Mrs. Griffith

Pre-Calculus

Unit 4

Exponential
and
Logarithmic Functions

Name:

Date:

Topic:

Class:

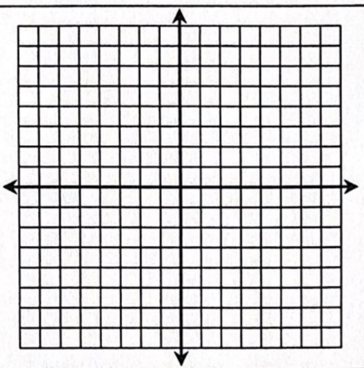
Main Ideas/Questions

Notes/Examples

EXPONENTIAL FUNCTION

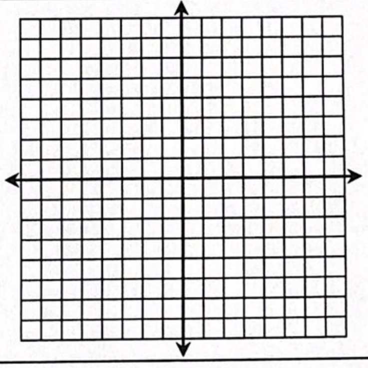
b is the **base** of the function

Graph $f(x) = 2^x$



When $a > 1$ and $b > 1$, the function is **increasing** and called an **exponential growth.**

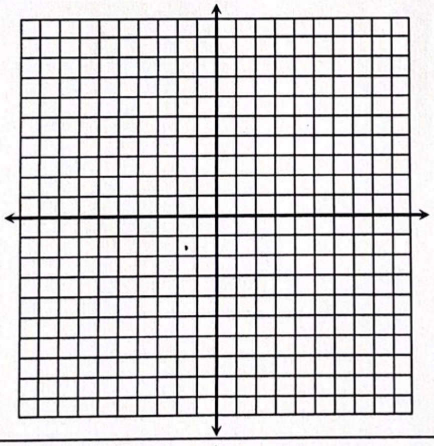
Graph $f(x) = \left(\frac{1}{2}\right)^x$



When $a < 1$ and $b < 1$, the function **decreasing** and called an **exponential decay.**

Directions: Classify the function as an exponential growth or decay, graph, then identify its key characteristics.

1. $f(x) = 3^x$



Growth or Decay?

Domain:

Range:

y-intercept:

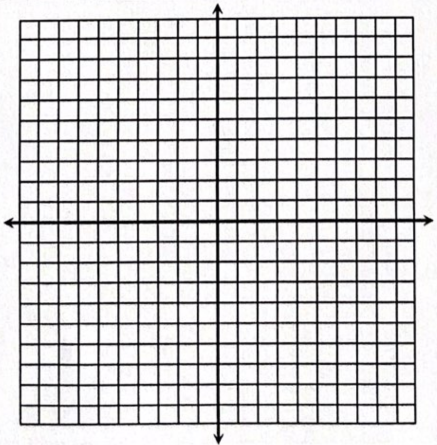
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

2. $f(x) = \left(\frac{1}{3}\right)^x$



Growth or Decay?

Domain:

Range:

y-intercept:

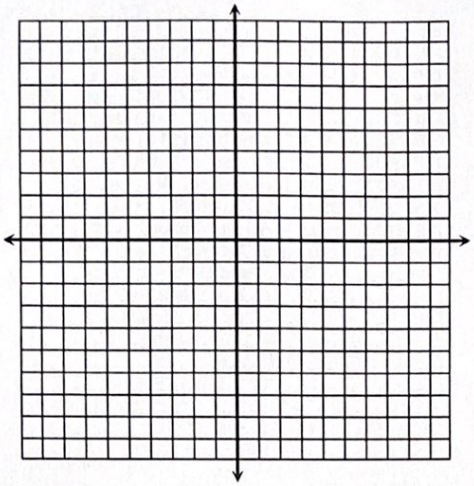
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

3. $f(x) = \left(\frac{2}{5}\right)^x$



Growth or Decay?

Domain:

Range:

y-intercept:

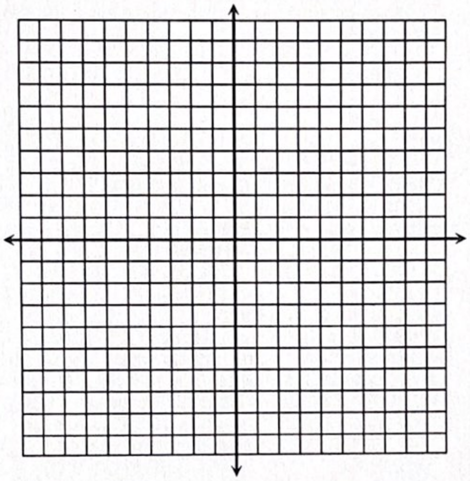
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

4. $f(x) = 4^x$



Growth or Decay?

Domain:

Range:

y-intercept:

Asymptote:

Increasing Interval:

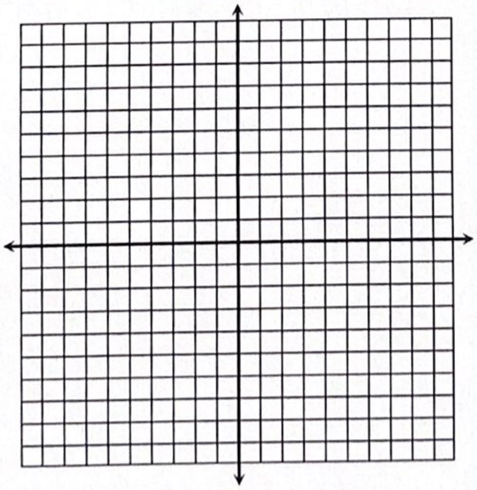
Decreasing Interval:

End Behavior:

Natural Base
EXPONENTIAL FUNCTION

- e is an ______________________ ______________________ with an approximate value of ______________________.
- Exponential functions with base e are called **natural base** exponential functions.
- Many real-world applications of exponential functions use base e.

Graph the function $f(x) = e^x$, then identify its key characteristics.



Growth or Decay?

Domain:

Range:

y-intercept:

Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

Name:

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Transformations of EXPONENTIAL FUNCTIONS

Recall the following transformations rules given a function $f(x)$:

Translations (Shifts)	**Reflections**	**Dilations** (compress/stretch)
$f(x+h)$ shifts left	$-f(x)$ reflects over the x-axis	$a \cdot f(x)$ is a vertical compression when $\lvert a \rvert < 1$ and a vertical stretch when $\lvert a \rvert > 1$
$f(x-h)$ shifts right		
$f(x)+k$ shifts up	$f(-x)$ reflects over the y-axis	$f(b \cdot x)$ is a horizontal stretch when $\lvert b \rvert < 1$ and a horizontal compression when $\lvert b \rvert > 1$
$f(x)-k$ shifts down		

Directions: (a) Identify the parent function, and **(b)** describe the transformations.

1. $f(x)=2^{x+1}-3$

2. $f(x)=-\left(\frac{1}{2}\right)^{x-5}+1$

3. $f(x)=-3\cdot 4^{x-2}-7$

4. $f(x)=\frac{4}{3}\cdot e^{-x}+5$

5. $f(x)=5\cdot 2^{-(x-4)}-9$

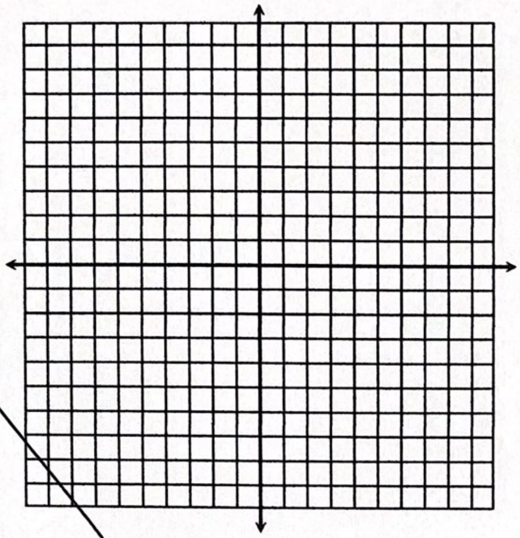
6. $f(x)=-\left(\frac{2}{3}\right)^{4x}$

7. $f(x)=-e^{\frac{1}{2}x}+4$

8. $f(x)=-\left(\frac{3}{4}\right)^{3(x+2)}-1$

Directions: Graph each function, then identify its key characteristics.

9. $f(x) = 3^{x-1} - 4$



Domain:

Range:

y-intercept:

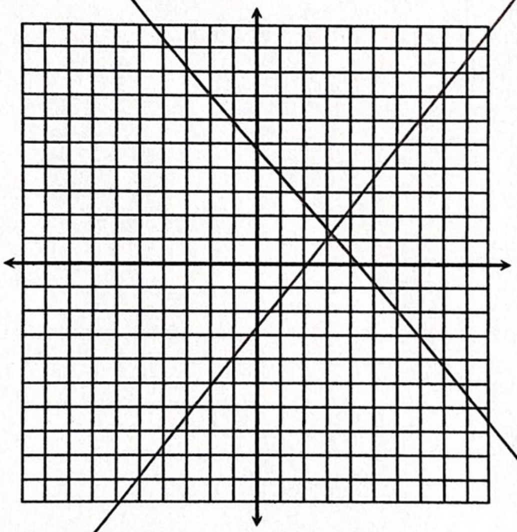
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

10. $f(x) = -\left(\frac{1}{2}\right)^{x} + 2$



Domain:

Range:

y-intercept:

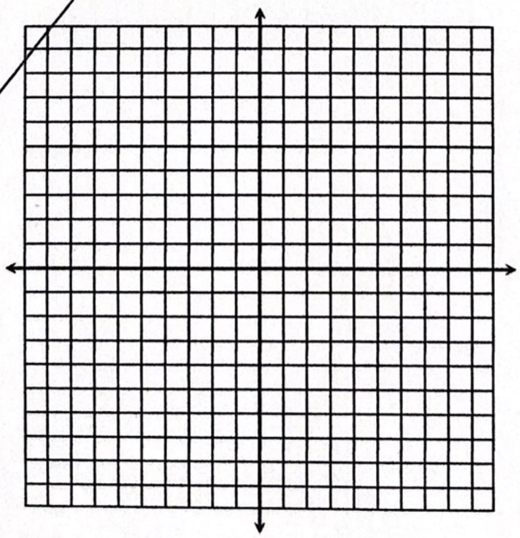
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

11. $f(x) = 3\left(\frac{1}{4}\right)^{x+5}$



Domain:

Range:

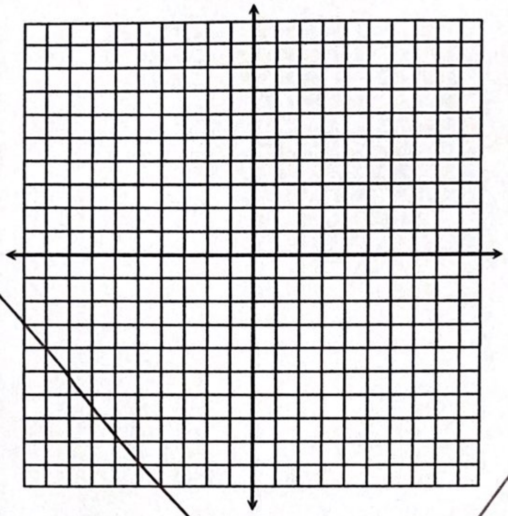
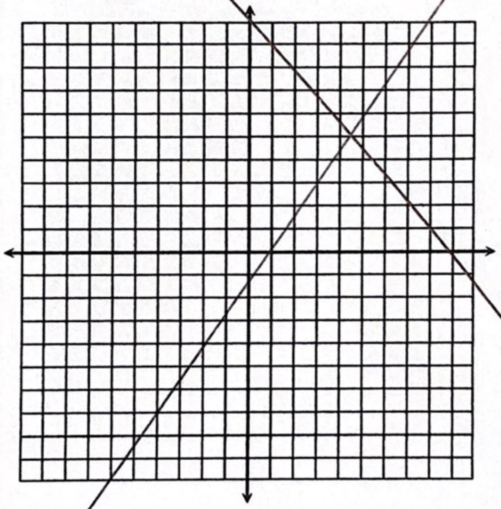
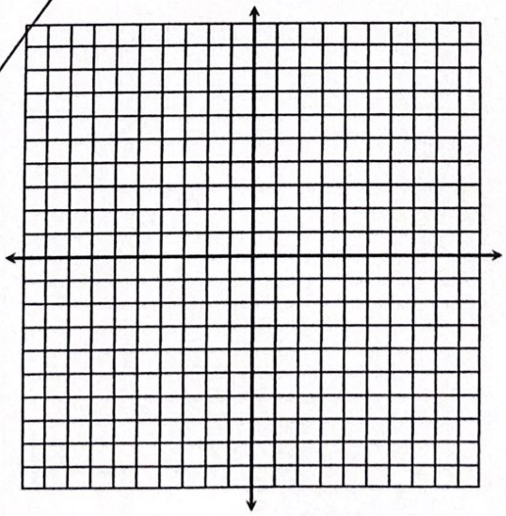
y-intercept:

Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

12. $f(x)=e^{-x}+3$		**Domain:** **Range:** **y-intercept:** **Asymptote:** **Increasing Interval:** **Decreasing Interval:** **End Behavior:**
13. $f(x)=-\left(\dfrac{1}{3}\right)^{2(x-1)}$		**Domain:** **Range:** **y-intercept:** **Asymptote:** **Increasing Interval:** **Decreasing Interval:** **End Behavior:**
14. $f(x)=e^{\frac{1}{3}(x+2)}-1$		**Domain:** **Range:** **y-intercept:** **Asymptote:** **Increasing Interval:** **Decreasing Interval:** **End Behavior:**

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Main Ideas/Questions	**Notes/Examples**

Exponential
GROWTH & DECAY

Exponential growth occurs when a quantity **exponentially increases** over time.	Exponential decay occurs when a quantity **exponentially decreases** over time.
EXPONENTIAL GROWTH FUNCTION:	**EXPONENTIAL DECAY FUNCTION:**

where a is the initial amount, r is the growth or decay rate (as a decimal), and t is the length of time

1. Brooke started her career with an annual salary of 32,000. Each year thereafter, her salary increased by 2.5%. Write and use an exponential growth function to find her salary when she retires after 30 years.

2. In 1995, a magazine had 14,000 subscribers. The number of subscribers increased by 40% each year thereafter. Write and use an exponential growth function to find the number of subscribers in 2016.

3. Kate drank an energy beverage with 150 milligrams of caffeine. Each hour the amount of caffeine in her system decreases by about 12%. Write and use an exponential decay function to find the amount of caffeine in her system after eight hours.

4. The half-life of Mercury-197 is 3 days. Write and use an exponential decay function to find the amount of Mercury-197 left from a 50-gram sample after 20 days.

Continuous GROWTH & DECAY

Sometimes a quantity is constantly increasing or decreasing at an exponential rate, and not just after each year, month, day, hour, etc. The formula to the right can be used to find the balance of the account in this case.

***r** is positive for growth models and negative for decay models

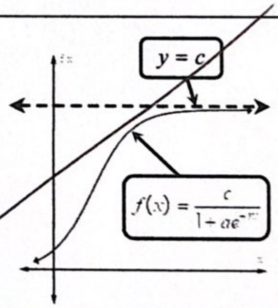
5. A garbage dumpster started with 4 pounds of garbage. The amount of garbage increased continuously by 35% each day from this point forward. Find the amount of garbage in the dumpster after two weeks.

6. The population of a town is declining at a continuous rate of 1.5%. If the current population is 16,000 people, find the population in 8 years.

LOGISTIC GROWTH Function

Sometimes a quantity exponential increases, but then levels out, approaching a horizontal asymptote. This is called a **logistic growth model.** The logistic growth function is given as:

$$f(x)=\frac{c}{1+ae^{-rx}}$$



7. A disease begins to spread in a town of 20,000 people. After t days, the number of people who have been infected by the disease is modeled by the function below. Using the function, find the number of people infected after 10 days.

$$f(t)=\frac{20{,}000}{1+1150e^{-0.95t}}$$

8. The population P, in millions, of a country from 1850 to 2000 is modeled by the equation below where t is the years since 1850. Using the function, find the population of the country in 1920.

$$P(t)=\frac{135}{1+58e^{-0.025t}}$$

Name:

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Main Ideas/Questions	**Notes/Examples**
compound interest	A common application of exponential growth is compound interest. Compound interest is interest paid on both the initial investment, called the principal, and on previously earned interest. **FORMULA:** A = ______ P = ______ r = ______ n = ______ t = ______
examples	**1.** Dave invests $300 in an account with a 5% interest rate. If he makes no other deposits or withdrawals, find his account balance after 15 years if the interest is compounded with the following frequencies. **a)** semiannually **b)** monthly **2.** If $2,500 is deposited into a savings account earning 8% annual interest, how much will be in the account at the end of 25 years if the interest is compounded with the following frequencies: **a)** quarterly **b)** daily **3.** When Amelia turned 6, her grandparents opened a college savings account for her with an initial deposit of $500. The account earns 3.2% interest compounded bimonthly. If her grandparents make no other deposits or withdrawals, how much money will be in the account when Amelia can access it at age 18?

