Madison Public Schools
Precalculus/Honors Precalculus

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Course Overview

Description

Precalculus is an advanced mathematics course that extends concepts introduced in Algebra 2 while at the same time preparing students to be critical thinkers and problem solvers in higher level mathematics classes. Students will utilize previously mastered algebraic concepts and skills and will extend them to solve challenging and real world problems. A graphing intensive course, students will use multiple learning modalities such as graphing technology, collaboration, and communication to develop a strong understanding of the behavior of functions which is required in Calculus and other higher level mathematics courses. Each unit provides an opportunity for students to master new skills and encourages them to develop and utilize appropriate mathematical terminology and notation. Major topics include functions from a calculus perspective, exponential and logarithmic functions, trigonometric functions, equations and applications, systems of equations and matrices, conic sections, parametric equations, vectors, polar coordinates, complex numbers, an introduction to calculus (limits and derivatives), and inferential statistics.

Goals

This course aims to:

- enable students to make sense of various types of problems and the reasonableness of their answers
- build student confidence with the various approaches to solving a problem and persevere in solving them
- encourage students to become abstract thinkers who make sense of quantities and their relationships in problem situations
- develop students' ability to cooperatively discuss, make conjectures and critique ideas of one another
- use, apply, and model mathematics to solve problems arising in everyday life, society, and the workplace
- consider the variety of available tools when solving a mathematical problem
- communicate mathematical ideas precisely and effectively to others
- determine a pattern or analyze structure within mathematical content to apply to related ideas
- use repeated reasoning to follow a multi-step process through to completion

Materials

Core: McGraw Hill Precalculus (2014)
Supplemental: Khan Academy, Various websites related to precalculus

Resources

Suggested activities and resources page

Benchmark Assessments

Common Benchmark Assessment are given for each unit with common types of questions across the levels including problems that focus on the main ideas and anchor standards of the course.
Modifications and Adaptations for Special Needs Learners (Gifted and Talented Students, English Language Learners, Special Education Students, At-Risk Students)

Scope and Sequence
(Pacing Guide)

Unit Number	Topic of Study	Duration (Lessons Taught)
1	Functions from a Calculus Perspective	6 lessons
2	Power, Polynomial, and Rational Functions	6 lessons
3	Exponential and Logarithmic Functions	7 lessons
4	Trigonometric Functions and Equations	20 lessons
5	Systems and Matrices	17 lessons
6	Conic Sections and Parametric Equations	14 lessons
7	Vectors	13 lessons
8	Polar Coordinates and Complex Numbers	11 lessons
9	Sequences and Series	8 lessons
10	Limits and Derivatives	15 lessons

Unit 1 Overview
Unit Title: Functions From a Calculus Perspective
Unit Summary: In this unit students will develop a more formal understanding of functions and notation as will be required in Calculus and higher level mathematics courses. Key characteristics of functions and function behaviors will be addressed from both an algebraic and graphical perspective.
Suggested Pacing: 6 lessons
Learning Targets
Unit Essential Questions:

- How can mathematical notation convey information clearly?
- Why is it important to understand a function's domain and range?
- How are continuity, end behavior and limits of function related?
- What type of information can extrema and average rates of change convey?

Unit Enduring Understandings:

- Reading and choosing appropriate notation is vital to clarity in mathematics.
- A graph conveys important information about the function or relation it represents.
- Graphs provide a quick way to understand the relationship between variables in a situation.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc …

Summative Assessment: Unit 1 Assessment
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: identifying, evaluating, and graphing functions, and analyzing functions (determining and describing continuity and end behavior, determining intervals where a function increases, decreases, and has a max or min, determining average rate of change, and performing compositions of functions).

