The Alphabet of Trigonometry

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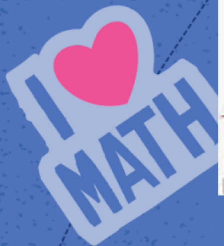
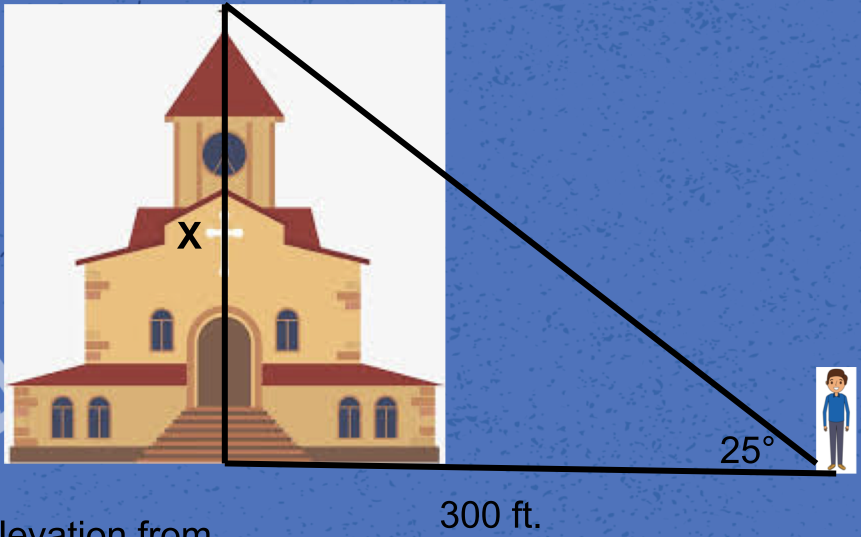
Angle of Elevation

The angle through which the eye moves up from the horizontal to look at something above.





Solution:
$\tan\Theta = \tan 25° = x/300$
$X = 300 \bullet \tan 25°$
X=139.89 feet

The angle of elevation from the top of the church from a person 300 feet away is 25°. Using the angle of elevation find the height of the church.

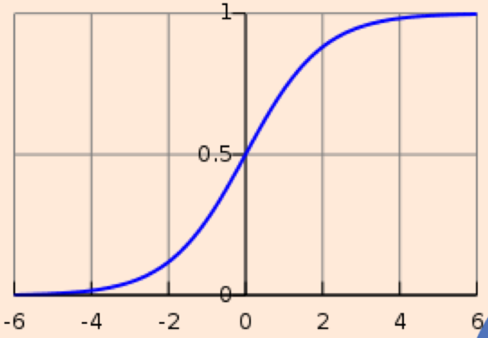
Basic Logistic Function

A function that grows or decays rapidly for a period of time and levels off.

Equation

$$f(x) = \frac{1}{1+e^{-(x)}}$$

Graph

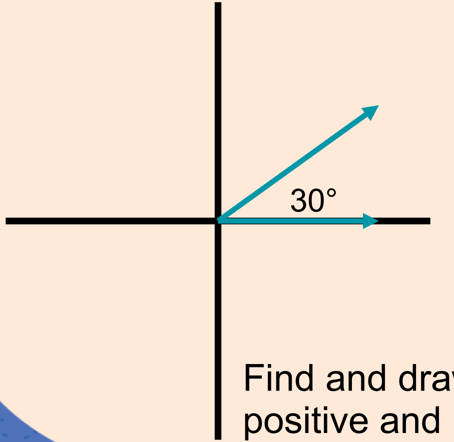


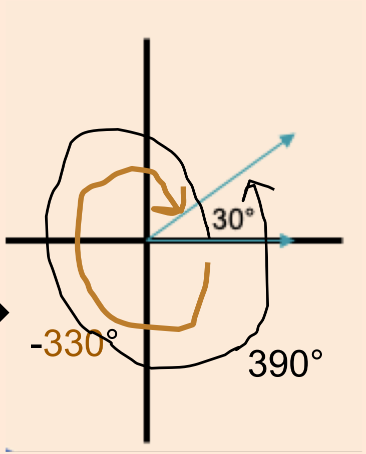
Domain= (-∞, ∞)

Range= (0, 1)

Coterminal Angles

Angle with the same side but have different measure.





Find and draw a positive and negative angle that is coterminal with the given angle.