4. Suppose a savings account offers a 0.4% interest rate compounded semiannually. If Samantha opens an account with $750 and makes no other deposits or withdrawals, how much interest will she have earned after 10 years?

5. In 1990, Carter deposited $1,000 in an investment account that earns $2\frac{3}{8}$% annual interest, compounded quarterly. If no other deposits or withdrawals were made, find the balance of his account in 2025.

CONTINUOUS COMPOUND INTEREST

In some cases, interest is compounded **continuously** meaning the account is constantly earning interest. The formula to the right can be used to find the balance of the account in this case.

FORMULA:

EXAMPLES

6. Suppose $800 is invested in an account at a 6% interest rate compounded continuously. If no other withdrawals or deposits are made, find the balance in the account after 20 years.

7. Find the balance of an account after 5 years if $1,200 is initially invested at an interest rate of 12.5% per year, compounded continuously and there are no other deposits or withdrawals.

Option A:
5.5% annual interest compounded monthly

Option B:
2.7% annual interest compounded continuously

8. Carla is investing $1,500 in a new 30-year retirement account. Determine which of the interest rates and compounding periods shown to the left would be her best investment option.

Name:

Date:

Topic:

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Main Ideas/Questions	**Notes/Examples**

What is a
LOGARITHM?

A logarithm (log) is another way of writing exponents.

Logarithmic Form

$\log_b a = x$

Exponential Form

Read as **"log base b of a equals x."**

COMMON LOGARITHM

A logarithm with base 10 is called a **common logarithm** and can be written without the base.

$\log_{10} x \rightarrow$

Converting
BETWEEN FORMS

Write each equation in exponential form.

1. $\log_7 49 = 2$

2. $\log_2 32 = 5$

3. $\log 1000 = 3$

4. $\log_4 \frac{1}{64} = -3$

5. $\log_8 2 = \frac{1}{3}$

6. $\log_9 27 = \frac{3}{2}$

Write each equation in logarithmic form.

7. $5^2 = 25$

8. $8^0 = 1$

9. $3^{-4} = \frac{1}{81}$

10. $12^{\frac{1}{2}} = 2\sqrt{3}$

11. $10^{-1} = \frac{1}{10}$

12. $16^{\frac{3}{4}} = 8$

Evaluating
LOGARITHMS

Evaluate the following logarithms using your knowledge of exponents.

13. $\log_6 36$

14. $\log_2 128$

15. $\log \frac{1}{100}$

16. $\log_{16} 2$

17. $\log_3 \frac{1}{27}$

18. $\log_8 1$

CHANGE OF BASE FORMULA

Some logarithms are not as easy to evaluate as those above, and will require the **change of base formula**.

$\log_b a =$

Choose BASE 10 because there is a calculator button for it! →

Approximate each logarithm using the change of base formula.

19. $\log_5 34$

20. $\log_2 98$

21. $\log_{20} 4$

22. $\log_6 2$

23. $\log_3 225$

24. $\log_8 \frac{1}{2}$

NATURAL LOGARITHM

A logarithm with base e is called a **natural logarithm** and is written as ln.

$\log_e x \rightarrow$

Write each equation in exponential form.

25. $\ln x = 4$

26. $\ln 10 = x$

Write each equation in logarithmic form.

27. $e^x = 50$

28. $e^{0.5} = x$

Approximate the value of each expression.

Use the ln button on the calculator to evaluate natural logarithms. →

29. $\ln 64$

30. $\ln \frac{1}{3}$

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples
PRODUCT Property $\log_b(m\cdot n)=$	**Condense into a single logarithm.** **1.** $\log_3 9+\log_3 5$ **2.** $\log 6+\log(x-3)$ **3.** $\ln 4x^2+\ln 3x^3$ **Expand using the product property.** **4.** $\log 72$ **5.** $\ln\frac{9}{10}$ **6.** $\log_5(x^2-4)$
QUOTIENT Property $\log_b\left(\frac{m}{n}\right)=$	**Condense into a single logarithm.** **7.** $\ln 96-\ln 6$ **8.** $\log(8x^{10})-\log(4x^2)$ **9.** $\log_7\sqrt{40}-\log_7\sqrt{5}$ **Expand using the quotient property.** **10.** $\log_3 8$ **11.** $\ln\frac{3}{4}$ **12.** $\log\left(\frac{x-7}{x+1}\right)$
POWER Property $\log_b m^n=$	**Condense into a single logarithm. Simplify if possible.** **13.** $3\cdot\log 6$ **14.** $\frac{1}{2}\cdot\log_4 81$ **15.** $(x-1)\cdot\ln 4$ **Expand using the power property.** **16.** $\log_2 5^x$ **17.** $\ln\sqrt[3]{2x+1}$ **18.** $\log_6\frac{1}{64}$

USING THE PROPERTIES OF LOGARITHMS

CONDENSING LOGS

Directions: Condense each expression into a single logarithm.

19. $2 \cdot \log_4 9 + 3 \cdot \log_4 2$

20. $\log 80 - 2 \cdot \log 4$

21. $3 \cdot \ln(pq) + 4 \cdot \ln(pq^2)$

22. $\frac{1}{2}(\log_5 x^6 + \log_5 x^3)$

23. $\frac{2}{3} \cdot \ln 64 - 2 \cdot \ln 8$

24. $-2\left(\log 15 + \frac{1}{2} \cdot \log \frac{1}{9}\right)$

25. $\log 72 - \frac{1}{3}(2 \cdot \log 4 + \log 32)$

26. $\frac{3}{2} \cdot \log_3 a - \frac{1}{4} \cdot \log_3 (16a^2)$

EXPANDING LOGS

Directions: Expand each logarithm completely.

27. $\log_7 (xy^3)$

28. $\ln\left(\frac{m^3}{n^7}\right)$

29. $\log\sqrt{a^3 b}$

30. $\log_4 \left(\frac{c^2}{d}\right)^4$

31. $\ln(5p^3q^4)^2$

32. $\log_2 \frac{\sqrt[3]{x}}{x^2 + x}$

LOGARITHMS *Reference Sheet*

COMMON LOGARITHM A **base 10** logarithm, written as:		**NAUTRAL LOGARITHM** A **base *e*** logarithm, written as:	
PROPERTY	**RULE**	**EXAMPLE 1**	**EXAMPLE 2**
BASIC PROPERTIES	$\log_b 1 =$ $\log_b b =$	**Simplify:** $\log_6 1 =$	**Simplify:** $\log_4 4 =$
PRODUCT PROPERTY	$\log_b (m \cdot n) =$	**Condense:** $\log_3 8 + \log_3 (3x) =$	**Expand:** $\ln(x^2 - x - 2) =$
QUOTIENT PROPERTY	$\log_b \left(\frac{m}{n}\right) =$	**Condense:** $\log 80 - \log 16 =$	**Expand:** $\log_2 \left(\frac{x}{5}\right)$
POWER PROPERTY	$\log_b m^n =$	**Condense:** $3 \cdot \ln 9 =$	**Expand:** $\log_7 3^{2x} =$
CHANGE OF BASE FORMULA	$\log_b a =$	**Evaluate:** $\log_5 138 =$	**Evaluate:** $\log_{14} 2 =$

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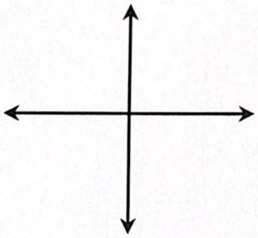
Main Ideas/Questions	**Notes/Examples**

LOGARITHMIC FUNCTION

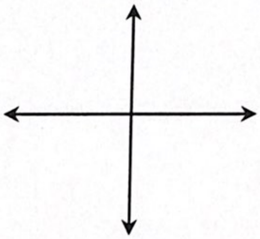
b is the **base** of the function

A logarithmic function is the **inverse** of an exponential function. Using your graphing calculator, sketch the following graphs:

$f(x) = \log x$



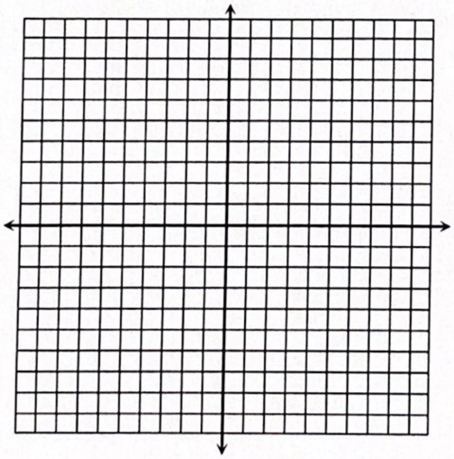
$f(x) = 10^x$



To graph a logarithmic function, you can use the inverse exponential function, then invert the values from the table to graph the logarithmic function.

Directions: Graph each function and identify its key characteristics.

1. $f(x) = \log_3 x$



Domain:

Range:

***x*-intercept:**

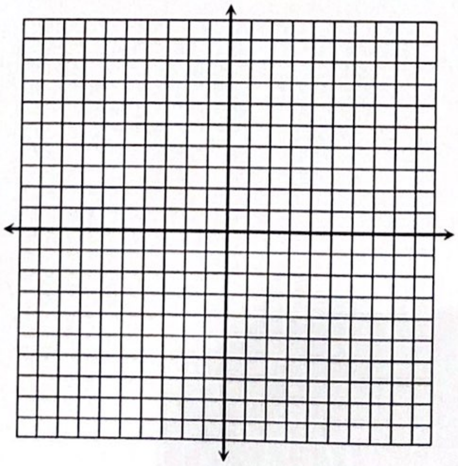
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

2. $f(x) = \log_{\frac{1}{2}} x$



Domain:

Range:

***x*-intercept:**

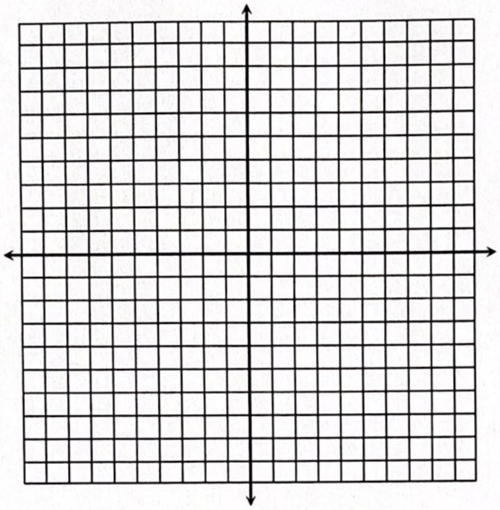
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

3. $f(x) = \ln x$



Domain:

Range:

x-intercept:

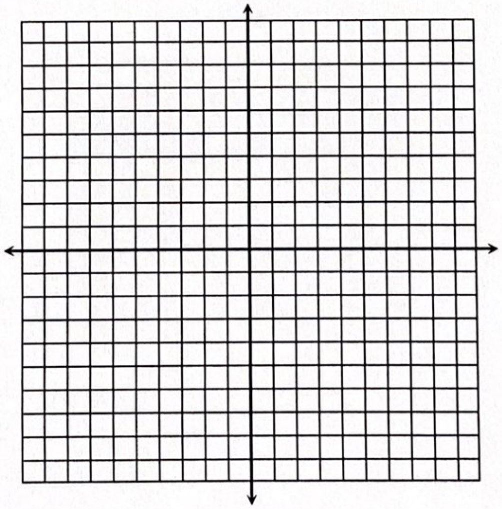
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

4. $f(x) = \log_{\frac{1}{3}} x + 1$



Domain:

Range:

x-intercept:

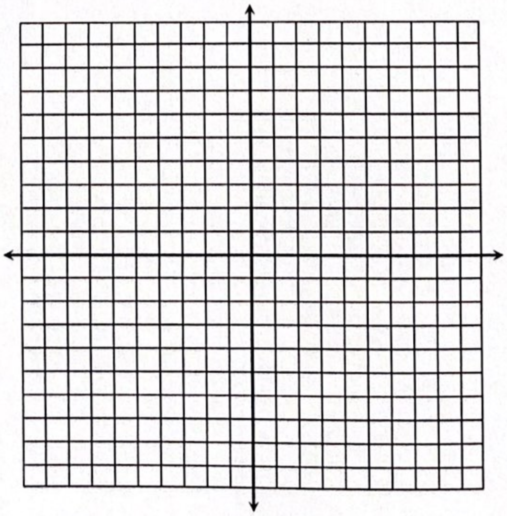
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

5. $f(x) = -\log_2(x-2)$



Domain:

Range:

x-intercept:

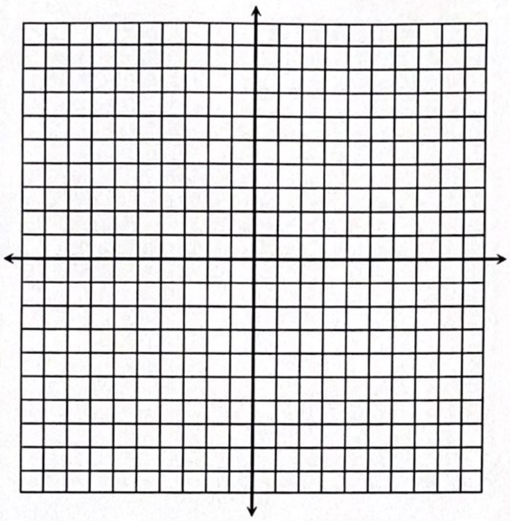
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

6. $f(x) = \log_4(x+5) - 2$



Domain:

Range:

x-intercept:

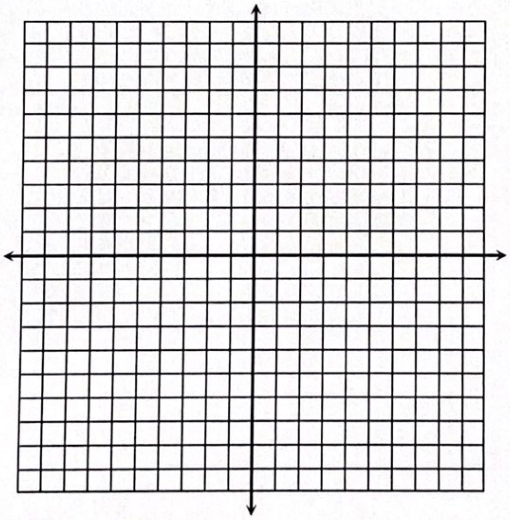
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

7. $f(x) = \log_{\frac{1}{2}}(-x) + 3$



Domain:

Range:

x-intercept:

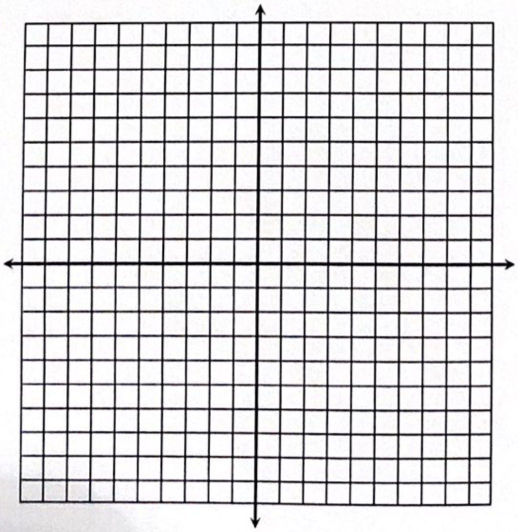
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

8. $f(x) = 3 \cdot \ln x - 1$



Domain:

Range:

x-intercept:

Asymptote:

Increasing Interval:

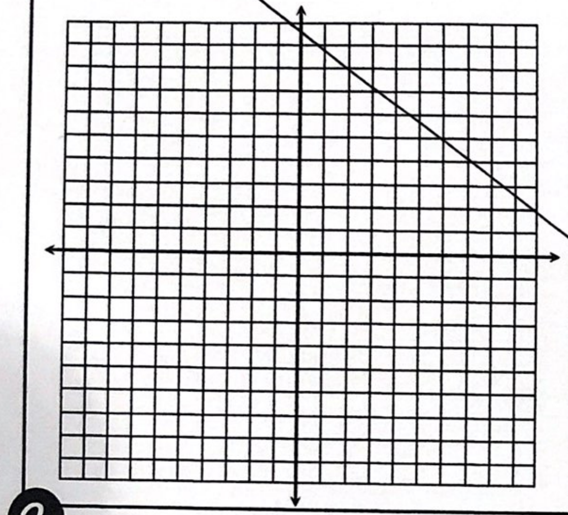
Decreasing Interval:

End Behavior:

EXPONENTIAL VS. LOGARITHMIC functions

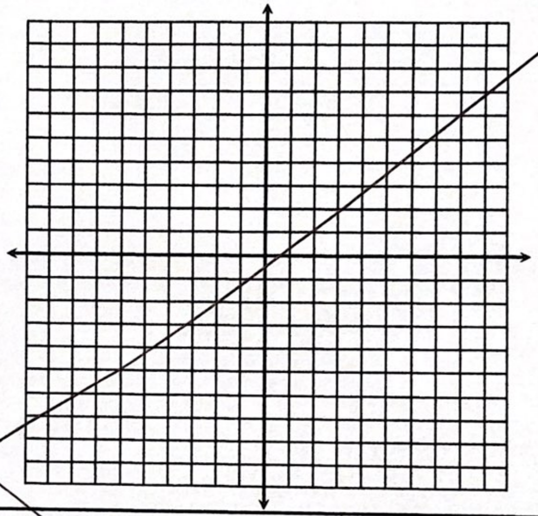
1

$f(x) = 3^{x-2} + 1$



Domain:
Range:
y-intercept:
Asymptote:
Inc. Interval:
Dec. Interval:
End Behavior:

$f(x) = \log_3(x-1) + 2$



Domain:
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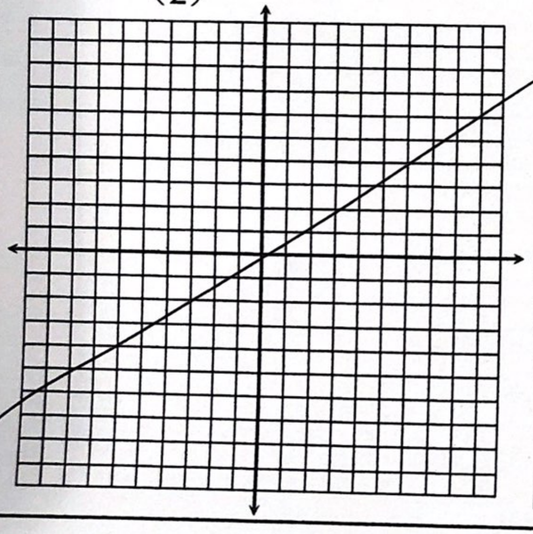
Asymptote:
Inc. Interval:
Dec. Interval:
End Behavior:

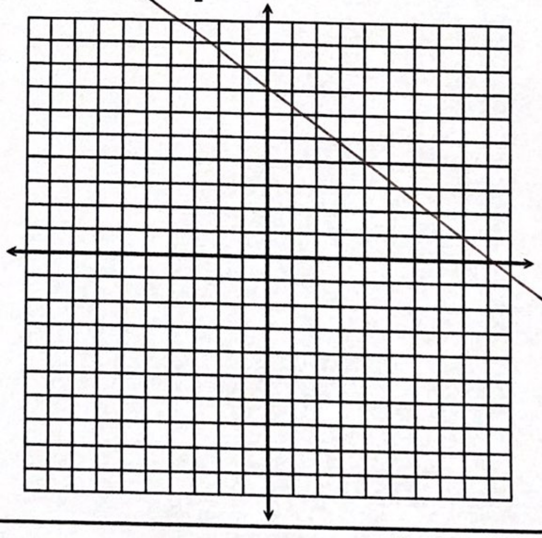
2

$f(x) = \left(\frac{1}{2}\right)^{-x} - 5$



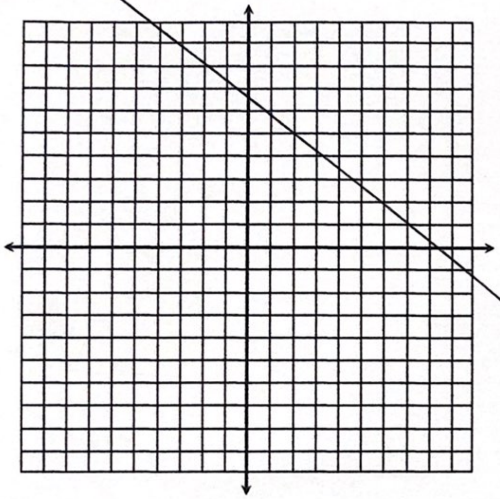
Domain:
Range:
y-intercept:
Asymptote:
Inc. Interval:
Dec. Interval:
End Behavior:

$f(x) = -\log_{\frac{1}{2}}(x+5)$



Domain:
Range:
x-intercept:
Asymptote:
Inc. Interval:
Dec. Interval:
End Behavior:

③ $f(x) = 2^{3(x+1)} - 4$



Domain:

Range:

y-Intercept:

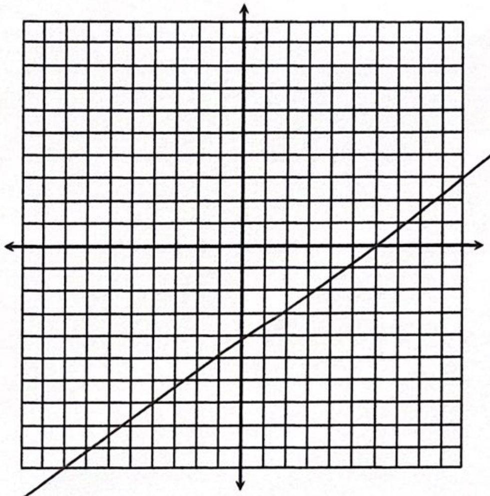
Asymptote:

Inc. Interval:

Dec. Interval:

End Behavior:

$f(x) = \frac{1}{3}\log_2(x+4) - 1$



Domain:

Range:

x-Intercept:

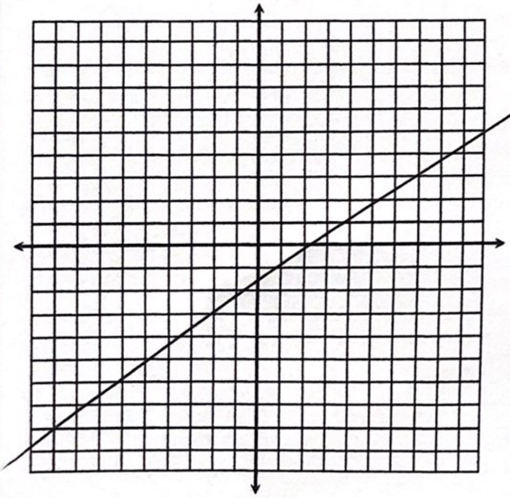
Asymptote:

Inc. Interval:

Dec. Interval:

End Behavior:

④ $f(x) = -\frac{1}{4}e^{x-3}$



Domain:

Range:

y-Intercept:

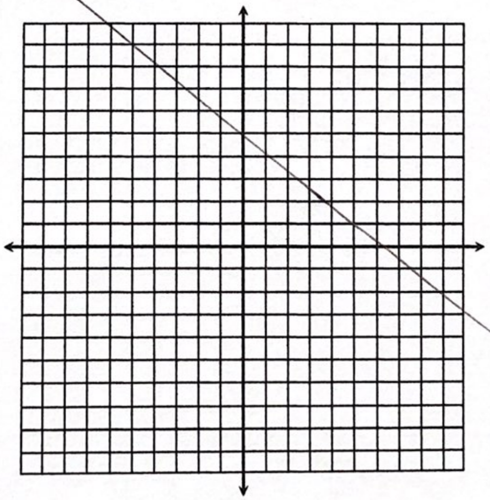
Asymptote:

Inc. Interval:

Dec. Interval:

End Behavior:

$f(x) = \ln(-4x) + 3$



Domain:

Range:

x-Intercept:

Asymptote:

Inc. Interval:

Dec. Interval:

End Behavior:

Name:

Date:

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Main Ideas/Questions	**Notes/Examples**
solving **EXPONENTIAL EQUATIONS** (using a common base)	**Steps to solve an exponential equation using a common base:** ❶ Rewrite the equation using a common base. ❷ Use the properties of exponents to simplify each side of the equation. ❸ Use the one-to-one property: **If** $b^x = b^y$**, then** ❹ Solve!
SET 1: WITH A COMMON BASE	**1.** $3^{2x-9} = 3^7$ **2.** $e^{4w-1} = e^{5-2w}$ **3.** $5^{c-1} \cdot 5^{3c+2} = 5^{7c+16}$ **4.** $8^{k^2+k} \cdot 8^{2k-9} = 8^{4k} \cdot 8^{11}$
SET 2: WITHOUT A COMMON BASE	**5.** $9^{4y-26} = 81$ **6.** $\frac{1}{64} = 4^{a^2-4}$ **7.** $2^{m-9} = 32^{m+3}$ **8.** $\left(\frac{1}{12}\right)^{4x+3} \cdot 12^{x^2} = 12^{2x+13}$

9. $343^{2n-4} = \left(\frac{1}{49}\right)^{n+4}$

10. $8^{2-v} = 128^{v+3}$

11. $9^{-p} = 243^{2p+6}$

12. $\left(\frac{1}{4}\right)^{2x^2-6} = \left(\frac{1}{64}\right)^{2x^2-5}$

13. $36^{2r} \cdot \frac{1}{36} = 216$

14. $25^{a} \cdot 625^{a+3} = \frac{1}{25}$

15. $\left(\frac{1}{16}\right)^{x+7} \cdot 64^{x+5} = \left(\frac{1}{32}\right)^{4}$

16. $512^{y^2} = \left(\frac{1}{64}\right)^{3y} \cdot \left(\frac{1}{8}\right)^{5y^2}$

Name:

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Main Ideas/Questions

Notes/Examples

Solving

LOGARITHMIC EQUATIONS

Type 1:
log = log

❶ Condense the logarithms on each side of the equation.

❷ Use the one-to-one property: **If $\log_b x = \log_b y$, then**

❸ Solve and check for extraneous solutions.

Directions: Solve each equation. Check for extraneous solutions.

1. $\log_3(7x-1) = \log_3(5x+17)$

2. $\ln(k^2-4k) = \ln(k+14)$

3. $\log 4 + \log_6(c+3) = \log 8$

4. $\log_7(w+6) - \log_7(5w-3) = \frac{1}{3}\cdot\log_7 8$

5. $\log_4(4p+3) = \frac{1}{2}\cdot\log_4(16p^4)$

6. $2\cdot\ln(a+1) = \frac{3}{2}(\ln 80 - \ln 5)$

solving

LOGARITHMIC EQUATIONS

Type 2:

log = number

➊ Condense and isolate the logarithm.

➋ Rewrite the equation in exponential form.

➌ Solve and check for extraneous solutions.

Directions: Solve each equation. Check for extraneous solutions.

7. $\log_2(3x-4)=7$

8. $\ln 2a=9$

9. $\log_6(w+7)-5=-3$

10. $2\cdot\log_9(k^2+2k)+4=5$

11. $\log_4(2v+3)+\log_4(2v-3)=2$

12. $\frac{1}{3}\cdot\ln 27+\ln(x-5)=4$

13. $\log_2(n-3)+\log_2(n+1)=5$

14. $\log_2 c^2-\log_2(3c-5)=2$

Name:

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Main Ideas/Questions	Notes/Examples
Solving EXPONENTIAL EQUATIONS (using logarithms)	**If using a common base is not possible, exponential equations can be solved using logarithms.** ① Isolate the exponential expression. ② Take the logarithm of each side. ③ Expand the logarithms if necessary using the power rule. ④ Solve and check for extraneous solutions.

EXAMPLES

1. $3^x = 80$

2. $e^x = 140$

3. $5^{x+1} = 18$

4. $\left(\frac{1}{3}\right)^{2x-5} = 120$

5. $e^{3x} + 25 = 108$

6. $-2 \cdot 8^{4-x} = -50$

	7. $\frac{2}{3}\cdot 2^{x-6}-1=41$	**8.** $-2\cdot 4^{2x+7}+9=-55$
	9. $2^{x+5}=3^{x-2}$	**10.** $8^{2x-1}=5^{x+3}$
	11. $4^{x-3}=11^{3x+2}$	**12.** $9^{-2x}=2^{5x+4}$

LOGARITHMIC & EXPONENTIAL EQUATIONS Review!

LOGARITHMIC EQUATIONS

1. $\log_7(x+13) = \log_7(3-x)$

2. $\log_2(n^2+13) = \log_2(n-1) + \log_2(n+3)$

3. $2 \cdot \ln(a+3) = \frac{1}{4} \cdot \ln 16 + \ln(a+7)$

4. $\log(3c+4) - \log(c-6) = \log(c+6)$

5. $\log_2(5v+23) - 9 = -2$

6. $\log_{16}(p+5) - \log_{16}(p-2) = \frac{1}{2}$

7. $\ln(r+1) + 3 \cdot \ln 2 = 7$

8. $\frac{1}{3} \cdot \log_9 64 + 2 \cdot \log_9 n = 2$

EXPONENTIAL EQUATIONS

9. $\left(\frac{1}{27}\right)^{2x-6} = 9^{x-1}$

10. $4^{3m+1} = \left(\frac{1}{8}\right)^{m+4} \cdot 32^{m-2}$

11. $5^{w-1} = 90$

12. $e^{3r-2} - 16 = 120$

13. $-4 \cdot 9^{2k+5} + 14 = 6$

14. $\frac{2}{3} \cdot 5^{m-8} - 9 = 21$

15. $3^{4x+1} = 8^{x-5}$

16. $4^{2x+3} = 7^{15-2x}$

Name:

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Main Ideas/Questions	**Notes/Examples**
EXPONENTIAL GROWTH & DECAY Applications	**1.** Mark started a new blog to write about his travels. In its initial week, the blog had 800 readers. From this point on, the number of readers each week increased by 25%. Use an exponential growth model to find the week in which the number of visitors reaches 10,000 people.
	2. A new stock entered the stock market in January 2012 at \$0.72 per share. Four years later, the price per share was \$3.85. Using a continuous exponential growth model, find the growth rate.
	3. A certain medicine has a half-life of 5 hours. If a patient is given 500-mg at noon, at what time will they have 100-mg remaining in their bloodstream?
LOGISTIC GROWTH Applications	**4.** The growth of a plant can be modeled by the equation below where h is the height of the plant (in centimeters) and t is the number of weeks since the seed was planted. How many weeks will it take the plant to reach a height of 50 centimeters? $h = \frac{250}{1+12e^{-0.72t}}$

5. A small lake is stocked with a certain species of fish. The fish population is modeled by the equation below, where P is the number of fish in thousands, and t is the number of years since the lake was stocked. How many years will it take the fish population to reach 6,000 fish?

$$P = \frac{8}{1+3e^{-0.7t}}$$

COMPOUND INTEREST
Applications

6. If $500 is deposited into a retirement account that pays an annual interest rate of 4% compounded quarterly, how long will it take the account to reach a balance of $1,200 if there are no other deposits and withdrawals?

7. Dave deposited $500 into a savings account. After 15 years, the account balance had tripled with no other deposits or withdrawals. Assuming the interest compounds continuously, find the interest rate.

8. Scarlet invested $8,000 in an account that pays 7.5% interest per year, compounded continuously. How long will it take the account to reach $10,000?

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Main Ideas/Questions

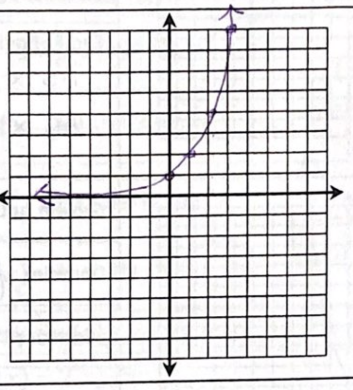
Notes/Examples

EXPONENTIAL FUNCTION

$f(x) = b^x$

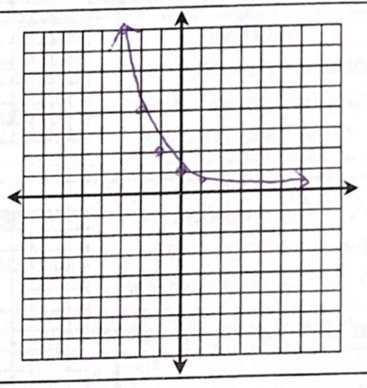
b is the **base** of the function

Graph $f(x) = 2^x$



When $a > 1$ and $b > 1$, the function is **increasing** and called an **exponential growth.**

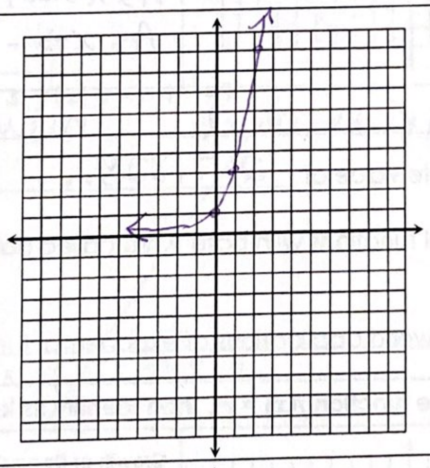
Graph $f(x) = \left(\frac{1}{2}\right)^x$



When $a < 1$ and $b < 1$, the function **decreasing** and called an **exponential decay.**

Directions: Classify the function as an exponential growth or decay, graph, then identify its key characteristics.