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Use graphs to analyze functions	Content: Symmetry Skills: Estimate function values Find domain and range Find intercepts and zeros Test for symmetry Classify functions as even, odd, or neither	Small group activity: provide students with the graphs of several different functions. (1) Give students several input values, ask students to evaluate the function value at those points given the graph. Include at least one real world example. (2) Ask students to identify intercepts, zeros, and symmetry. Emphasize correct mathematical notation.	F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities MP.1-MP.8 CRP.8 Utilize critical thinking to make sense of problems and persevere in solving them 9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences.	1 lesson
Describe and graph transformations of greatest integer functions and piecewise functions	Content: Greatest Integer Function Piecewise functions	Group activity: see above Class activity: Provide students with a piecewise function. For	F-IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions**[H]**.	1 lesson

	Skills: Describe transformations and the effect on the parent function's key characteristics Sketch transformations of the parent function Write equations for transformations Graph transformations using greatest integer functions Graph piecewise functions	example $f(x) = \{x + 2, x > 1\} \cup \{5 - \frac{1}{2}x^2, x \leq 1\}$. Students graph on mini-whiteboards for quick check. Repeat with other piecewise functions. Include with multiple function types and domain restrictions and differentiate by ability. Real world example and guided practice 6, p. 50, McGraw Hill eStudent edition Quiz question: Create the equation of a piecewise function that is cubic for any domain value no more than -2 and is absolute value for any greater than or equal to -2. Graph this piecewise function.	F-BF.3 Build new functions from existing functions. Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x+k) for specific values of k. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them MP.1-MP.8	
Use limits to determine continuity and describe end behavior of functions	Content: Limits Continuity Infinite Discontinuity Removable Discontinuity Continuity test Intermediate Value Theorem End Behavior and Infinity Skills: Determine if a function is continuous at a point Identify types of discontinuity Use the Intermediate Value Theorem to estimate zeros of a function Use graphs to describe end behavior of a function Use functions defined algebraically to describe end behavior of a function.	Word problem practice 1-3 Continuity, End Behavior, and Limits, www.connectED.mcgraw-hill.com, p. 19 Quiz question: provide students with several functions. Ask them to identify and classify all points of discontinuity. Technology activity: Provide students with several different functions and have them use a table via graphing technology such as the TI-84 to investigate continuity at a point. Journal/Pair-Share: Describe how the Intermediate Value Theorem could be used in a real world situation.	F-IF.2 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities. MP.1-MP.8 CRP.8 Utilize critical thinking to make sense of problems and persevere in solving them CRP.11 Use technology to enhance productivity 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs. NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. NJSLSA.W4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.	1 lesson

Determine the average rate of change of a function	Content: Average rate of change Secant Line Average Speed Difference quotient **[H]** Skills: Calculate the average rate of change of a function on a given interval. Use average rate of change to answer questions about real world situations Approximate the average rate of change at a point **[H]**.	Journal/Pair-Share: Using mathematically mature vocabulary, explain how average rate of change is the same and/or different than slope of a line. Quiz question: The formula for the distance traveled by falling objects on the Moon is $d(t) = 2.7t^2$, where d(t) is the distance in feet and t is the time in seconds. Find the average speed of a falling object on the Moon between 1 and 2 seconds. How do you think this would compare to falling object on the earth? Why? Small Group activity: Connect to AP Calculus, Rate of Change at a Point. p. 82-83 www.connectED.mcgraw-hill.com **[H]**.	F-IF.6 Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph. **[H]** MP.1-MP.8 CRP.8 Utilize critical thinking to make sense of problems and persevere in solving them	2 lessons
Perform operations on functions	Content: Sum, Product, Difference, Quotient Composition Inverses Horizontal Line test Inverse function notation Skills: Compose two or more functions Find compositions Find compositions including restricted domain **[H]** Decompose a function **[H]** Find inverse functions algebraically Find inverses graphically Use inverses to solve problems	Group activity: see above Self-check quiz 1-6 www.connectED.mcgraw-hill.com Word problem practice 1-6 Function Operations and Composition of Functions, www.connectED.mcgraw-hill.com Quickwrite/journal: Describe how to use the horizontal line test to tell if two functions are inverses. Provide an example. Guided Practice 5, McGraw Hill McGraw Hill eTeacher edition p. 69 Quiz question: sketch the graph of a *function* that has an inverse only if it's domain is restricted. Explain why **[H]**.	F.BF.1 Build a function that models a relationship between two quantities. F.BF.1c Compose functions F.BF.4b Verify by composition that one function is the inverse of another F.BF.4c Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.4d Produce an invertible function from a non-invertible function by restricting the domain **[H]** A.APR.1 Perform arithmetic operations on polynomials. MP.1-MP.8	1 lesson

Unit 2 Overview

Unit Title: Power, Polynomial, and Rational Functions

Unit Summary:
This unit provides a summary of functions, graphs, and algebraic skills fundamental to success throughout the Precalculus curriculum. Students will use information about transformations, powers and radicals, polynomials, and rational functions to solve problems and will extend their knowledge by performing more advanced skills such as partial fraction decomposition. A mastery of algebraic skills presented is fundamental to success in all future Precalculus units.