Damping Oscillation

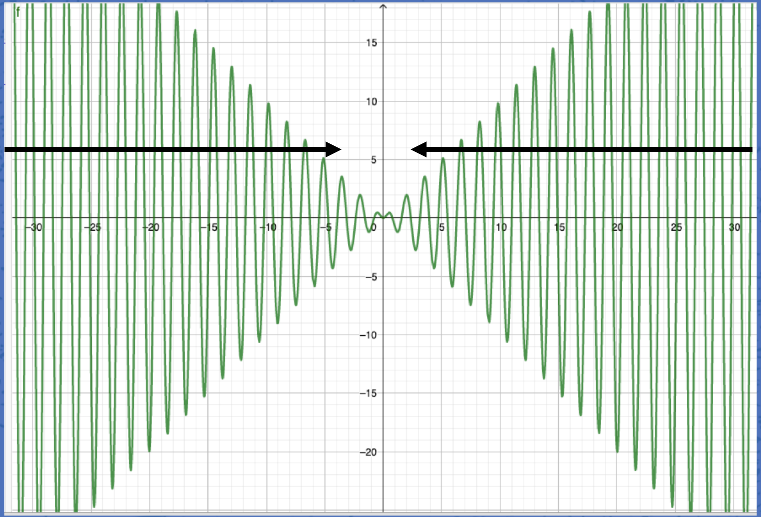
When the amplitude of the waves reduces over time.

Tell whether the function exhibits damped oscillation and then identify the damping factor and tell whether the damping occurs as x approaches 0 or as x approaches ∞

$F(x) = x\sin 4x$

The x is the damping factor

It has damped oscillation as shown on the graph and the damping occurs when x approaches 0



Here is the graph of f(x)=xsin4x

Eccentricity

A nonnegative number that specifies how off center the focus of a conic is.

Find the eccentricity of the equation

$$\frac{(y-2)^2}{4} - \frac{(x+3)^2}{16}$$

a^2=4 a=2

b^2=16 b=4

4+16=20=c^2

c=$\sqrt{20}$

e=$\frac{\sqrt{20}}{2}$ = $\sqrt{5}$

e = c ÷ a

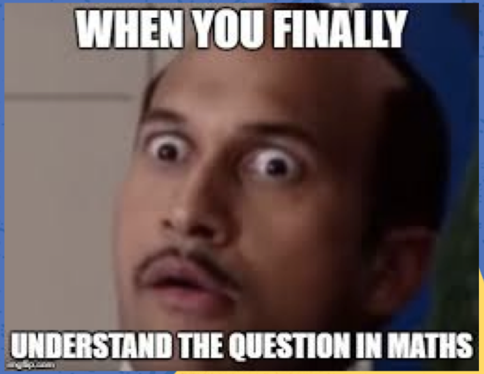
e = eccentricity

a = semimajor axis

b = semi minor axis

c = distance from the center of the focus to either focus

$a^2 + b^2 = c^2$



Focal Width of a Parabola

The length of the chord through the focus and perpendicular to the axis.

Focal Width = | 4p |

Find the focal width of the parabola.

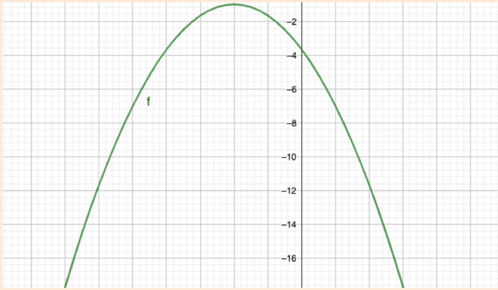
$(x+4)^2$= -6(y+1)

4p = -6

$4 \cdot \frac{-3}{2}$ = -6

p = $\frac{-3}{2}$

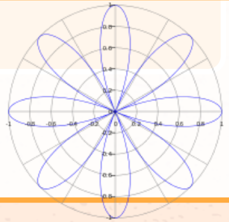
|4p| = 6 = focal width



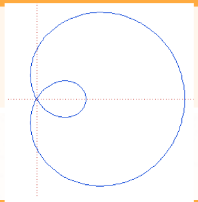
Graphs of Polar Equations

The set of all points in the polar coordinate system corresponding to the ordered pairs (r, Θ) that are solutions of the polar equation.