1. $f(x) = 3^x$



Growth or Decay? growth

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

***y*-intercept:** 1

Asymptote: $y = 0$

Increasing Interval: $(-\infty, \infty)$

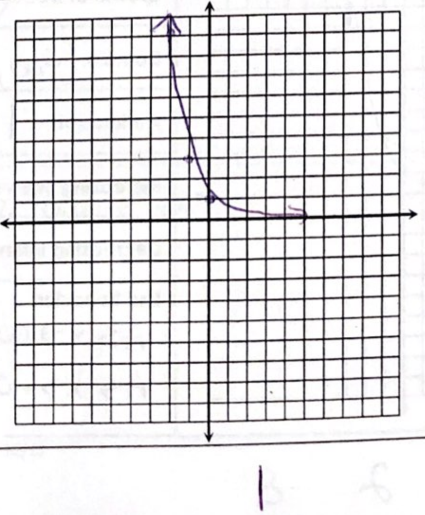
Decreasing Interval: none

End Behavior:

As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $F(x) \to 0$

2. $f(x) = \left(\frac{1}{3}\right)^x$



Growth or Decay? Decay

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

***y*-intercept:** 1

Asymptote: $y = 0$

Increasing Interval: none

Decreasing Interval: $(-\infty, \infty)$

End Behavior:

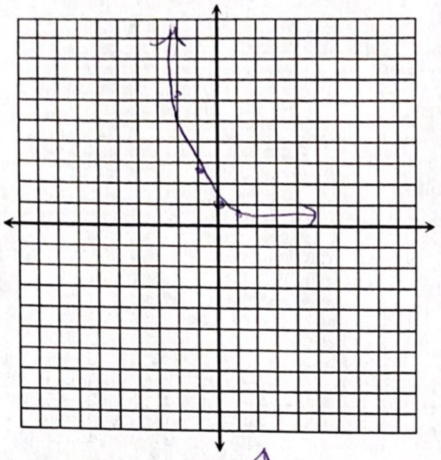
As $x \to \infty$, $y \to 0$

As $x \to -\infty$, $y \to \infty$

1

3. $f(x) = \left(\frac{2}{5}\right)^x$

$\frac{25}{4} = 6\frac{1}{4}$



Growth or Decay? Decay

Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$

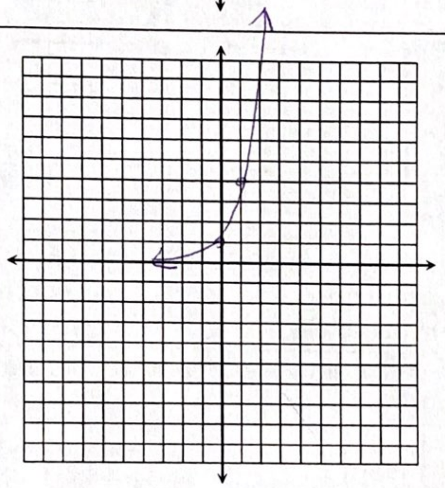
***y*-intercept:** 1 **Asymptote:** $y=0$

Increasing Interval: none

Decreasing Interval: $(-\infty, \infty)$

End Behavior:
As $x \to \infty, y \to 0$
As $x \to -\infty, y \to \infty$

4. $f(x) = 4^x$



Growth or Decay? Growth

Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$

***y*-intercept:** 1 **Asymptote:** $y=0$

Increasing Interval: $(-\infty, \infty)$

Decreasing Interval: none

End Behavior:
As $x \to \infty, y \to \infty$
As $x \to -\infty, y \to 0$

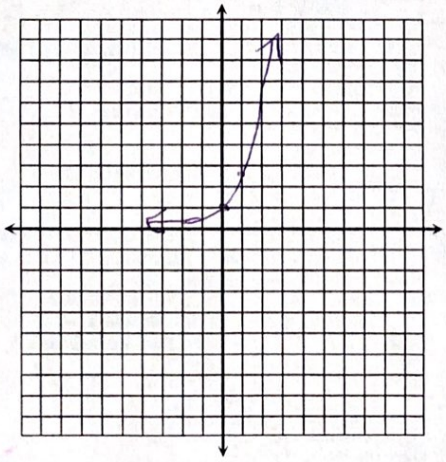
Natural Base

EXPONENTIAL FUNCTION

$\boxed{f(x) = e^x}$

- *e* is an irrational number with an approximate value of 2.71828....
- Exponential functions with base *e* are called **natural base** exponential functions.
- Many real-world applications of exponential functions use base *e*.

Graph the function $f(x) = e^x$, then identify its key characteristics.



Growth or Decay? Growth

Domain: $(-\infty, \infty)$ **Range:** $(0, \infty)$

***y*-intercept:** 1 **Asymptote:** $y=0$

Increasing Interval: $(-\infty, \infty)$

Decreasing Interval: none

End Behavior:
As $x \to \infty, y \to \infty$
As $x \to -\infty, y \to 0$

Conti

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Main Ideas/Questions	**Notes/Examples**

Exponential **GROWTH & DECAY**

Exponential growth occurs when a quantity **exponentially increases** over time.	Exponential decay occurs when a quantity **exponentially decreases** over time.
EXPONENTIAL GROWTH FUNCTION: $f(t)=a(1+r)^t$	**EXPONENTIAL DECAY FUNCTION:** $f(t)=a(1-r)^t$

where ***a*** is the initial amount, ***r*** is the growth or decay rate (as a decimal), and ***t*** is the length of time

1. Brooke started her career with an annual salary of 32,000. Each year thereafter, her salary increased by 2.5%. Write and use an exponential growth function to find her salary when she retires after 30 years.

$f(t)=32{,}000(1+.025)^t$

$f(30)=32{,}000(1+.025)^{30}$

$=\$67{,}122.16$

2. In 1995, a magazine had 14,000 subscribers. The number of subscribers increased by 40% each year thereafter. Write and use an exponential growth function to find the number of subscribers in 2016.

$f(t)=14{,}000(1+.4)^t$

$f(21)=14{,}000(1+.4)^{21}$

$=16{,}398{,}978$ students

3. Kate drank an energy beverage with 150 milligrams of caffeine. Each hour the amount of caffeine in her system decreases by about 12%. Write and use an exponential decay function to find the amount of caffeine in her system after eight hours.

$f(t)=150(1-.12)^t$

$f(8)=150(1-.12)^8$

$=53.95$ mg

4. The half-life of Mercury-197 is 3 days. Write and use an exponential decay function to find the amount of Mercury-197 left from a 50-gram sample after 20 days.

$f(t)=50(1-.5)^t$

$f(20/3)=50(1-.5)^{20/3}$

$=.49$ g

Continuous GROWTH & DECAY

Sometimes a quanitity is constantly increasing or decreasing at an exponential rate, and not just after each year, month, day, hour, etc. The formula to the right can be used to find the balance of the account in this case.

$A = Pe^{rt}$

*r is positive for growth models and negative for decay models

5. A garbage dumpster started with 4 pounds of garbage. The amount of garbage increased continuously by 35% each day from this point forward. Find the amount of garbage in the dumpster after two weeks.

$A = 4e^{.35t}$

$A = 4e^{.35(14)}$

$A = 537.16$ lb

6. The population of a town is declining at a continuous rate of 1.5%. If the current population is 16,000 people, find the population in 8 years.

$A = 16,000e^{-.015t}$

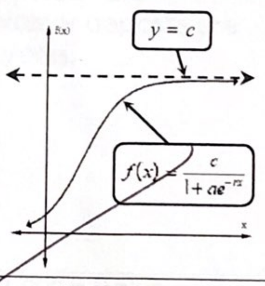
$A = 16,000e^{-.015(8)}$

$A = 14190$ people

LOGISTIC GROWTH Function

Sometimes a quantity exponential increases, but then levels out, approaching a horizontal asymptote. This is called a **logistic growth model**. The logistic growth function is given as:

$$f(x) = \frac{c}{1 + ae^{-rx}}$$



7. A disease begins to spread in a town of 20,000 people. After t days, the number of people who have been infected by the disease is modeled by the function below. Using the function, find the number of people infected after 10 days.

$$f(t) = \frac{20,000}{1 + 1150e^{-0.95t}}$$

8. The population P, in millions, of a country from 1850 to 2000 is modeled by the equation below where t is the years since 1850. Using the function, find the population of the country in 1920.

$$P(t) = \frac{135}{1 + 58e^{-0.025t}}$$

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Main Ideas/Questions	**Notes/Examples**
COMPOUND INTEREST	A common application of exponential growth is compound interest. Compound interest is interest paid on both the initial investment, called the principal, and on previously earned interest.
	FORMULA: $A=P(1+\frac{r}{n})^{nt}$
	A = total balance P = principal (initial) amount r = rate n = # times compounded yearly t = time in years
EXAMPLES	**1.** Dave invests \$300 in an account with a 5% interest rate. If he makes no other deposits or withdrawals, find his account balance after 15 years if the interest is compounded with the following frequencies.
	a) semiannually $A=300(1+\frac{.05}{2})^{2(15)}$ $A=\$629.27$
	b) monthly $A=300(1+\frac{.05}{12})^{12(15)}$ $A=\$634.11$
	2. If \$2,500 is deposited into a savings account earning 8% annual interest, how much will be in the account at the end of 25 years if the interest is compounded with the following frequencies:
	a) quarterly $A=2500(1+\frac{.08}{4})^{4(25)}$ $A=\$18111.62$
	b) daily $A=2500(1+\frac{.08}{365})^{365(25)}$ $A=\$1 18468.58$
	3. When Amelia turned 6, her grandparents opened a college savings account for her with an initial deposit of \$500. The account earns 3.2% interest compounded bimonthly. If her grandparents make no other deposits or withdrawals, how much money will be in the account when Amelia can access it at age 18? $A=500(1+\frac{.032}{24})^{24(12)}$ $A=\$733.88$

4. Suppose a savings account offers a 0.4% interest rate compounded semiannually. If Samantha opens an account with $750 and makes no other deposits or withdrawals, how much interest will she have earned after 10 years?

$A = 750\left(1+\frac{.004}{2}\right)^{2(10)}$

$A = \$780.58$

780.58
-750.00

$\boxed{\$30.58 \text{ interest}}$

5. In 1990, Carter deposited $1,000 in an investment account that earns $2\frac{3}{8}$% annual interest, compounded quarterly. If no other deposits or withdrawals were made, find the balance of his account in 2025.

$A = 1000\left(1+\frac{.02375}{4}\right)^{4(35)}$

$A = \$2290.55$

CONTINUOUS COMPOUND INTEREST

In some cases, interest is compounded **continuously** meaning the account is constantly earning interest. The formula to the right can be used to find the balance of the account in this case.

FORMULA:

$A = Pe^{rt}$

EXAMPLES

6. Suppose $800 is invested in an account at a 6% interest rate compounded continuously. If no other withdrawals or deposits are made, find the balance in the account after 20 years.

$A = 800e^{.06(20)}$

$A = \$2656.09$

7. Find the balance of an account after 5 years if $1,200 is initially invested at an interest rate of 12.5% per year, compounded continuously and there are no other deposits or withdrawals.

$A = 1200e^{.025(5)}$

$A = \$2241.90$

Option A:
5.5% annual interest compounded monthly

Option B:
2.7% annual interest compounded continuously

8. Carla is investing $1,500 in a new 30-year retirement account. Determine which of the interest rates and compounding periods shown to the left would be her best investment option.

Option A

$A = 1500\left(1+\frac{.055}{12}\right)^{12(30)}$

$A = \$7781.08$

Option B

$A = 1500e^{.027(30)}$

$A = 3371.86$

Option A

15. $\log \frac{1}{100} = x$

$10^x = \frac{1}{100}$ $x = -2$

16. $\log_{16} 2 = x$

$16^x = 2$

$x = \frac{1}{4}$

17. $\log_3 \frac{1}{27} = x$

$3^x = \frac{1}{27}$ $x = -3$

18. $\log_8 1 = x$

$8^x = 1$

$x = 0$

CHANGE OF BASE FORMULA

Some logarithms are not as easy to evaluate as those above, and will require the **change of base formula.**

$$\log_b a = \frac{\log a}{\log b}$$

Choose BASE 10 because there is a calculator button for it! →

Approximate each logarithm using the change of base formula.

19. $\log_5 34$

$\frac{\log 34}{\log 5}$ 2.1911

20. $\log_2 98$

$\frac{\log 98}{\log 2}$ 6.6147

21. $\log_{20} 4$

$\frac{\log 4}{\log 20}$ .4628

22. $\log_6 2$

$\frac{\log 2}{\log 6}$ .3869

23. $\log_3 225$

$\frac{\log 225}{\log 3}$ 4.9299

24. $\log_8 \frac{1}{2}$

$\frac{\log \frac{1}{2}}{\log 8}$ $-\frac{1}{3}$

NATURAL LOGARITHM

A logarithm with base e is called a **natural logarithm** and is written as ln.

$\log_e x \rightarrow \ln x$

Write each equation in exponential form.

25. $\ln_e x = 4$

$e^4 = x$

26. $\ln_e 10 = x$

$e^x = 10$

Write each equation in logarithmic form.

27. $e^x = 50$

$\log_e 50 = x$

$\ln 50 = x$

28. $e^{0.5} = x$

$\log_e x = .5$

$\ln x = .5$

Approximate the value of each expression.

Use the ln button on the calculator to evaluate natural logarithms. →

29. $\ln 64$

4.1589

30. $\ln \frac{1}{3}$

-1.0986

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Main Ideas/Questions	Notes/Examples
What is a **LOGARITHM?**	**A logarithm (log) is another way of writing exponents.** **Logarithmic Form** $\log_b a = x$ → **Exponential Form** $b^x = a$ Read as "**log base b of a equals x.**"
COMMON LOGARITHM	A logarithm with base 10 is called a **common logarithm** and can be written without the base. $\log_{10} x \rightarrow \log x$
Converting **BETWEEN FORMS**	**Write each equation in exponential form.** **1.** $\log_7 49 = 2$ — $7^2=49$ **2.** $\log_2 32 = 5$ — $2^5=32$ **3.** $\log 1000 = 3$ — $10^3=1000$ **4.** $\log_4 \frac{1}{64} = -3$ — $4^{-3}=\frac{1}{64}$ **5.** $\log_8 2 = \frac{1}{3}$ — $8^{\frac{1}{3}}=2$ **6.** $\log_9 27 = \frac{3}{2}$ — $9^{\frac{3}{2}}=27$ **Write each equation in logarithmic form.** **7.** $5^2 = 25$ — $\log_5 25=2$ **8.** $8^0 = 1$ — $\log_8 1=0$ **9.** $3^{-4} = \frac{1}{81}$ — $\log_3\left(\frac{1}{81}\right)=-4$ **10.** $12^{\frac{1}{2}} = 2\sqrt{3}$ — $\log_{12}(2\sqrt{3})=\frac{1}{2}$ **11.** $10^{-1} = \frac{1}{10}$ — $\log_{10}\left(\frac{1}{10}\right)=-1$ **12.** $16^{\frac{3}{4}} = 8$ — $\log_{16} 8=\frac{3}{4}$
Evaluating **LOGARITHMS**	**Evaluate the following logarithms using your knowledge of exponents.** **13.** $\log_6 36 = x$ $6^x=36$ $x=2$ **14.** $\log_2 128 = x$ $2^x=128$ $x=7$

10

Name: | **Date:**

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Main Ideas/Questions	Notes/Examples

PRODUCT

Property

$\log_b(m \cdot n) =$ $\log_b m + \log_b n$

2|72
2|36
2|18
3|9
3

Condense into a single logarithm.

1. $\log_3 9 + \log_3 5$
$\log_3(9 \cdot 5)$
$\log_3 45$

2. $\log 6 + \log(x-3)$
$\log 6(x-3)$
$\log(6x-18)$

3. $\ln 4x^2 + \ln 3x^3$
$\ln(4x^2)(3x^3)$
$\ln(12x^5)$

Expand using the product property.