Suggested Pacing: 6 lessons

Learning Targets

Unit Essential Questions:

- What does it mean to simplify an expression?
- How are transformations of different functions related to one another?
- What are the most important characteristics of a function?
- How and why should we solve equations?

Unit Enduring Understandings:

- Different situations call for use of expressions in various forms.
- Characteristics of functions, such as zeros and intercepts model real world situations.
- Many different types of information can be gathered about a situation given its graph.
- Solving equations is fundamental to answering questions about mathematical models.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment: assessment for Unit 2
This assessment contains a variety of open-ended and short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment contains questions that address the following concepts: families of functions, graphs of functions, and algebraic skills (operations on expressions, simplifying expressions, and solving equations), as well as transformations of functions (polynomial and rational).

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Graph and analyze power and radical functions	Content: Zeros Y-intercept Power Function Maximum/Minimum values or Extrema Domain/Range Continuity Radical Function Skills: Find intercepts Describe domain and range Identify continuous intervals List increasing/decreasing intervals Identify extreme values Graph power functions Graph radical functions Graph functions involving rational and negative exponents	Group activity: see above Partner activity: Give students several different functions including at least one of each power or radical. Students should identify the type of function, major characteristics, and graph. Check work via TI-84 graphing calculator and/or www.desmos.com Quickwrite/Journal: What is the best method to solve the radical equation $2x = \sqrt{(100 - 12x)} - 2$? Why? Quiz question: Use limit notation to describe the end behavior of the function $f(x) = 2x^{-4}$. Justify with a graph of the function.	F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F-IF.7a Graph quadratic functions and show intercepts, maxima, and minima F-IF.7b Graph *square root, cube root functions*, and piecewise-defined functions, including step functions and absolute value functions. CRP4 Communicate clearly and effectively with reason CRP11 Use technology to enhance productivity 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs. MP.1-MP.8	1 lesson
Analyze and graph polynomial functions Factor and solve a polynomial equation Solve real world problems involving characteristics of polynomial functions such as maximums, minimums, and zeros.	Content: Polynomial Long division Synthetic division The Remainder Theorem The Factor Theorem The Fundamental Theorem of Algebra Degree Repeated roots End behavior Turning points Skills: Factor a polynomial Solve a polynomial equation to find its roots Find the equation of a polynomial given its zeros Solve a polynomial equation	Group activity: see below 2-2 Graphing Technology Lab: Behavior of Graphs, McGraw Hill eStudent edition p. 96 Real World example 7: Model Data Using Polynomial Functions and Additional Example 7, McGraw Hill McGraw Hill eTeacher edition p. 103 Word Problem Practice: The Remainder and Factor Theorems worksheet 2-3 www.connectED.mcgraw-hill.com Self-check quiz 2-4 www.connectED.mcgraw-hill.com	F-IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them MP.1-MP.8	2 lessons

	Describe end behavior Graph polynomial functions			
Analyze and graph rational functions	Content: Rational equation Rational function Points of discontinuity Removable discontinuity (hole in the graph) Vertical asymptotes Horizontal asymptotes Oblique asymptotes (H) Skills: Find points of discontinuity Find removable discontinuities Find vertical and horizontal asymptotes Sketch the graph of a rational function Solve rational equations	Group activity: see below Quickwrite/Journal: Write the equation of a rational function with 1 vertical asymptote, 1 removable discontinuity, and horizontal asymptote y = 1. Explain how you know. Quiz question: Determine domain, vertical and horizontal asymptotes, holes, intercepts, and sketch a graph of $h(x) = \frac{x^2-9}{x^2-x-6}$.	F-IF.7d Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. CRP4 Communicate clearly and effectively with reason CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them MP.1-MP.8	1 lesson
Solve non-linear inequalities **[H]**	Content **[H]**: Polynomial inequality Sign chart Rational inequality Skills **[H]**: Solve polynomial inequalities using a sign chart Solve rational inequalities using a sign chart Solve inequalities graphically	Quickwrite/journal **[H]**: describe how to use a sign chart to test intervals when solving an inequality Quiz question **[H]**: Describe the mathematical procedures you would use to solve $\frac{x-3}{x+2} > 2$. Graphing Technology activity **[H]**: Sketch the graph of y1 =x^3+3x^2 and y2 = 4. Demonstrate using shading/highlighting where the solution set to $x^2 + 3x \leq 4$ then list in interval (set) notation.	A-REI.10 Understand that the graph of an equation in two variables is a set of all its solutions plotted in the coordinate plane, often forming a curve **[H]**. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them **[H].** CRP 11 Use technology to enhance productivity **[H].** 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs. MP.1-MP.8 NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. NJSLSA.W4. Produce clear and coherent writing in which the development,	2 lessons