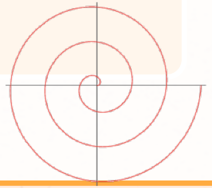
Rose Curve



Limacon Curve



Spiral of Archimedes



$r = a \cdot \cos(n\Theta)$

$r = a \cdot \sin(n\Theta)$

If n is even then the number of petals is 2n.

If n is odd the number of petals in n.

$r = a \pm b\cos\Theta$

$r = a \pm b\sin\Theta$

$r = k(\Theta)$

Heron's Formula

The area of $\triangle ABC$ with semi perimeter s is given by

$$\sqrt{s(s-a)(s-b)(s-c)}$$

Find the area of a triangle with sides of 13, 15, 18.

Semi perimeter = (13+15+18)/2 = 23

Area = $\sqrt{23(23-13)(23-15)(23-18)}$

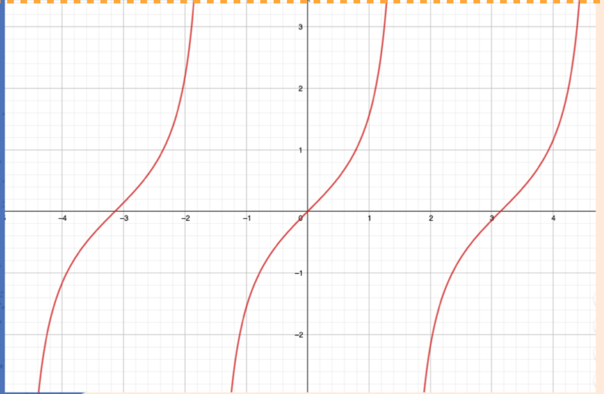
$$\sqrt{23(10)(8)(5)} = \sqrt{9200} = 20\sqrt{23}$$

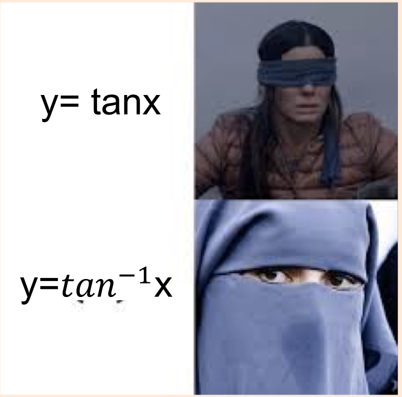
Inverse Tangent Function

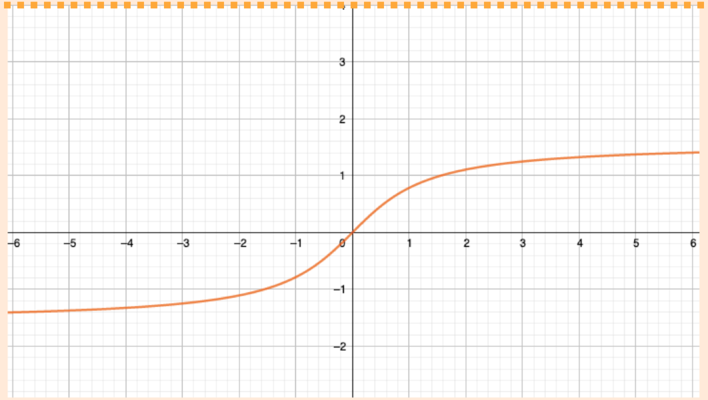
The function y = tan^{-1}

Tangent

Inverse Tangent







y= tanx

y=tan^{-1}x

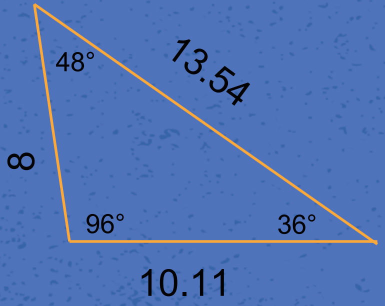
Law of Sins

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Solve △ ABC given that
angle A = 36°, angle B = 48°, a = 8

$$\frac{\sin(36°)}{8} = \frac{\sin(48°)}{b}$$
$$\frac{\sin(36°)}{8} = \frac{\sin(96°)}{c}$$

Cross multiply each equation
b = 10.11
c = 13.54



Maximum R Value

The value of |r| at the point on the graph of a polar equation that has the maximum distance from the pole.