4. $\log 72$
$\log 2^3 + \log 2^2$

5. $\ln \frac{9}{10}$
$\ln 9 - \ln 10$

⑥ $\log_5(x^2-4)$
1

QUOTIENT

Property

$\log_b\left(\frac{m}{n}\right) =$ $\log_b m - \log_b n$

Condense into a single logarithm.

7. $\ln 96 - \ln 6$
$\ln \frac{96}{6}$
$\ln 16$

8. $\log(8x^{10}) - \log(4x^2)$
$\log \frac{8x^{10}}{4x2}$
$\log 2x^8$

9. $\log_7 \sqrt{40} - \log_7 \sqrt{5}$
$\log_7 \frac{\sqrt{40}}{\sqrt{5}}$
$\log_7 \sqrt{8}$

Expand using the quotient property.

⑩ $\log_3 8$
$\log_3$ [illegible]

11. $\ln \frac{3}{4}$
$\ln 3 - \ln 4$

12. $\log\left(\frac{x-7}{x+1}\right)$
$\log(x-7) - \log(x+1)$

POWER

Property

$\log_b m^n =$ $n \log_b m$

Condense into a single logarithm. Simplify if possible.

13. $3 \cdot \log 6$
$\log 6^3$
$\log 216$

14. $\frac{1}{2} \cdot \log_4 81$
$\log_4 81^{1/2}$
$\log_4 9$

15. $(x-1) \cdot \ln 4$
$\ln 4^{x-1}$

Expand using the power property.

16. $\log_2 5^x$
$x \log_2 5$

17. $\ln \sqrt[3]{2x+1}$
$\ln(2x+1)^{1/3}$
$\frac{1}{3}\ln(2x+1)$

18. $\log_6 \frac{1}{64}$
$\log_6 2^{-6}$
$-6 \log_6 2$

USING THE PROPERTIES OF LOGARITHMS

CONDENSING LOGS

Directions: Condense each expression into a single logarithm.

19. $2 \cdot \log_4 9 + 3 \cdot \log_4 2$

$\log_4 9^2 + \log_4 2^3$

$\log_4 81 + \log_4 8$

$\log_4 81(8)$

$\boxed{\log_4 648}$

20. $\log 80 - 2 \cdot \log 4$

$\log 80 - \log 4^2$

$\log \frac{80}{16}$

$\boxed{\log 5}$

21. $3 \cdot \ln(pq) + 4 \cdot \ln(pq^2)$

$\ln(pq)^3 + \ln(pq^2)^4$

$\ln p^3 q^3 p^4 q^8$

$\boxed{\ln p^7 q^{11}}$

22. $\frac{1}{2}(\log_5 x^6 + \log_5 x^3)$

$\frac{1}{2}(\log_5 x^6 \cdot x^3)$

$\frac{1}{2}(\log_5 x^9)$

$\log_5 x^{9(\frac{1}{2})}$

$\log_5 x^{9/2}$

$\boxed{\log_5 \sqrt{x^9}}$

23. $\frac{2}{3} \cdot \ln 64 - 2 \cdot \ln 8$

$\ln 64^{2/3} - \ln 8^2$

$\ln 16 - \ln 64$

$\ln \frac{16}{64}$

$\boxed{\ln \frac{1}{4}}$

24. $-2\left(\log 15 + \frac{1}{2} \cdot \log \frac{1}{9}\right)$

$-2(\log 15 + \log(\frac{1}{9})^{1/2})$

$-2(\log 15 \cdot (\frac{1}{3}))$

$-2(\log 5)$

$\log 5^{-2}$

$\boxed{\log \frac{1}{25}}$

25. $\log 72 - \frac{1}{3}(2 \cdot \log 4 + \log 32)$

$\log 72 - \frac{1}{3}(\log 4^2 + \log 32)$

$\log 72 - \frac{1}{3}(\log 16(32))$

$\log 72 - \log 512^{1/3}$

$\log \frac{72}{8}$

$\boxed{\log 9}$

26. $\frac{3}{2} \cdot \log_3 a - \frac{1}{4} \cdot \log_3(16a^2)$

$\log_3 \frac{a^{\frac{3}{2}}}{(16a^2)^{1/4}} \rightarrow \log_3 \frac{a^{3/2}}{2a^{1/2}} \rightarrow \boxed{\log_3 \frac{a}{2}}$

EXPANDING LOGS

Directions: Expond each logarithm completely.

27. $\log_7(xy^3)$

$\log_7 x + 3\log_7 y$

28. $\ln\left(\frac{m^3}{n^7}\right)$

$3\ln m - 7\ln n$

29. $\log\sqrt{a^3 b}$

$\log(a^3 b)^{1/2}$

$\log a^{3/2} b^{1/2}$

$\frac{3}{2}\log a + \frac{1}{2}\log b$

30. $\log_4\left(\frac{c^2}{d}\right)^4$

$\log_4 \frac{c^8}{d^2}$

$8\log_4 c - 2\log_4 d$

31. $\ln(5p^3q^4)^2$

$\ln 25p^6q^8$

$\ln 25 + 6\ln p + 8\ln q$

$2\ln 5 + 6\ln p + 8\ln q$

32. $\log_2 \frac{\sqrt[3]{x}}{x^2 + x}$

$\log_2 x^{1/3} - \log_2(x^2 + x)$

$\frac{1}{3}\log_2 x - \log_2 x(x+1)$

$\frac{1}{3}\log_2 x - (\log_2 x + \log_2(x+1))$

LOGARITHMS Reference Sheet

H

COMMON LOGARITHM A **base 10** logarithm, written as:		NAUTRAL LOGARITHM A **base *e*** logarithm, written as:	
PROPERTY	**RULE**	**EXAMPLE 1**	**EXAMPLE 2**
BASIC PROPERTIES	$\log_b 1 = 0$ $\log_b b = 1$	**Simplify:** $\log_6 1 = 0$	**Simplify:** $\log_4 4 = 1$
PRODUCT PROPERTY	$\log_b (m \cdot n) =$ $\log_b m + \log_b n$	**Condense:** $\log_3 8 + \log_3 (3x) =$ $\log_3 8(3x)$ $\log_3 24x$	**Expand:** $\ln(x^2 - x - 2) =$ $\ln(x-2)(x+1)$ $\ln(x-2) + \ln(x+1)$
QUOTIENT PROPERTY	$\log_b \left(\frac{m}{n}\right) =$ $\log_b m - \log_b n$	**Condense:** $\log 80 - \log 16 =$ $\log \frac{80}{16}$ $\log 5$	**Expand:** $\log_2 \left(\frac{x}{5}\right)$ $\log_2 x - \log_2 5$
POWER PROPERTY	$\log_b m^n =$ $n \log_b m$	**Condense:** $3 \cdot \ln 9 =$ $\ln 9^3$ $\ln 729$	**Expand:** $\log_7 3^{2x} =$ $2x \log_7 3$
CHANGE OF BASE FORMULA	$\log_b a = \frac{\log a}{\log b}$	**Evaluate:** $\log_5 138 =$ $\frac{\log 138}{\log 5} = 3.0615$	**Evaluate:** $\log_{14} 2 =$ $\frac{\log 2}{\log 14} = .2626$

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Main Ideas/Questions	Notes/Examples

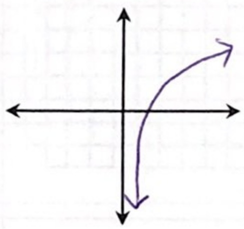
LOGARITHMIC FUNCTION

$f(x)=\log_b x$

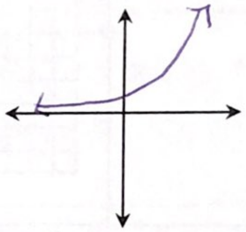
b is the **base** of the function

A logarithmic function is the **inverse** of an exponential function. Using your graphing calculator, sketch the following graphs:

$f(x) = \log x$



$f(x) = 10^x$

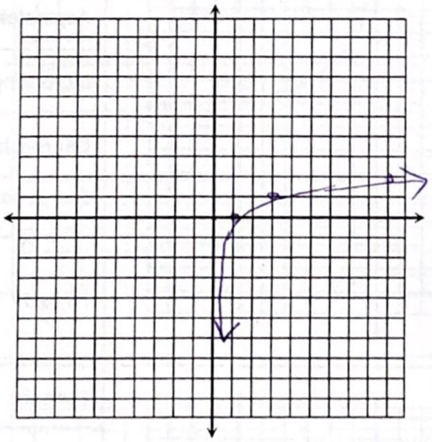


To graph a logarithmic function, you can use the inverse exponential function, then invert the values from the table to graph the logarithmic function.

Directions: Graph each function and identify its key characteristics.

1. $f(x) = \log_3 x$

x	y
1	0
3	1
9	2



Domain: $(0,\infty)$ [redacted]

Range: [redacted] $(-\infty,\infty)$

***x*-intercept:** $(1,0)$

Asymptote: $X=0$

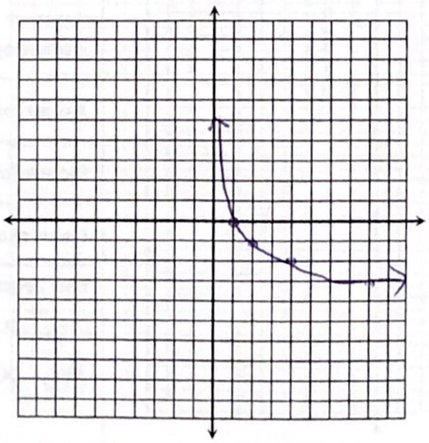
Increasing Interval: $(0,\infty)$

Decreasing Interval: None

End Behavior:
As $x\to\infty, f(x)\to\infty$
As $x\to-\infty, y\to-\infty$

2. $f(x) = \log_{\frac{1}{2}} x$

x	y
1	0
2	-1
4	-2
8	-3



Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

***x*-intercept:** 1

Asymptote: $X=0$

Increasing Interval: none

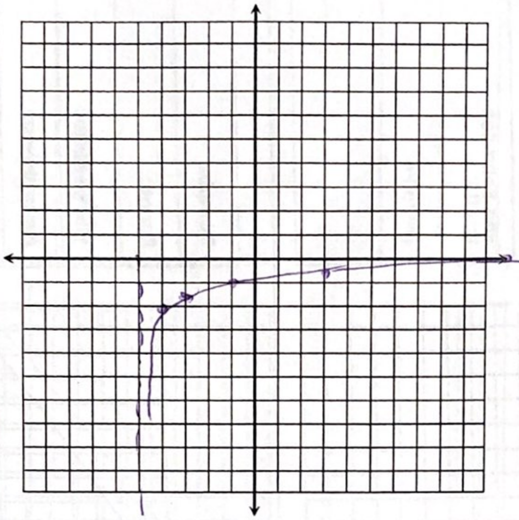
Decreasing Interval: [redacted] $(0,\infty)$

End Behavior:
As $x\to\infty, y\to-\infty$
As $x\to-\infty, y\to\infty$

left 5 down 2

$f(x) = \log_4(x+5) - 2$

x	y
-4	-2
-3	-1.5
-1	-1
3	-.5
11	0



Domain: $(-5, \infty)$

Range: $(-\infty, \infty)$

x-intercept: [redacted] $(11, 0)$

Asymptote: $x = -5$

Increasing Interval: $(-5, \infty)$

Decreasing Interval: none

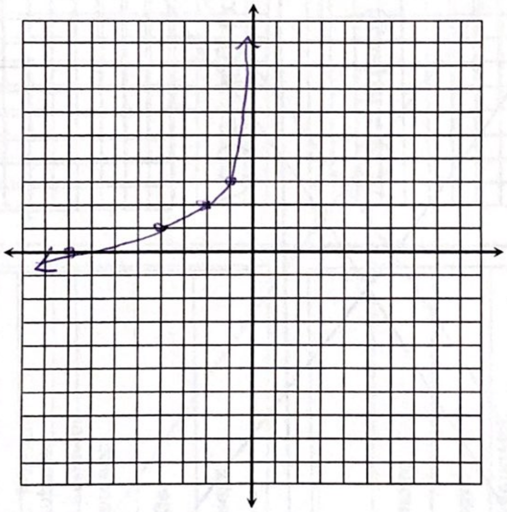
End Behavior:

As $x \to \infty, y \to \infty$

As $x \to -\infty, y \to -\infty$

7. $f(x) = \log_{\frac{1}{2}}(-x) + 3$

x	y
-8	0
-4	1
-2	2
-1	3



Domain: $(-\infty, 0)$

Range: $(-\infty, \infty)$

x-intercept: [redacted] $(-8, 0)$

Asymptote: $X = 0$

Increasing Interval: $(-\infty, 0)$

Decreasing Interval: none

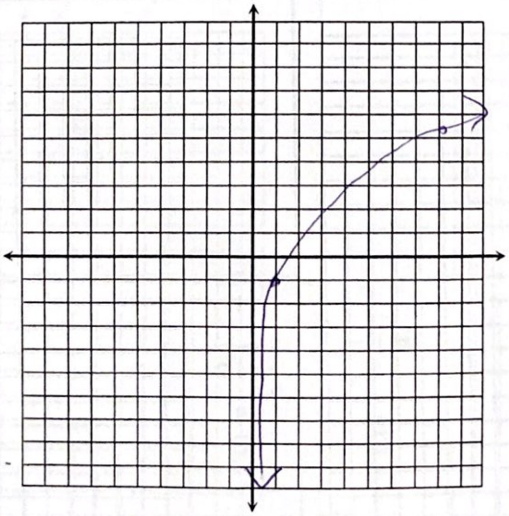
End Behavior:

As $x \to \infty, y \to \infty$

As $x \to -\infty, y \to -\infty$

8. $f(x) = 3 \cdot \ln x - 1$

x	y
1	-1
8	5.2



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(1.5, 0)$

Asymptote: $X = 0$

Increasing Interval: $(0, \infty)$

Decreasing Interval: none

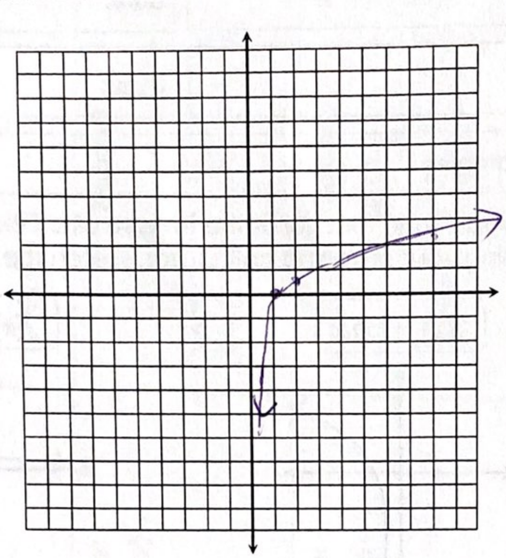
End Behavior:

As $x \to \infty, y \to \infty$

As $x \to -\infty, y \to -\infty$

3. $f(x) = \ln x$

x	y
1	0
2	.7
8	2.1



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

***x*-intercept:** $(1, 0)$

Asymptote: $x = 0$

Increasing Interval: $(0, \infty)$

Decreasing Interval: none

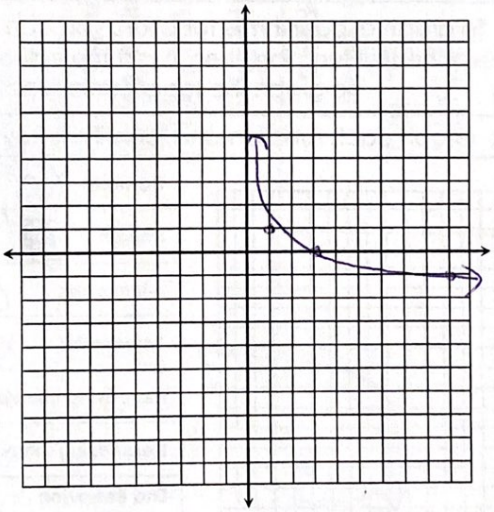
End Behavior:

As $x \to \infty, y \to \infty$

As $\to -\infty, y \to -\infty$

4. $f(x) = \log_{\frac{1}{3}} x + 1$

x	y
1	1
3	0
9	-1



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

***x*-intercept:** $(3, 0)$

Asymptote: $x = 0$

Increasing Interval: none

Decreasing Interval: $(0, \infty)$

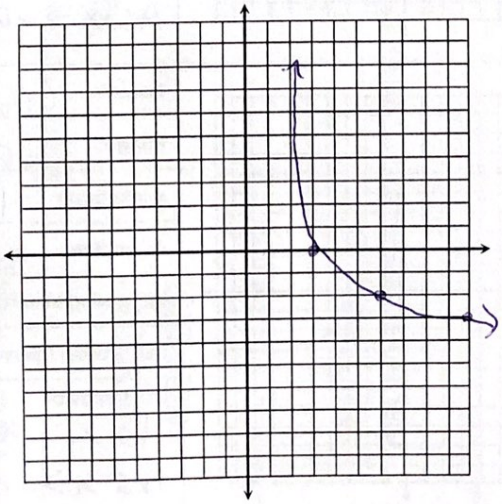
End Behavior:

As $x \to \infty, y \to -\infty$

As $x \to -\infty, y \to \infty$

5. $f(x) = -\log_2(x - 2)$

x	y
3	0
6	-2
10	-3



Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

***x*-intercept:** $(3, 0)$

Asymptote: $x = 2$

Increasing Interval: none

Decreasing Interval: $(2, \infty)$

End Behavior:

As $x \to \infty, y \to -\infty$

As $x \to -\infty, y \to \infty$

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Main Ideas/Questions	**Notes/Examples**

Solving

EXPONENTIAL EQUATIONS

(using a common base)

Steps to solve an exponential equation using a common base:

❶ Rewrite the equation using a common base.

