about mathematics (This assesses and impacts students’ mindsets.)     - Solving a Math Problem: Writing to explain the procedures and steps the student used to solve a specific problem     - Explaining Mathematical Ideas: Writing about math concepts (e.g. How are addition and subtraction alike?”)     - Application sentence that shows a connection to real world use (e.g. “I will use a ratio of guests to cookies to help me decide how many cookies to buy for the party.”)     - Whenever possible, ask students to cite the evidence from the text that supports their thinking. Writing justification for an answer by citing the source is an example of text dependent analysis in mathematics.   + [Ways to implement writing](https://drive.google.com/drive/folders/1YLnE6GiWNVDnFwyhmlhbvlPnyQikn7xP?usp=sharing):     - [Journals](https://drive.google.com/drive/folders/1KxkKQexSg45Hv659nQd5S7odRfdlZ4C3?usp=sharing)     - Note Making (e.g. [Cornell Notes](https://drive.google.com/drive/folders/1b2b9UkLm2posBPkUX856whW81SM0Q0id?usp=sharing))     - [Entrance/Exit tickets](https://drive.google.com/drive/folders/1wDnJS9L37rDbOQMz_w8gv3XGU9G_8CKo?usp=sharing)     - Quick writes     - Summary writing     - Think-Ink-Pair-Share * Vocabulary   + [Six Step](https://drive.google.com/drive/folders/1y0vVCsCEWkwr6rm8MswHmWcs5bx70paG?usp=sharing)   + Ongoing emphasis on use and meaning of mathematical terms   + Precise use of mathematical terms, vocabulary, and notation   + [Interactive word wall](https://drive.google.com/drive/folders/1wMpOMShIjJCSKQKcJt1uVjv4BdCW55ss?usp=sharing) * Reading used as anticipatory set, problem to solve, summarize and make connections   + Fiction texts (e.g. *Sir Circumference and the Dragon of Pi, Meatballs and Spaghetti for All*)   + Non-fiction texts (e.g. newspapers, magazines, textbooks) | * Stimulates children to think through their own ideas and to approach objectivity when sharing with others * Discussing mathematics helps students organize and consolidate their thinking, communicate coherently and clearly, analyze and evaluate the thinking and strategies of others, and use the language of mathematics * Acts as a formative assessment to drive targeted instruction. Utilizing math terminology in math talk, reading and writing takes students from progressing to proficient * Speaking, writing, and reading mathematics increase generalized use of math * Verbalizing and writing about mathematics create students who internalize the learning and are better able to think about mathematics and comprehend a process * Increases the understanding of the steps in a process by watching and listening to others (including teachers) think aloud * Writing provides an additional exposure and opportunity to recall content and reflect on new learning and enhances retention |
| **Mathematics embedded in real-world contexts** | Effective mathematics instruction connects to real-world situations relevant to the students in order to engage them in the mathematics of the problem.   * Non-fiction texts * Photographs * Research * Cross-curricular connections (e.g., social studies, science, physical education, family consumer sciences) * [Project-based learning](https://drive.google.com/drive/folders/1xusPKPsyPV0W93rbk4Tklh-A0TyqrvW4?usp=sharing) * Life skills * Careers * Data from current events * Sports * Authentic coursework * Use of technology that is beyond procedural practice * Music | * Maximizes understanding and retention of knowledge when students have applied math to a practical setting relevant to their own point of reference * Builds and strengthens memory pathways helping students make connections * Establishes a purpose for computation practice and fluency |
| **This component of fluency. . .** | **. . . is developed conceptually through** | **. . .using** |
| **Small numbers (basic facts)** | **Phase 1:** Counting (e.g. counting all, counting on, skip counting, etc.)  **Phase 2:** Reasoning strategies (e.g. doubles, doubles plus 1, making ten, breaking apart, repeated addition, arrays, etc.)  **Phase 3:** Automaticity | Concrete  Representational  Concrete  Representational  Abstract  Abstract |
| **Multi-digit numbers, fractions, decimals, percents, and integers** | **Phase 1:** Developing understanding through models (e.g. base ten blocks, fraction strips , arrays, etc.)  **Phase 2:** Relating models to procedures and algorithms (e.g. adding by place value, distributive property to multiply, compensation, etc.) | Concrete  Representational  Concrete  Representational  Abstract |