			organization, and style are appropriate to task, purpose, and audience. RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	

Unit 3 Overview

Unit Title: Exponential and Logarithmic Functions

Unit Summary:
In this unit students will analyze and graph the exponential function and its inverse, the logarithmic function. They will use properties of logarithms and will solve exponential equations using various methods. Ultimately, they will make decisions and answer questions about growth and decay situations modelled by exponential and logarithmic functions, including those in chemistry, physics, biology, business, finance and many others.

Suggested Pacing: 7 lessons

Learning Targets

Unit Essential Questions:
- What types of situations can exponential functions model?
- How can you choose the most appropriate method for solving an exponential equation?
- What is a logarithm and how can it be helpful?
- How can the knowledge about exponential and logarithmic functions help you make good decisions?

Unit Enduring Understandings:
- Exponential functions model growth and decay.
- Logarithmic functions and their properties are related to exponential functions.
- Exponential and logarithmic equations can be solved using various techniques.
- Mathematical models can be used to predict outcomes and to make decisions about important populations and/or events.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment: assessment for Unit 3
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: analyzing and graphing the exponential function and its inverse, the logarithmic function, properties of logarithms, solving exponential equations using various methods, and drawing conclusions from growth and decay situations modelled by exponential and logarithmic functions.

Alternative Assessment: Create a multimedia presentation demonstrating an appropriate exponential growth or decay model for a real world problem.

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Evaluate, analyze and graph exponential functions	Content: Transcendental function Exponential function Exponential growth Exponential decay The natural base, *e* Skills: Recognize a function as exponential Identify exponential functions as growth or decay Sketch the graph of exponential functions with any base, $b > 1$, including 2, 10, and *e* Sketch the graph of an exponential function with base b such that $0 < b < 1$ Sketch the graph of exponential functions with transformations Find the domain and range of an exponential function Identify equations of asymptotes for an exponential function Describe the end behavior and continuity of an exponential function Understand and explain why *e* is called the natural base.	Gallery walk/small group activity: Graph $f(x) = 2^x$, $f(x) = ½^x$, $f(x) = 2^{-x}$. Each group should be given a different base, including at least base 10 and base e. Graphs should be done on a display sheet or the board. Put up around the room, jigsaw and have students walk-around the room to observe. Next to each graph have students list similarities and differences. Optionally, provide a record sheet for students to copy graphs and take notes. Debrief as a whole class. Quiz question: Sketch the graph of each function and describe its domain, range, intercepts, asymptotes, and end behavior. Ex: $f(x) = 4^x$, $f(x) = 5^{-x}$. Differentiate by ability by providing different functions. Include transformations of the exponential function such as $f(x) = 5^{x-3}+4$, etc. Include a mix of dilations and reflections at an appropriate level.		1 lesson
Evaluate expressions involving logarithms	Content:	Quickwrite: Explain the relationship between a	F.BF.5	1 lesson