❷ Use the properties of exponents to simplify each side of the equation.

❸ Use the one-to-one property: **If $b^x = b^y$, then**

❹ Solve!

SET 1:

WITH A COMMON BASE

1. $3^{2x-9} = 3^7$

$2x-9=7$

$2x=16$

$\boxed{x=8}$

2. $e^{4w-1} = e^{5-2w}$

$4w-1=5-2w$

$6w-1=5$

$6w=6$

$\boxed{w=1}$

3. $5^{c-1} \cdot 5^{3c+2} = 5^{7c+16}$

$5^{c-1+3c+2}=5^{7c+16}$

$5^{4c+1}=5^{7c+16}$

$4c+1=7c+16$

$-15=3c$

$\boxed{c=-5}$

4. $8^{k^2+k} \cdot 8^{2k-9} = 8^{4k} \cdot 8^{11}$

$8^{K^2+3K-9}=8^{4K+11}$

$K^2+3K-9=4K+11$

$K^2-K-20=0$

$(K-5)(K+4)=0$

$\boxed{K=5 \quad K=-4}$

SET 2:

WITHOUT A COMMON BASE

5. $9^{4y-26} = 81$

$9^{4y-26}=9^2$

$4y-26=2$

$4y=28$

$\boxed{y=7}$

6. $\frac{1}{64} = 4^{a^2-4}$

$4^{-3}=4^{a^2-4}$

$-3=a^2-4$

$a^2=1$

$\boxed{a=\pm 1}$

7. $2^{m-9} = 32^{m+3}$

$2^{m-9}=2^{5(m+3)}$

$m-9=5m+15$

$-24=4m$

$\boxed{m=-6}$

8. $\left(\frac{1}{12}\right)^{4x+3} \cdot 12^{x^2} = 12^{2x+13}$

$12^{-1(4x+3)+x^2}=12^{2x+13}$

$-4x-3+x^2=2x+13$

$x^2-6x-16=0$

$(x-8)(x+2)=0$

$\boxed{x=8 \quad x=-2}$

9. $343^{2n-4} = \left(\frac{1}{49}\right)^{n+4}$

$7^{3(2n-4)} = 7^{-2(n+4)}$

$6n-12 = -2n-8$

$8n = 4$

$\boxed{n = \frac{1}{2}}$

10. $8^{2-v} = 128^{v+3}$

$2^{3(2-v)} = 2^{7(v+3)}$

$6-3v = 7v+21$

$-15 = 10v$

$\boxed{v = -\frac{3}{2}}$

11. $9^{-p} = 243^{2p+6}$

$3^{2(-p)} = 3^{5(2p+6)}$

$-2p = 10p+30$

$-12p = 30$

$p = -\frac{30}{12}$

$\boxed{p = -\frac{5}{2}}$

12. $\left(\frac{1}{4}\right)^{2x^2-6} = \left(\frac{1}{64}\right)^{2x^2-5}$

$4^{-1(2x^2-6)} = 4^{-3(2x^2-5)}$

$-2x^2+6 = -6x^2+15$

$4x^2 = 9$

$x^2 = \frac{9}{4}$

$\boxed{x = \pm\frac{3}{2}}$

13. $36^{2r} \cdot \frac{1}{36} = 216$

$6^{2(2r)+-2} = 6^3$

$4r-2 = 3$

$4r = 5$

$\boxed{r = \frac{5}{4}}$

14. $25^{a} \cdot 625^{a+3} = \frac{1}{25}$

$25^{a} \cdot 25^{2(a+3)} = 25^{-1}$

$25^{a+2(a+3)} = 25^{-1}$

$a+2a+6 = -1$

$3a = -7$

$\boxed{a = -\frac{7}{3}}$

15. $\left(\frac{1}{16}\right)^{x+7} \cdot 64^{x+5} = \left(\frac{1}{32}\right)^{4}$

$2^{-4(x+7)} \cdot 2^{6(x+5)} = 2^{-5(4)}$

$2^{-4x-28+6x+30} = 2^{-20}$

$2x+2 = -20$

$2x = -22$

$\boxed{x = -11}$

16. $512^{y^2} = \left(\frac{1}{64}\right)^{3y} \cdot \left(\frac{1}{8}\right)^{5y^2}$

$8^{3y^2} = 8^{-2(3y)} \cdot 8^{-1(5y^2)}$

$8^{3y^2} = 8^{-6y+(-5y^2)}$

$3y^2 = -6y-5y^2$

$8y^2+6y = 0$

$2y(4y+3) = 0$

$\boxed{y = 0}$

$4y+3 = 0$

$4y = -3$

$\boxed{y = -\frac{3}{4}}$

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Main Ideas/Questions

Solving

LOGARITHMIC EQUATIONS

Type 1:

log = log

Notes/Examples

❶ Condense the logarithms on each side of the equation.

❷ Use the one-to-one property: **If $\log_b x = \log_b y$, then**

❸ Solve and check for extraneous solutions.

Directions: Solve each equation. Check for extraneous solutions.

1. $\log_3(7x-1) = \log_3(5x+17)$

$7x-1=5x+17$

$2x=18$

$\boxed{x=9}$

2. $\ln(k^2-4k) = \ln(k+14)$

$k^2-4k=k+14$

$k^2-5k-14=0$

$(k-7)(k+2)=0$

$\boxed{k=7 \quad k=-2}$

3. $\log 4 + \log_6(c+3) = \log 8$

$4(c+3)=8$

$4c+12=8$

$4c=-4$

$\boxed{c=-1}$

4. $\log_7(w+6) - \log_7(5w-3) = \frac{1}{3}\cdot\log_7 8$

$\log_7 \frac{(w+6)}{5w-3} = \log_7 8^{1/3}$

$\frac{w+6}{5w-3} = \frac{2}{1}$

$2(5w-3)=w+6$

$10w-6=w+6$

$9w=12$

$\boxed{w=\frac{4}{3}}$

* **5.** $\log_4(4p+3) = \frac{1}{2}\cdot\log_4(16p^4)$

$\log_4(4p+3)=\log_4(16p^4)^{1/2}$

$4p+3=4p^2$

$4p^2-4p-3=0$

$(2p-3)(2p+1)=0$

$2p=3 \qquad 2p=-1$

$\boxed{p=\frac{3}{2} \qquad p=-\frac{1}{2}}$

6. $2\cdot\ln(a+1) = \frac{3}{2}(\ln 80 - \ln 5)$

$\ln(a+1)^2=(\ln 80-\ln 5)^{3/2}$

$\ln(a+1)^2=\ln\left(\frac{80}{5}\right)^{3/2}$

$\ln(a+1)^2=\ln 16^{3/2}$

$(a+1)^2=64$

$a^2+2a+1=64$

$a^2+2a-63=0$

$(a+9)(a-7)=0$

$\cancel{a=-9} \quad \boxed{a=7}$

Solving

LOGARITHMIC EQUATIONS

Type 2:
log = number

❶ Condense and isolate the logarithm.

❷ Rewrite the equation in exponential form.

❸ Solve and check for extraneous solutions.

Directions: Solve each equation. Check for extraneous solutions.

7. $\log_2(3x-4)=7$

$2^7=3x-4$

$128=3x-4$

$132=3x$

$x=\frac{132}{3}$

$\boxed{x=44}$

8. $\ln_e 2a=9$

$e^9=2a$

$a=\frac{e^9}{2}$

$\boxed{a\approx 4051.5}$

9. $\log_6(w+7)-5=-3$

$\log_6(w+7)=2$

$6^2=w+7$

$36=w+7$

$\boxed{29=w}$

10. $2\cdot\log_9(k^2+2k)+4=5$

$2\log_9(K^2+2K)^{\cancel{8}}=1$

[illegible]

$\log_9(K^2+2K)=\frac{1}{2}$

$9^{1/2}=K^2+2K$

$K^2+2K-3=0$

$(K+3)(K-1)=0$

$\boxed{\begin{array}{l}K=-3\\K=1\end{array}}$

11. $\log_4(2v+3)+\log_4(2v-3)=2$

$\log_4(2v+3)(2v-3)=2$

$\log_4(4v^2-9)=2$

$4^2=4v^2-9$

$4v^2=25$

$v^2=\frac{25}{4}$

$v=\pm\frac{5}{2}$

$\boxed{v=\frac{5}{2}}$

12. $\frac{1}{3}\cdot\ln 27+\ln(x-5)=4$

$\ln 27^{1/3}+\ln(x-5)=4$

$\ln 3(x-5)=4$

$e^4=3x-15$

$e^4+15=3x$

$x=\frac{e^4}{3}+5$

$\boxed{x\approx 23.20}$

13. $\log_2(n-3)+\log_2(n+1)=5$

$\log_2(n-3)(n+1)=5$

$\log_2(n^2-2n-3)=5$

$2^5=n^2-2n-3$

$32=n^2-2n-3$

$n^2-2n-35=0$

$(n-7)(n+5)=0$

$\boxed{n=7}$ $\boxed{\cancel{n=-5}}$ 23

14. $\log_2 c^2-\log_2(3c-5)=2$

$\log_2\frac{c^2}{3c-5}=2$

$2^2=\frac{c^2}{3c-5}$

$4(3c-5)=c^2$

$12c-20=c^2$

$c^2-12c+20=0$

$(c-10)(c-2)=0$

$\boxed{c=10\quad c=2}$

Name:

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Main Ideas/Questions

Notes/Examples

Solving

EXPONENTIAL EQUATIONS

(using logarithms)

If using a common base is not possible, exponential equations can be solved using logarithms.

❶ Isolate the exponential expression.

❷ Take the logarithm of each side.

❸ Expand the logarithms if necessary using the power rule.

❹ Solve and check for extraneous solutions.

EXAMPLES

(1.) $3^x = 80$

$$\log 3^x = \log 80$$
$$x \log 3 = \log 80$$
$$x = \frac{\log 80}{\log 3}$$
$$\boxed{x \approx 3.9887}$$

2. $e^x = 140$

$$\ln e^x = \ln 140$$
$$x = \ln 140$$
$$\boxed{x \approx 4.9416}$$

(3.) $5^{x+1} = 18$

$$\log 5^{x+1} = \log 18$$
$$(x+1)\log 5 = \log 18$$
$$x+1 = \frac{\log 18}{\log 5}$$
$$x = \frac{\log 18}{\log 5} - 1$$
$$\boxed{x \approx .7959}$$

4. $\left(\frac{1}{3}\right)^{2x-5} = 120$

$$\log\left(\frac{1}{3}\right)^{2x-5} = \log 120$$
$$(2x-5)\log\left(\frac{1}{3}\right) = \log 120$$
$$2x-5 = \frac{\log 120}{\log \frac{1}{3}}$$
$$x = \frac{\frac{\log 120}{\log 1/3} + 5}{2}$$
$$\boxed{x \approx .3211}$$

(5.) $e^{3x} + 25 = 108$

$$e^{3x} = 83$$
$$\ln e^{3x} = \ln 83$$
$$3x = \ln 83$$
$$x = \frac{\ln 83}{3}$$
$$\boxed{x \approx 1.4729}$$

(6.) $-2 \cdot 8^{4-x} = -50$

$$8^{4-x} = 25$$
$$\log 8^{4-x} = \log 25$$
$$(4-x)\log 8 = \log 25$$
$$4-x = \frac{\log 25}{\log 8}$$
$$x = \frac{\frac{\log 25}{\log 8} - 4}{-1}$$
$$\boxed{x \approx 2.4520}$$

7. $\frac{2}{3}\cdot 2^{x-6} - 1 = 41$

$(\frac{3}{2})\frac{2}{3}\cdot 2^{x-6} = 42(\frac{3}{2})$

$2^{x-6} = 63$

$(x-6)\log 2 = \log 63$

$x-6 = \frac{\log 63}{\log 2}$

$x = \frac{\log 63}{\log 2} + 6$

$\boxed{x \approx 11.9773}$

8. $-2\cdot 4^{2x+7} + 9 = -55$

$-2\cdot 4^{2x+7} = -64$

$4^{2x+7} = 32$

$(2x+7)\log 4 = \log 32$

$2x+7 = \frac{\log 32}{\log 4}$

$2x = \frac{\frac{\log 32}{\log 4} - 7}{2}$

$\boxed{x \approx -2.25}$

9. $2^{x+5} = 3^{x-2}$

$(x+5)\log 2 = (x-2)\log 3$

$x\log 2 + 5\log 2 = x\log 3 - 2\log 3$

$x\log 2 - x\log 3 = -2\log 3 - 5\log 2$

$x(\log 2 - \log 3) = -2\log 3 - 5\log 2$

$x = \frac{-2\log 3 - 5\log 2}{\log 2 - \log 3}$

$\boxed{x = 13.9666}$

10. $8^{2x-1} = 5^{x+3}$

$(2x-1)\log 8 = (x+3)\log 5$

$2x\log 8 - \log 8 = x\log 5 + 3\log 5$

$2x\log 8 - x\log 5 = 3\log 5 + \log 8$

$x(2\log 8 - \log 5) = 3\log 5 + \log 8$

$x = \frac{3\log 5 + \log 8}{2\log 8 - \log 5}$

$\boxed{x \approx 2.7100}$

11. $4^{x-3} = 11^{3x+2}$

$(x-3)\log 4 = (3x+2)\log 11$

$x\log 4 - 3\log 4 = 3x\log 11 + 2\log 11$

$x\log 4 - 3x\log 11 = 2\log 11 + 3\log 4$

$x(\log 4 - 3\log 11) = 2\log 11 + 3\log 4$

$x = \frac{2\log 11 + 3\log 4}{\log 4 - 3\log 11}$

$\boxed{x \approx -1.5420}$

12. $9^{-2x} = 2^{5x+4}$

$-2x\log 9 = (5x+4)\log 2$

$-2x\log 9 = 5x\log 2 + 4\log 2$

$-2x\log 9 - 5x\log 2 = 4\log 2$

$x(-2\log 9 - 5\log 2) = 4\log 2$

$x = \frac{4\log 2}{-2\log 9 - 5\log 2}$

$\boxed{x \approx -.3527}$

LOGARITHMIC & EXPONENTIAL EQUATIONS Review!