Find the maximum R Value for the Limacon Curve

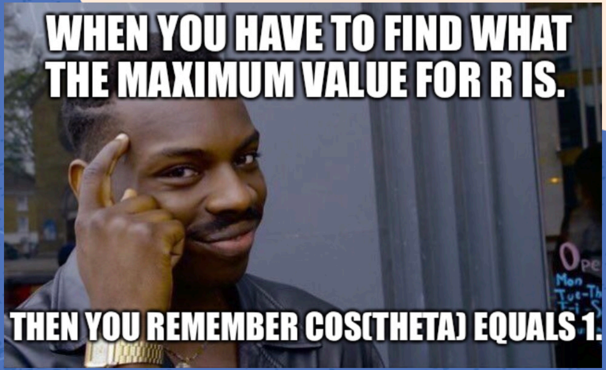
$r = 2+3\cos\theta$

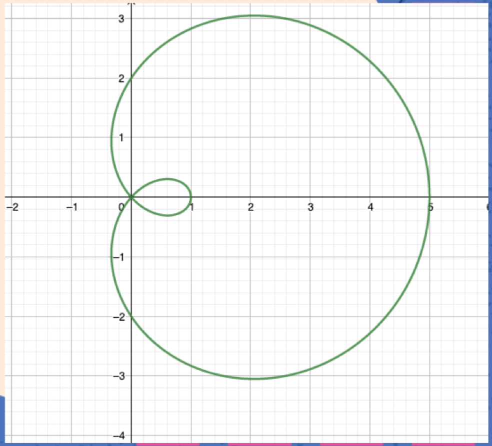
We know that $\cos\theta=1$

Substitute 1 in for $\cos\theta$

$r = 2+3(1) = 5$

$|r| = 5$





Nth Root of a Complex Number

A complex number v such that v^n=z

Equation

$$\sqrt[n]{r} = (cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n})$$

Where k= 0,1,2,……,n-1

Find the nth root of the complex number.
2+2i, n=3
r= $\sqrt{2^2 + 2^2} = \sqrt{8}$
$\sqrt[3]{\sqrt{8}}(cos\frac{45+2\pi k}{3} + i\sin\frac{45+2\pi k}{3}$
The number of answers is = to n

Answers

k= 0
$\sqrt{2}(cos15° + isin15°)$

k= 1
$\sqrt{2}(cos135° + isin135°)$
=(-1,+i)

k= 2
$\sqrt{2}(cos255° + isin255°)$

finally found the square root!

Orthogonal Vectors

Two Vectors **u** and **v** that u times v = 0

Prove that vectors u and v are orthogonal.

u = $< 2{,}3 >$

v = $< -6, 4 >$

Find their dot product

u · v = <2,3> · <-6, 4>

= -12 +12 = 0

The two vectors are orthogonal

Phase Shift

How far the function is shifted horizontally from the usual position.

If the **horizontal shift** is positive, the shifting moves to the right. If the horizontal shift is negative, the shifting moves to the left.

From the sinusoidal equation,

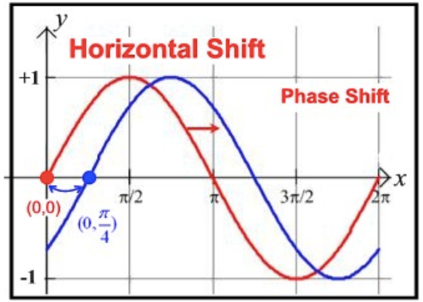
$$y = A\sin(B(x-C)) + D$$

the horizontal shift is obtained by determining the change being made to the x-value.

The **horizontal shift** is C.

In mathematics, a horizontal shift may also be referred to as a **phase shift**.*(see page end)

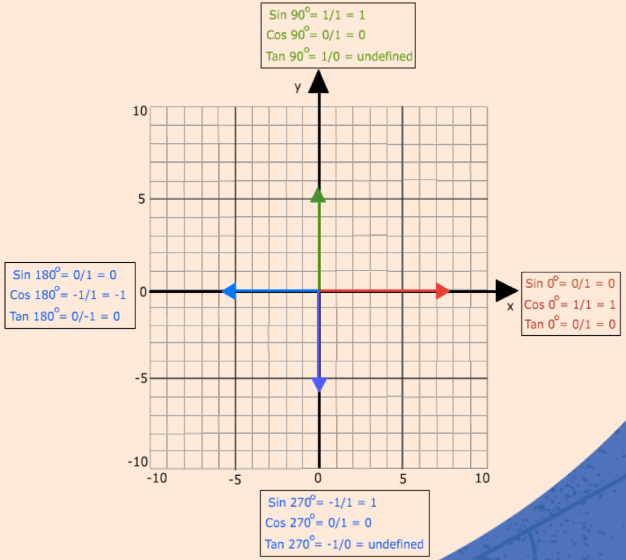
The easiest way to determine horizontal shift is to determine by how many units the "starting point" (0,0) of a standard sine curve, $y = \sin(x)$, has moved to the right or left.