	Logarithmic form Basic properties of logarithms The common logarithm The natural logarithm Skills: Rewrite exponential expressions in logarithmic form Rewrite log expressions in exponential form Evaluate log expressions Apply basic properties of logarithms and natural logarithms	logarithmic expression and an exponential expression. Demonstrate by use of example. Foldable: Create a brochure with the basic properties of logarithms. Include specific examples with numbers. Add to this foldable as students learn graphs and more log properties. See Dina Zikes Teaching Mathematics with Foldables at https://blogs.edutech.nodak.edu/badlandsreadingcouncil/files/2012/03/math-foldables.pdf for more information	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	
Analyze and graph logarithmic functions	Content: The graph of $y = \log_b x$ The domain of $y = \log_b x$ The range of $y = \log_b x$ Transformations of the graph $y = \log_b x$ Skills: Sketch the graph of $y = \log_b x$ for various values of b Sketch transformations of $y = \log_b x$ State the domain, range, and asymptotes of log graphs Use log graphs to answer real world questions.	Guided Practice 7: Technology www.connectED.mcgraw-hill.com p. 177	F.IF.7.e *Graph exponential and logarithmic functions, showing intercepts, and end behavior*, and trigonometric functions, showing period, midline, and amplitude F.BF.1.b Combine standard function types using arithmetic operations. F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. CRP8 Utilize critical thinking to make sense of problems and persevere in solving them.	1 lesson
Understand and apply log properties in an algebraic context	Content: Product property Quotient property Power property Change of base formula Skills: Condense logarithms to one log Expand logarithms Apply the change of base formula to transform logs from one base to another	Foldable: Add to the brochure created above. Include specific examples with numbers. See Dina Zikes Teaching Mathematics with Foldables at https://blogs.edutech.nodak.edu/badlandsreadingcouncil/files/2012/03/math-foldables.pdf for more information Quick Write/Pair-share: how can the log properties be helpful when solving log equations. Demonstrate by using log properties to simplify a log equation. Project: Have students create a simplifying logs puzzle that matches two different forms of logarithmic expressions. Include at least 10 different	F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or *e*; evaluate the logarithm using technology. F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. CRP8 Utilize critical thinking to make sense of problems and persevere in solving them.	1 lesson

		problems and at least one of each different rule.		
Solve exponential and logarithmic equations	Content: Exponential equation One-to-One property Quadratic form of an Exponential equation Logarithmic equation Extraneous solution Skills: Solve exponential equations by rewriting both sides with the same base (One-to-one property) Solve logarithmic equations by rewriting both sides with the same base (One-to-one property) Solve log equations by simplifying using log properties Check for and exclude extraneous solutions to log equations Solve exponential equations using logarithms Solve exponential equations in quadratic form Solve Exponential and Logarithmic Inequalities **[H]**	Technology activity: Provide students with several different exponential and logarithmic equations. Have them solve each by hand, showing work, then check their solutions by graphing both sides of the equation and calculating the intersection point(s). Be sure to include at least one logarithmic equation that has an extraneous solution. Graph using the TI-84 calculator or www.desmos.com Journal/Quickwrite: Describe an exponential equation that can be solved without the use of logarithms. How do you know? Compare this to an equation that requires the use of logarithms. Graphing Technology Lab, Extend 3-4, Solving Exponential and Logarithmic Inequalities, www.connectED.mcgraw-hill.com p. 199A-199B **[H]**	F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. CRP8 Utilize critical thinking to make sense of problems and persevere in solving them.	1 lesson
Use math models to solve problems involving exponential growth and decay	Content: Compound interest Continuously compounded interest Exponential growth or decay model equation Continuous exponential growth or decay Exponential Regression Logarithmic Regression Logistic Regression Linearizing Data Skills: Use compound interest formulas to solve problems. Compare continuously compounded interest to other compounding intervals. Choose and apply the appropriate exponential growth or decay model to a real world problem.	Graphing Technology Lab 3-1: Financial Literacy: Exponential Functions, www.connectED.mcgraw-hill.com p. 170-171 Additional Example 7, Deer population, McGraw Hill eTeacher edition p. 165 Real world example 7 and Additional example 7: Earthquakes, www.connectED.mcgraw-hill.com p. 177 and McGraw Hill eTeacher edition p. 177 Follow this by Example 4 on p. 203 Additional Example 3, Advertising, McGraw Hill eTeacher edition p. 202 **[H]**. Additional examples can be found in Interactive Classroom at www.connectED.mcgraw-hill.com	F.LE.1.c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.IF.8.b Use the properties of exponents to interpret expressions for exponential functions. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. MP.1-MP.8 CRP3 Attend to personal health and financial well-being. CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]** CRP11 Use technology to enhance productivity 8.1.12.F.1	2 lessons

			Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs.	