**Additional Resources**

[Academic Action Plan - Math Resources](https://drive.google.com/drive/folders/1CsM3k1hQxHNtXIW_c8FoGU7xNDRaKEvQ?usp=sharing)

[NCDPI Math Wiki](http://maccss.ncdpi.wikispaces.net/) (temporary link - will expire no later than 12/31/18)

[New K-8 Standards Documents](https://drive.google.com/drive/folders/1SfbhuzgpE5PBu9tsdTXmRsFhURmCH6i3?usp=sharing) (Standards, Crosswalk, Unpacking, Major Revisions)

<http://www.tools4ncteachers.com/> (Resources aligned with frameworks)

<https://www.nc2ml.org/> (Frameworks (pacing guides)research briefs for K-8)

References:

Adding It Up (2009). Washington, DC. National Academy Press

Audet, Lauren. “6 Characteristics Of Rich Math Tasks.” 11 July 2016, blog.heinemann.com/6-characteristics-rich-math-tasks.

Bellock, S. & Willingham, D. (2014). Ask the Cognitive Scientist Math Anxiety: Can Teachers Help Students Reduce It? American Educator

Billmeyer. R. (2006). Strategies to Engage the Mind of the Learner: Building Strategic Learners. Omaha, NE: Printco Graphics

Boaler, J. (2015). Fluency Without Fear: Research Evidence on the Best Ways to learn Math Facts. [www.youtubed.org](http://www.youtubed.org)

Boston, M. & Wolf, M.K. (2006). Assessing academic rigor in mathematics instruction: Center for the Study of Evaluation, Standards and Student Testing, UCLA

Burns, C. & Sillbey, R. (2000). So You Have to Teach Math? Sound Advice for K-6 Teachers. Sausalito, CA. Math Solutions Publications

Burns, M. (2007). About Teaching Mathematics: A K-8 Resource (3rd ed.). Sausalito, CA: Math Solutions Publications

Chappuis, J. (2009). Seven Strategies for Assessment for Learning . Portland, OR Educational Testing Service

Fisher, D. (2010). Guided Instruction: How to Develop Confident and Successful Learners. Alexandria VA ASCD

Fisher, D. & Frey, N. (2014). Better Learning Through Structured Learning: A Framework For the Gradual Release of Responsibility. Alexandria, VA ASCD

Gray, E. & Tall, D. (1994). Duality, Ambiguity, and Flexibility: A “Proceptual” View of Simple Arithmetic. Journal for Research in Mathematics Education

Hierbert, J. & Grouws, D.A. (2007). The effects of classroom mathematics teaching on student learning. Charlotte, NC Information Age Publishing.

Jackson, K., Garrison, A., Wilson, J. Gibbons, L. & Shahan, E. (2013). *Journal for Research in Mathematics Education,* 44(4), 646-682

Leiwand, S. (2009). Accessible Mathematics: 10 Instructional Skills That Raise Student Achievement. Portsmouth, NH Heinemann

NC2ML (2018) The Role of Mathematical Tasks. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. <http://nc2ml.org/>

NC2ML (2018) Launching a Task. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. <http://nc2ml.org/>

Marzano, R. (2001). Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement. Alexandria, VA ASCD

National Council of Teachers of Mathematics (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: Author

O’Connell, S. & SanGiovanni, J. (2011). Mastering the Basic Math Facts in Addition and Subtraction. Portsmouth, NH Heinemann

Schmoker, M. J. (2011). Focus: Elevating the Essentials to Radically Improve Student Learning. Alexandria, VA ASCD

Seeley, C. (2009). Faster Isn’t Smarter. Sausalito, CA: Math Solutions

Seeley, C. (2014). Smarter Than We Think. Sausalito, CA: Math Solutions

Sousa, D. A. (2008). How the Brains Learns Mathematics. Thousand Oaks, CA: Corwin