$y = \sin(x)$ $\quad y = \sin\left(x - \frac{\pi}{4}\right)$

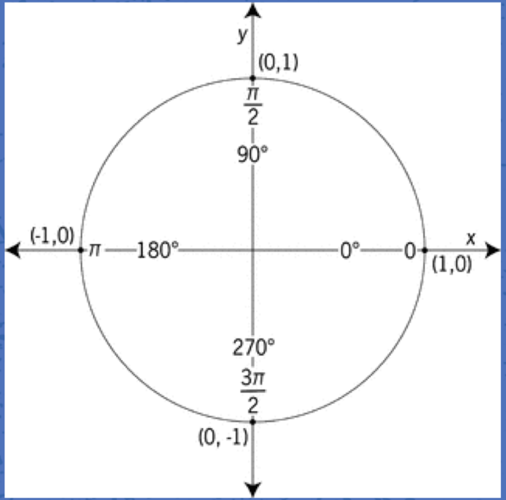
$$\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$$ shift sine to the left to create cosine

$$\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$$ shift cosine to the right to create sine



Quadrantal Angle

An angle in standard position whose terminal side lies on on an axis

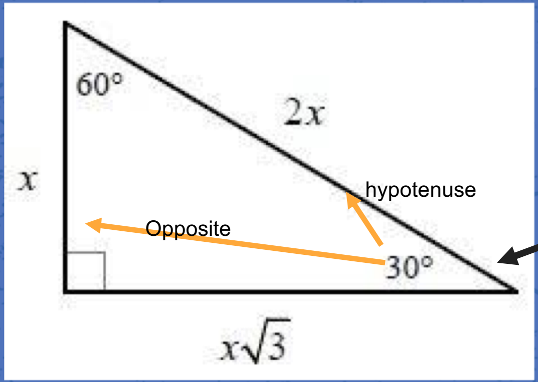


A quadrantal angle is one that is in the standard position and has a measure that is a multiple of 90° or π/2 radians. A quadrantal angle will have its terminal lying along an x or y axis.

Some examples of quadrantal angles are those at 0°, 90°, 180°, 270°, or 360°

Reference Triangles

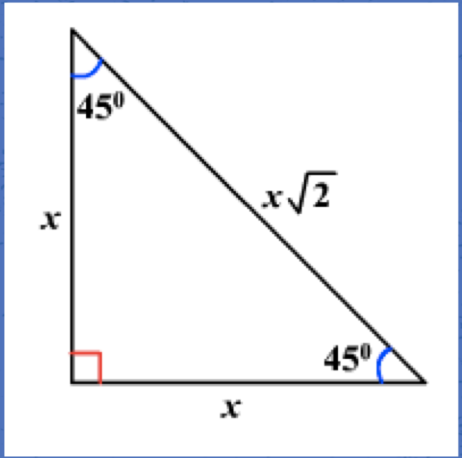
A triangle formed by the terminal side of angle θ, the x-axis, and a perpendicular dropped from point on the terminal side to the x-axis. The angle reference triangle at the origin is the reference angle.



Using one of the reference triangles find the sin(30°) where x equals 1

Using SOHCAHTOA we know that sin is opposite over hypotenuse.

$\sin(30°) = \frac{1}{2}$



Symmetric about the Y-axis

A graph in which $(-r,-\theta)$ or $(r,\pi-\theta)$ is on the graph whenever (r, θ) is

Test this equation to see if it is symmetric about the x-axis.