LOGARITHMIC EQUATIONS

1. $\log_7(x+13) = \log_7(3-x)$

$x+13 = 3-x$

$2x = -10$

$\boxed{x=-5}$

2. $\log_2(n^2+13) = \log_2(n-1) + \log_2(n+3)$

$\log_2(n^2+13) = \log_2(n-1)(n+3)$

$n^2+13 = n^2+2n-3$

$13 = 2n-3$

$2n = 16$

$\boxed{n=8}$

3. $2 \cdot \ln(a+3) = \frac{1}{4} \cdot \ln 16 + \ln(a+7)$

$\ln(a+3)^2 = \ln 16^{1/4} \cdot (a+7)$

$(a+3)^2 = 2(a+7)$

$a^2+6a+9 = 2a+14$

$a^2+4a-5=0$

$(a+5)(a-1)=0$

$\cancel{a=-5}$ $\boxed{a=1}$

4. $\log(3c+4) - \log(c-6) = \log(c+6)$

$\log \frac{3c+4}{c-6} = \log(c+6)$

$\frac{3c+4}{c-6} = \frac{c+6}{1}$

$(c-6)(c+6) = 3c+4$

$c^2-36 = 3c+4$

$c^2-3c-40=0$

$(c-8)(c+5)=0$

$\boxed{c=8}$ $\cancel{c=-5}$

5. $\log_2(5v+23) - 9 = -2$

$\log_2(5v+23) = 7$

$2^7 = 5v+23$

$128 = 5v+23$

$105 = 5v$

$\boxed{v=21}$

6. $\log_{16}(p+5) - \log_{16}(p-2) = \frac{1}{2}$

$\log_{16} \frac{p+5}{p-2} = \frac{1}{2}$

$16^{\frac{1}{2}} = \frac{p+5}{p-2}$

$\frac{4}{1} = \frac{p+5}{p-2}$

$4(p-2) = p+5$

$4p-8 = p+5$

$3p = 13$

$\boxed{p = \frac{13}{3}}$

7. $\ln(r+1) + 3 \cdot \ln 2 = 7$

$\ln(r+1) + \ln 2^3 = 7$

$\ln(r+1)(8) = 7$

$e^7 = 8r+8$

$\frac{e^7-8}{8} = r$

$\boxed{r \approx 136.0791}$

8. $\frac{1}{3} \cdot \log_9 64 + 2 \cdot \log_9 n = 2$

$\log_9 64^{\frac{1}{3}} + \log_9 n^2 = 2$

$\log_9 64^{\frac{1}{3}} n^2 = 2$

$9^2 = 4n^2$

$81 = 4n^2$

$n^2 = \frac{81}{4}$

$n = \pm \frac{9}{2}$

$\boxed{n = \frac{9}{2}}$

XPONENTIAL EQUATIONS

9. $\left(\frac{1}{27}\right)^{2x-6} = 9^{x-1}$

$3^{-3(2x-6)} = 3^{2(x-1)}$

$-6x+18 = 2x-2$

$-8x = -20$

$x = \frac{20}{8}$

$\boxed{x = \frac{5}{2}}$

10. $4^{3m+1} = \left(\frac{1}{8}\right)^{m+4} \cdot 32^{m-2}$

$2^{2(3m+1)} = 2^{-3(m+4)} \cdot 2^{5(m-2)}$

$6m+2 = -3m-12+5m-10$

$6m+2 = 2m-22$

$4m = -24$

$\boxed{m = -6}$

11. $5^{w-1} = 90$

$\log 5^{w-1} = \log 90$

$(w-1)\log 5 = \log 90$

$w-1 = \frac{\log 90}{\log 5}$

$w = \frac{\log 90}{\log 5} + 1$

$\boxed{w \approx 3.7959}$

12. $e^{3r-2} - 16 = 120$

$e^{3r-2} = 136$

$\ln e^{3r-2} = \ln 136$

$3r-2 = \ln 136$

$3r = \frac{\ln 136 + 2}{3}$

$\boxed{r \approx 2.3042}$

13. $-4 \cdot 9^{2k+5} + 14 = 6$

$-4 \cdot 9^{2K+5} = -8$

$9^{2K+5} = 2$

$\boxed{K \approx -2.3423}$

$(2K+5)\log 9 = \log 2$

$2K+5 = \frac{\log 2}{\log 9}$

$2K = \frac{\frac{\log 2}{\log 9} - 5}{2}$

14. $\frac{2}{3} \cdot 5^{m-8} - 9 = 21$

$\left(\frac{3}{2}\right)\frac{2}{3} \cdot 5^{m-8} = 30\left(\frac{3}{2}\right)$

$5^{m-8} = 45$

$(m-8)\log 5 = \log 45$

$m-8 = \frac{\log 45}{\log 5}$

$\boxed{m \approx 10.3652}$

$m = \frac{\log 45}{\log 5} + 8$

15. $3^{4x+1} = 8^{x-5}$

$(4x+1)\log 3 = (x-5)\log 8$

$4x\log 3 + \log 3 = x\log 8 - 5\log 8$

$4x\log 3 - x\log 8 = -5\log 8 - \log 3$

$x(4\log 3 - \log 8) = -5\log 8 - \log 3$

$x = \frac{-5\log 8 - \log 3}{4\log 3 - \log 8}$

$\boxed{x \approx -4.9658}$

16. $4^{2x+3} = 7^{15-2x}$

$(2x+3)\log 4 = (15-2x)\log 7$

$2x\log 4 + 3\log 4 = 15\log 7 - 2x\log 7$

$2x\log 4 + 2x\log 7 = 15\log 7 - 3\log 4$

$x(2\log 4 + 2\log 7) = 15\log 7 - 3\log 4$

$x = \frac{15\log 7 - 3\log 4}{2\log 4 + 2\log 7}$

$\boxed{x \approx 3.7557}$

Name:

Date:

Topic:

Class:

Main Ideas/Questions | **Notes/Examples**

EXPONENTIAL GROWTH & DECAY
Applications

$f(t)=a(1+r)^t$
a = initial amount
r = growth rate
t = time
f(t) = final amount

$A=Pe^{rt}$
A = ending amount
P = initial
r = rate of growth
t = time (years)

$A=A_0\left(\frac{1}{2}\right)^{t/h}$
A = final amount
A_0 = initial amount
t = time
h = half-life

1. Mark started a new blog to write about his travels. In its initial week, the blog had 800 readers. From this point on, the number of readers each week increased by 25%. Use an exponential growth model to find the week in which the number of visitors reaches 10,000 people.

$10{,}000=800(1+.25)^t$
$\frac{10{,}000}{800}=\frac{800(1.25)^t}{800}$
$12.5=1.25^t$
$\log 12.5=\log 1.25^t$
$\log 12.5=t\log 1.25$
$t=\frac{\log 12.5}{\log 1.25}$
$t=11.3$
$\boxed{12\text{ weeks}}$

2. A new stock entered the stock market in January 2012 at \$0.72 per share. Four years later, the price per share was \$3.85. Using a continuous exponential growth model, find the growth rate.

$3.85=.72e^{r(4)}$
$3.85=.72e^{4r}$
$e^{4r}=5.3472$
$\ln e^{4r}=\ln 5.3472$
$4r=\ln 5.3472$
$r=\frac{\ln 5.3472}{4}$
$r=.4191$
41.91%

3. A certain medicine has a half-life of 5 hours. If a patient is given 500-mg at noon, at what time will they have 100-mg remaining in their bloodstream?

$100=500(.5)^{t/5}$
$.2=.5^{t/5}$
$\log .2=\log .5^{t/5}$
$\log .2=\frac{t}{5}\log .5$
$5\log .2=t\log .5$
$t=\frac{5\log .2}{\log .5}$
t = 11.6 hours
$\boxed{11:36\text{ pm}}$

LOGISTIC GROWTH
Applications

4. The growth of a plant can be modeled by the equation below where *h* is the height of the plant (in centimeters) and *t* is the number of weeks since the seed was planted. How many weeks will it take the plant to reach a height of 50 centimeters?

$$h=\frac{250}{1+12e^{-0.72t}}$$

$\frac{50}{1}=\frac{250}{1+12e^{-.72t}}$
$50(1+12e^{-.72t})=250$
$1+12e^{-.72t}=5$
$12e^{-.72t}=4$
$e^{-.72t}=\frac{1}{3}$
$\ln e^{-.72t}=\ln\left(\frac{1}{3}\right)$
$-.72t=\ln\left(\frac{1}{3}\right)$
$t=\frac{\ln\left(\frac{1}{3}\right)}{-.72}$
$t=1.5$
$\boxed{2\text{ weeks}}$

5. A small lake is stocked with a certain species of fish. The fish population is modeled by the equation below, where P is the number of fish in thousands, and t is the number of years since the lake was stocked. How many years will it take the fish population to reach 6,000 fish?

$$P = \frac{8}{1+3e^{-0.7t}}$$

$\frac{6}{1} = \frac{8}{1+3e^{-.7t}}$

$6(1+3e^{-.7t}) = 8$

$1+3e^{-.7t} = \frac{4}{3}$

$3e^{-.7t} = \frac{1}{3}$

$e^{-.7t} = \frac{1}{9}$

$\ln e^{-.7t} = \ln(\frac{1}{9})$

$-.7t = \ln(\frac{1}{9})$

$t = \frac{\ln(\frac{1}{9})}{-.7}$

$t = 3.2$

4 days

COMPOUND INTEREST

Applications

$A = P(1+\frac{r}{n})^{nt}$

A = amount

P = principal

r = rate

t = time in years

n = number of times compounded in one year

$A = Pe^{rt}$

compound continuously

6. If \$500 is deposited into a retirement account that pays an annual interest rate of 4% compounded quarterly, how long will it take the account to reach a balance of \$1,200 if there are no other deposits and withdrawals?

$1200 = 500(1+\frac{.04}{4})^{4t}$

$2.4 = (1.01)^{4t}$

$\log 2.4 = \log(1.01)^{4t}$

$\log 2.4 = 4t \log 1.01$

$\frac{\log 2.4}{4\log 1.01} = t$

$t \approx 22$ yrs

7. Dave deposited \$500 into a savings account. After 15 years, the account balance had tripled with no other deposits or withdrawals. Assuming the interest compounds continuously, find the interest rate.

$1500 = 500e^{15r}$

$3 = e^{15r}$

$\ln 3 = \ln e^{15r}$

$\ln 3 = 15r$

$\frac{\ln 3}{15} = r$

$r = .073$

$r = 7.3\%$

8. Scarlet invested \$8,000 in an account that pays 7.5% interest per year, compounded continuously. How long will it take the account to reach \$10,000?

$10{,}000 = 8000e^{.075t}$

$1.25 = e^{.075t}$

$\ln 1.25 = \cancel{\ln e}^{.075t}$

$\ln 1.25 = .075t$

$t = \frac{\ln 1.25}{.075}$

$t = 3$yrs

Name: ______________________________

Date: ____________________ **Per:** _______

Unit 4: Exponential & Logarithmic Functions

Homework 1: Graphing Exponential Functions

**** This is a 2-page document! ****

Directions: Classify each function as an exponential growth or an exponential decay. Sketch the curve.

1. $f(x)=\frac{1}{7}\cdot 6^x$

2. $f(x)=\frac{3}{2}\cdot\left(\frac{1}{4}\right)^x$

3. $f(x)=3\cdot\left(\frac{5}{2}\right)^x$

Directions: (a) Identify the parent function and **(b)** describe the transformations.

4. $f(x)=-\left(\frac{4}{3}\right)^{2(x-3)}+1$

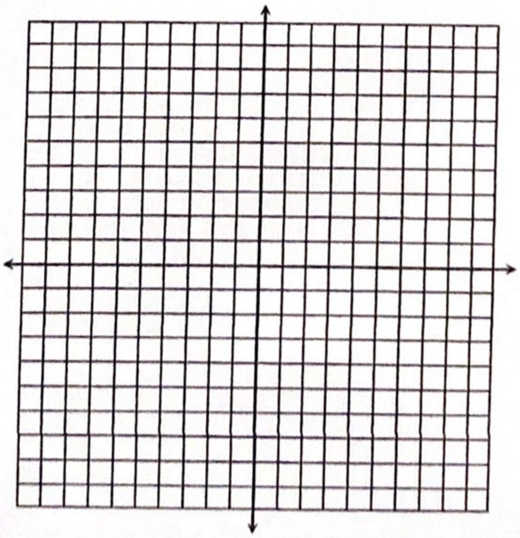
5. $f(x)=7^{-\frac{3}{2}x}+3$

6. $f(x)=\frac{1}{3}\cdot e^{-x}-9$

7. $f(x)=5\cdot\left(\frac{4}{5}\right)^{x+3}$

Directions: Graph each function, then identify its key characteristics.

8. $f(x)=5^x$



Domain:

Range:

y-intercept:

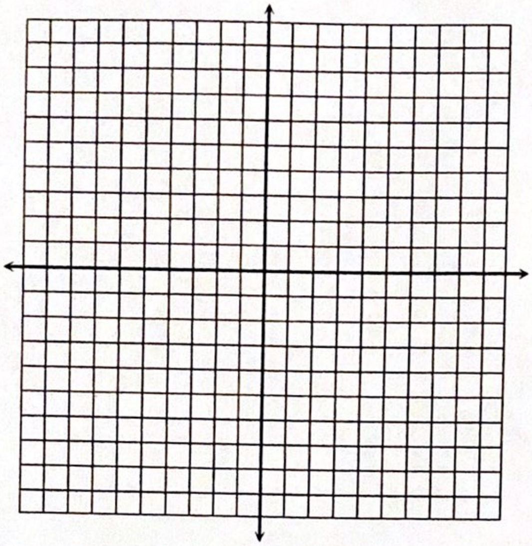
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

9. $f(x)=\left(\frac{2}{3}\right)^x$



Domain:

Range:

***y*-intercept:**

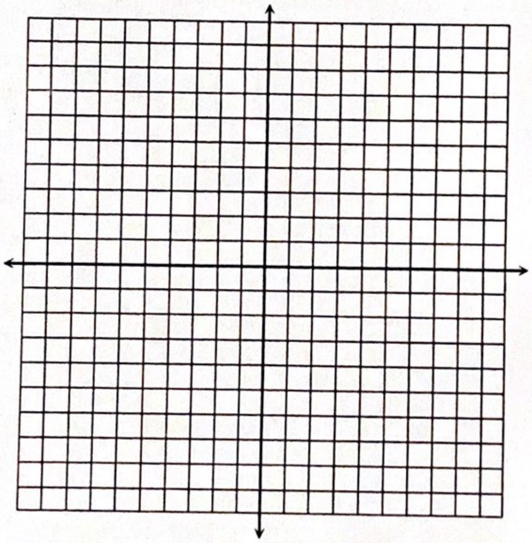
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

10. $f(x)=\left(\frac{1}{2}\right)^x$



Domain:

Range:

***y*-intercept:**

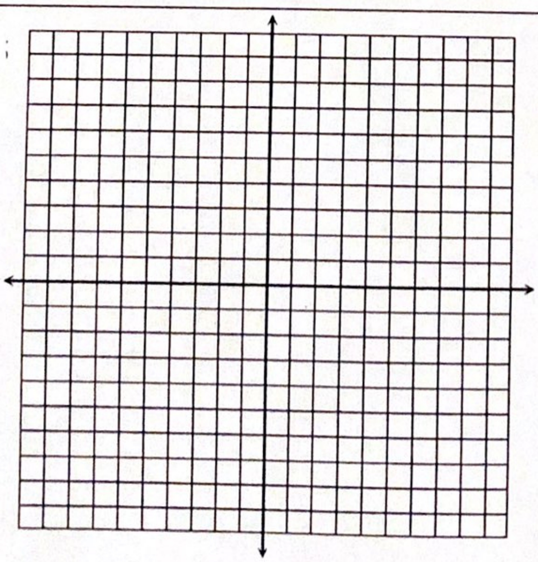
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

11. $f(x)=3^x$



Domain:

Range:

***y*-intercept:**

Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

Name: ______________________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ______________________ **Per:** _____ **Homework 2:** Exponential Growth & Decay

**** This is a 2-page document! ****

Exponential Growth and Decay

1. Aaron owns a rare baseball card. He bought the card for $7.50 in 1987 and its value increases by 6% each year. Write and use an exponential growth function to find the baseball card's value in 2015.

2. Jennifer started working at her job earning $6.25 per hour. Every six months, she gets a 3.25% raise. If Jennifer has worked at the job for 14 years, what is her hourly rate?

3. In 2005, the Summerville Journal had 110,000 subscriptions. The number of subscriptions subsequently decreased by 8% each year. Write and use an exponential decay function to find the number of subscriptions in 2022.

4. In November, 26 students at Monarch High School had contracted the flu. Each month, the number of students who have contracted the flu increases by 36%. Write and use an exponential growth function to find the total number of students who have contracted the flu by May.

5. Ian bought a new truck for $35,000 in 2015. Each year, the value of the truck depreciates by 9%. Write and use an exponential growth function to find the value of his truck at the end of his 60-month loan.

6. A certain compound has a half-life of four days. Write and use an exponential decay function to find the amount of compound remaining from a 75-ounce sample after three weeks.

Continuous Growth and Decay

7. A 4-foot tree was planted in 1984. The tree grows continuously by 22% each year from this point forward. Find the height of the tree after 8 years.

8. An ice sculpture measures 52 inches and melts continuously by 3% per minute. Find the height of a sculpture after 15 minutes.

9. Sam shaved his head down to ½" for a swim meet. If his hair grows continuously at a rate of one-quarter of an inch per month, find the length of his hair after a year.

Logistic Growth

10. The population of fish in a pond from 2001 to 2014 is modeled by the function below, where t is the years since 2001. Using the function, find the number of fish in the pond in 2014.

$$P(t)=\frac{1125}{1+12e^{-0.17t}}$$

11. The bears in Alaska are limited to a certain area to live due to the resources available for food and shelter. After t years, the number of bears living in the area is modeled by the function below. Using the function, find the number of bears after 17 years.

$$f(t)=\frac{103}{1+26e^{-0.31t}}$$

Name: ______________________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ____________________ **Per:** ______ **Homework 3:** Compound Interest

**** This is a 2-page document! ****

Compound Interest

1. If $1,800 is deposited into an account earning 6% interest, how much will be in the account at the end of 18 years if the interest is compounded with the following frequencies:

a) quarterly	**b)** weekly

2. Erica was given $300 for her birthday and decided to put it in a savings account that earns 3.75% interest. If she makes no other deposits or withdrawals, find her account balance after ten years if the interest is compounded with the following frequencies.

a) semiannually	**b)** daily

3. A $2,750 deposit was made to an account earning $2\frac{3}{4}\%$ annual interest compounded weekly. If no other deposits or withdrawals are made, find the balance of the account after nine years.

4. Jason saved money over the summer, accumulating $1,700. He opened a savings account that earns 4% annual interest compounded monthly. If Jason does not deposit or withdrawal from this account for 12 years, find its balance.

5. In January of 2003, Jaylen deposited $1,450 into an investment account earning 5% interest, compounded semiannually. If there are no other deposits or withdrawals from the account, find the total interest earned by the end of December in 2017.

6. In second grade, Eliza was given $500 from her grandparents. This money was deposited into a savings account which earns 3% annual interest, compounded quarterly. Find the account balance at graduation if there are no other deposits or withdrawals from the account.

Continuous Compound Interest

7. Moises was given a $1,500 signing bonus at his new job. He is going to invest this money in an account that earns 6% interest, compounded continuously. Find the account balance after ten years.

8. Suppose $2,800 is deposited into an account at a 2.5% interest rate, compounded continuously. If there are no other deposits or withdrawals, find the account balance after 25 years.

9. Find the balance of an account after seven years if $600 is deposited and the interest rate is 11.25% per year, compounded continuously and no other deposits or withdrawals are made.

10. Jacquie wants to invest $2,000 into an 18-year college fund for her new child. Option A has a 6% annual interest rate, compounded bimonthly. Option B has a 7.5% interest rate, compounded continuously. Determine which account is the better investment, and find how much more money she will earn by using that option.

Name: ______________________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ______________________ **Per:** _____ **Homework 4:** Introduction to Logarithms

Directions: Write each equation in exponential form.

1. $\log_3 27 = 3$

2. $\ln 8 = x$

3. $\log_{49} 343 = \frac{3}{2}$

4. $\log \frac{1}{10} = -1$

5. $\log_{24} 2\sqrt{6} = \frac{1}{2}$

6. $\ln x = \frac{2}{5}$

Directions: Write each equation in logarithmic form.

7. $81^{\frac{1}{2}} = 9$

8. $8^{-2} = \frac{1}{64}$

9. $e^7 = x$

10. $\left(3\sqrt{2}\right)^4 = 324$

11. $e^x = 21$

12. $\left(\frac{1}{5}\right)^{-3} = 125$

Directions: Evaluate each logarithm. Use the change of base formula when necessary.

13. $\log_3 81$

14. $\log_{11} 1$

15. $\ln 74$

16. $\log_{49} \frac{1}{7}$

17. $\log_{32} 2$

18. $\log_{\frac{1}{4}} 64$

19. $\log_{18} 124$

20. $\ln 12$

21. $\log_9 63$

22. $\ln 247$

23. $\log_{14} 3$

24. $\log_{64} 256$

Name: ______________________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ______________________ **Per:** _____ **Homework 5:** Properties of Logarithms

**** This is a 2-page document! ****

Directions: Condense each expression into a single logarithm.

1. $3 \cdot \log_2 p + 7 \cdot \log_2 q$

2. $\frac{1}{3} \cdot \ln 64 - \ln 12$

3. $\frac{1}{2}\left(\log_6 k^9 - \log_6 k^3\right)$

4. $5 \cdot \log_8(c^3 d^2) + 3 \cdot \log_8(cd^7)$

5. $5 \cdot \log a + \frac{1}{2} \cdot \log b - 4 \cdot \log c$

6. $\frac{2}{3} \cdot \log_2 27 + \frac{5}{2} \cdot \log_2 4$

7. $4 \cdot \log x - (2 \cdot \log v + 3 \cdot \log y)$

8. $\frac{1}{3}\left(\ln(64) - \ln(8x^3)\right)$

9. $\frac{1}{2} \cdot \log_3(9x) + \frac{3}{2} \cdot \log_3(4x^3)$

10. $\log(4x) + 2 \cdot \log(x-3)$

Directions: Expand each logarithm completely.

11. $\log_7 \sqrt[4]{5w+2}$

12. $\ln \frac{3r^2}{r-8}$

13. $\log_5 \frac{a\sqrt[3]{b}}{c^4}$

14. $\log_3(x^2+10x-24)$

15. $\log_4 \left(\frac{2j}{k^5}\right)^3$

16. $\log_6 \sqrt{3mn^8}$

17. $\log_3(7g^9h^5)^2$

18. $\log \frac{\sqrt{y}}{4y+8}$

19. $\log_2 \frac{\sqrt[3]{x^4y^7}}{z}$

20. $\ln(u^3-8)$

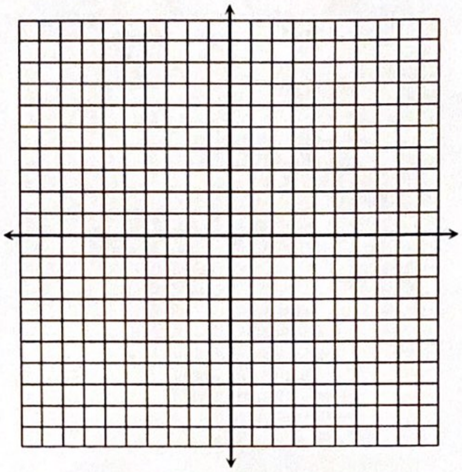
Name: ______________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ________________ **Per:** ______ **Homework 6:** Graphing Logarithmic Functions

** This is a 2-page document! **

Directions: Graph each function, then identify its key characteristics.

1. $f(x) = \log_2 x$



Domain:

Range:

x-intercept:

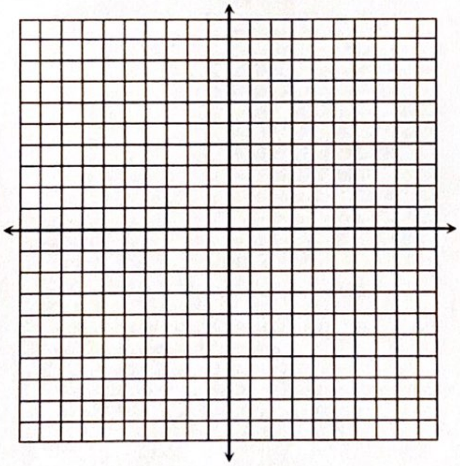
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

2. $f(x) = 2 \cdot \log_{\frac{1}{4}} x$



Domain:

Range:

x-intercept:

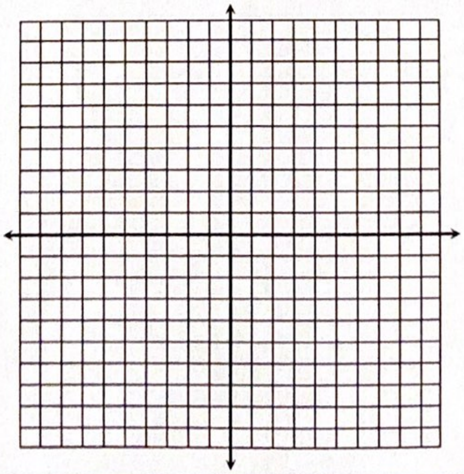
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

3. $f(x) = -\ln(x+5)$



Domain:

Range:

x-intercept:

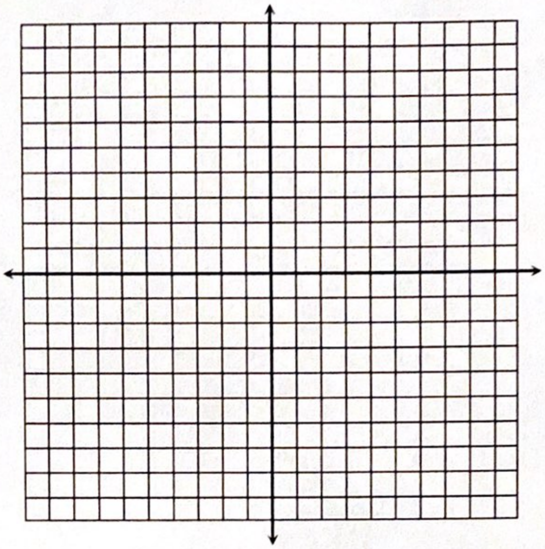
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

4. $f(x) = \log_3(x-2)+4$



Domain:

Range:

***x*-intercept:**

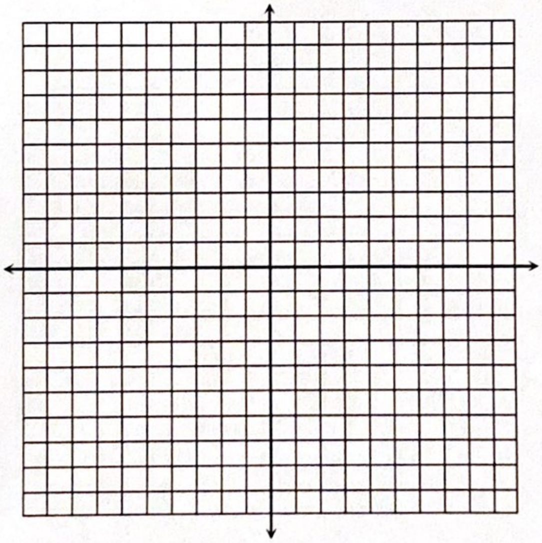
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

5. $f(x) = 5 \cdot \log(-x)$



Domain:

Range:

***x*-intercept:**

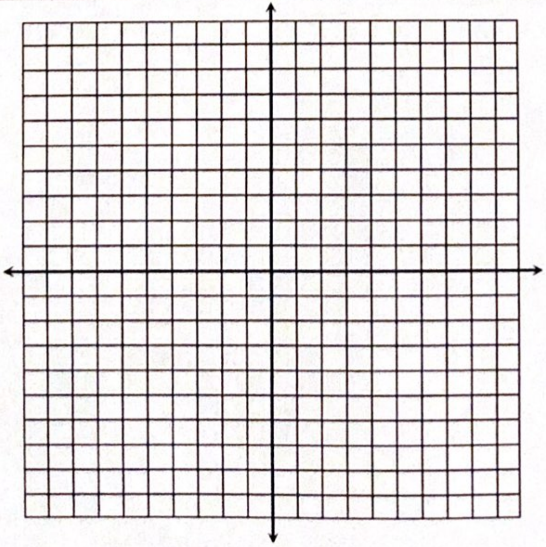
Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

6. $f(x) = \log_{\frac{1}{3}}(x+1)-2$



Domain:

Range:

***x*-intercept:**

Asymptote:

Increasing Interval:

Decreasing Interval:

End Behavior:

Name: ______________________________

Date: ____________________ **Per:** _______

Unit 4: Exponential & Logarithmic Functions

Homework 7: Solving Exponential Equations (using a common base)

**** This is a 2-page document! ****

Directions: Solve each equation using a common base.

1. $7^{3y-8} = 7^{13}$

2. $10^{5x+6} = 10^{x-12}$

3. $3^{p-7} \cdot 3^{2p+1} = 3^{8p-36}$

4. $e^{k^2+9} \cdot e^{-k} = e^{2k+5} \cdot e^{3k-4}$

5. $4^{3w+7} = 16^{11}$

6. $8^{9-2x} = 32^{3-x}$

7. $\left(\frac{1}{5}\right)^{u^2+8} = 125^{u^2-3}$

8. $\left(\frac{1}{27}\right)^{c+4} = 3^4 \cdot 3^{m-9}$

9. $9^{-4a} = 243^{2a+3}$

10. $16^{m+1} = 64^{-3m}$

11. $\left(\frac{1}{216}\right)^{-2k} = \left(\frac{1}{36}\right)^{k+2}$

12. $49^{2x^2+3} = 343^{x^2+5}$

13. $16^{-3y} \cdot \frac{1}{8} = \left(\frac{1}{32}\right)^{2y-3}$

14. $25^{7p-2} \cdot 625^{3-2p} = 1$

15. $\left(\frac{1}{81}\right)^{2w} \cdot 27 = 81^{\frac{1}{2}}$

16. $4^{a^2} = 16^{2a} \cdot 64^{2-a}$

Name: ______________________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ____________________ **Per:** _______ **Homework 8:** Solving Logarithmic Equations

**** This is a 2-page document! ****

Directions: Solve each equation. Check for extraneous solutions.

1. $\log_7(6a+4)=\log_7(9a-5)$

2. $\log_6(3x-11)+\log_6 2=\log_6(4x-8)$

3. $\ln(m+3)-\ln(2m-1)=\ln 4$

4. $\log_4(2p^2+3p)=\log_4(p^2+10)$

5. $\frac{1}{4}\cdot\ln(16q^8)-\ln 3=\ln 12$

6. $\log 6-\log(-2y-3)=\log 4$

7. $\log_5(3k+12)=\frac{3}{4}\cdot(\log_5 405-\log_5 5)$

8. $\log_8(w+12)+\log_8(w)=3\cdot\log_8 4$

9. $\log_6(11p+18)=3$

10. $2\cdot\log_2(3x-7)=10$

11. $\ln(4u)+3=5$

12. $\frac{1}{2}\cdot\log_{27}(5c+6)=\frac{2}{3}$

13. $2\cdot\log(2a-1)=0$

14. $\ln 2+\ln(k-4)=\frac{1}{3}$

15. $\log_2(2w^2)-\log_2(w+3)=3$

16. $\log_{12}(n+14)+\frac{1}{2}\cdot\log_{12}(4n^2)=2$

Name: ______________________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ____________________ **Per:** _______ **Homework 10:** Logarithmic & Exponential Equations

**** This is a 2-page document! ****

Directions: Solve each equation. Check for extraneous solutions.

1. $\log_{11}(4y-7)=\log_{11}(15+2y)$

2. $\log_4(p+5)=\frac{1}{2}\cdot\log_4(1-p)$

3. $\log_7(2x-1)-\log_7(3x-8)=\log_7 5$

4. $\ln(n+7)+\ln(n+2)=\ln 6$

5. $\log_5(7k-3)+4=6$

6. $2\cdot\{\log_4(c+1)+\log_4 2\}=3$

7. $\log_{16} 9+2\cdot\log_{16} a=\frac{3}{2}$

8. $\frac{1}{4}\cdot\ln 256+\ln u=6$

Directions: Solve each exponential equation using logarithms.

9. $243^{2p} = 9^{p+2}$

10. $\left(\frac{1}{625}\right)^{-3n} = 125^{n-2}$

11. $3^{2r-5} = 14$

12. $5 \cdot 17^{1-3x} = 20$

13. $\frac{2}{5} \cdot 4^{5v} - 8 = 4$

14. $-3 \cdot 4^{2y+9} + 11 = -4$

15. $3^{c+6} = 4^{2-c}$

16. $8^{2k+3} = 6^{3k-1}$

Name: ______________________________ **Unit 4:** Exponential & Logarithmic Functions

Date: ____________________ **Per:** _______ **Homework 11:** Applications with Equations

**** This is a 2-page document! ****

Directions: Solve each exponential equation using logarithms.

1. An online sales store started its business with 15 sales per week. If their sales increased by 18% each week, use an exponential growth model to find the week in which they exceeded 1,000 sales per week.

2. A certain chemical has a half-life of 3 days. If 750 ounces are initially used and 100 ounces are remaining, how many days have passed?

3. The average price of gas in 2006 in a Texas city was $3.92. In 2017, the average price was $2.36. Using a continuous exponential decay model, find its decay rate.

4. A flu epidemic has hit a local day care facility. The population of sick children is represented in the equation below where P is the number of sick children, and t is the number of days since the first child was diagnosed. How many days will it take for 50 children to catch the flu?

$$P = \frac{110}{1 + 7e^{-0.22t}}$$

5. The weight of a Doberman puppy can be modeled by the equation below where w is the weight of the puppy (in pounds) and t is the number of weeks since the puppy was born. How many weeks will it take the Doberman to reach a weight of 75 pounds?

$$w = \frac{95}{1 + 23e^{-0.15t}}$$

6. A deposit of $1000 is made to an account that earns 7% interest compounded continuously. After 14 years, the account has a balance of $2857.65. If there are no additional deposits or withdrawals, find the interest rate.

7. A retirement account was opened with a $900 deposit. If the account earns 4.25% interest compounded continuously and has no other deposits or withdrawals, how long will it take the value to double?

8. Sarah opened a savings account with a $725 deposit. This account earns 3.5% annual interest compounded bimonthly. How long will It take her account to reach a balance of $2000 if there are no other deposits or withdrawals?