$r = 4\sin 3\theta$

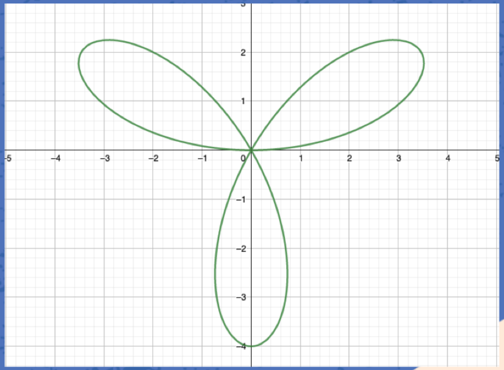
Replace (r, θ) with $(-r, -\theta)$

$-r = 4\sin 3(-\theta)$

$-r = -4\sin 3\theta$

$r = 4\sin 3\theta$

Since the bottom equation matches the original is it symmetric about the y-axis



Trigonometric Form of a Complex Number

r(cosΘ + isinΘ)

Find the trigonometric form of the complex number.

1 - $\sqrt{3}i$

First find what r equals

r^2 = $(1)^2 + (\sqrt{3})^2$ = 4

r = 2

Find what θ equals

θ = $\tan^{-1}\frac{b}{a}$ = $\tan^{-1}\frac{-\sqrt{3}}{1}$ = -60°

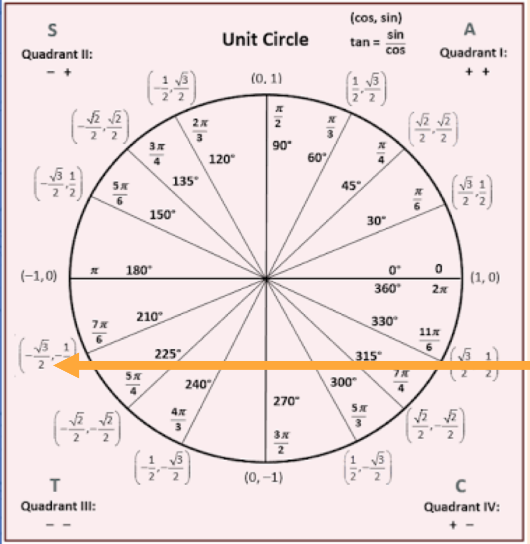
Now we can add 360°

$\theta = 300°$

Trigonometric Form Equals
2cos(300°) + 2isin(300°)

Unit Circle

A circle with radius 1 centered at the origin



Using the unit circle, figure out the cos($\frac{7\pi}{6}$)

The cos($\frac{7\pi}{6}$) = 210°

We know that 210° is a 30,60,90 triangle

Cos = adjacent over hypotenuse

Cos = 210° = $\frac{-\sqrt{3}}{2}$

Vertices of an Ellipse

The points where an ellipse intersects its focal axis

Center of (0,0)

- Vertices =
- (±a,0) or (0, ±a)

Center with (h,k)

- Vertices =
- (h±a, k) or (h, k±a)

If x is first   If y is first

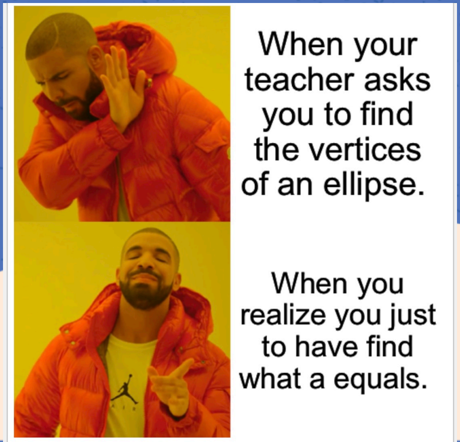
Find the vertices of the ellipse $\frac{y^2}{25} + \frac{x^2}{21} = 1$

The ellipse has a center of (0,0)

$a^2 = 25$

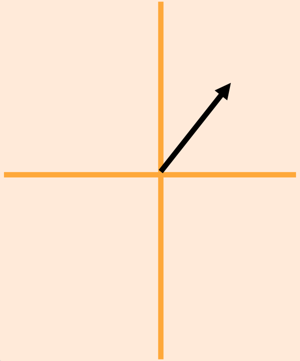
a = 5

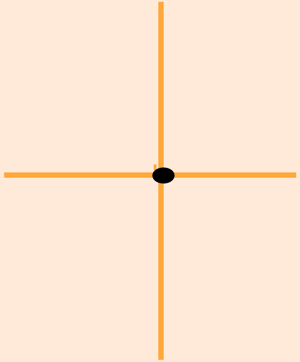
Vertices = (0, 5); (0, -5)



Zero Vector

The vector <0,0>, has zero length and no direction


Vector


Zero Vector


Now you know your Trig
ABC's, next time you can
solve with me