Unit 4 Overview

Unit Title: Trigonometric Functions and Equations

Unit Summary:
This unit provides full exposure to right triangle trigonometry, trigonometric graphs, trigonometric identities, inverse trig functions, and trig equations. Students will build upon basic knowledge of trigonometry to solve more challenging problems related to real world situations.

Suggested Pacing: 20 lessons

Learning Targets

Unit Essential Questions:

- How is trigonometry useful in answering questions about the world?
- Why would it be helpful to replace a trigonometric expression with a different, but equivalent expression?
- How can you be sure that two expressions are equivalent?

Unit Enduring Understandings:

- Trigonometry relates algebra and geometry to one another through right triangle relationships.
- Different characteristics of trig graphs represent various physical phenomena.
- Trigonometric identities are helpful in solving equations and answering questions about different situations.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment: assessment for Unit 4
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: right triangle trigonometry, trigonometric graphs, trigonometric identities, inverse trig functions, and trig equations.

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Use right triangle and unit circle trig to solve problems	Content: Sine Cosine Tangent Cotangent Secant Cosecant Reciprocal function Radian measure Coterminal angles Arc length Unit circle Reference angle Skills: Find trig values Evaluate trig functions for special angles Find arc length Find angular and linear speeds Convert between degrees and radians Find and draw coterminal angles Find reference angles Find trig values at Evaluate trig functions for any angle using the unit circle	Group activity: Unit circle poster project (2-4 students.) Students are to use sections 4-1 through 4-3 to create a poster which includes the Unit Circle including all special coordinates labelled, one 1st quadrant 30-60-90 and on 45-45-90 triangle labelled with side length formulas. It should also include definitions of all 6 trig functions including reciprocal identities, special angle values, degree-radian conversion formulas, etc. The poster should be creative (ex: pizza "PI", etc.) This can act as an assessment of all content and skills in this unit. Individual quiz should follow. Posters can be displayed as study aids throughout the rest of the unit. Practice 4-3, Trigonometric functions on the unit circle. www.connectED.mcgraw-hill.com, p. 17 e Teacher's edition p. 252	F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. MP.1-MP.8 WHST.11-12.1. Write arguments focused on discipline-specific content. NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. NJSLSA.W4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	2 lessons
Graph the six basic trig functions with transformations	Content: Sinusoid Periodic function Period Amplitude Frequency Phase shift Vertical shift Midline Tangent graph Reciprocal functions Harmonic motion Damped harmonic motion **[H]** Skills: Graph y = sin(x) and y = cos(x) Apply transformations to y = A sin(Bx-C)+D.	Group activity: On the back of the unit circle project include two full periods of each trig function. Include secant superimposed on the graph of y = cos(x) and cosecant superimposed on the graph of y = sin(x). Journal/quickwrite: Given f(x) = A sin(Bx-C)+D, explain what each of A, B, C, and D do to the graph of y = sin(x). Provide an example. TI-84 transformation activity: Getting Triggy With It. All materials can be found at https://education.ti.com/en/84activitycentral/us/detail?id=ABC472729AE148B5821DFCF79F3D05F1&t=C68BCC2B94734976BBEF0911A0EFD45C	F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]** 9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences.	4 lessons

	Graph y = tan(x) with transformations Graph y = sec(x) and y = csc(x) as the reciprocal of cosine and sine. Identify domain, range, intercepts, asymptotes, symmetry and extrema of graphs. Solve problems involving damping constants **[H]** Graph the sine function parametrically **[H]**	Quiz question: Graph two full periods of sine, cosine, and tangent. Translate each to the right by $\pi/4$ and include the equation. Real world Example 6, Damped Harmonic Motion and additional example Music, www.connectED.mcgraw-hill.com eTeacher's edition p. 276 **[H]** Graphing technology lab: Explore 4-4, Graphing the Sine Function Parametrically, www.connectEd.mcgraw-hill.com , pp. 254-255 **(H)**		
Graph inverse trig functions and use them to solve problems	Content: Angle of elevation Angle of depression Arcsine function Arccosine function Arctangent function Restricted range Inverse trig properties Compositions of inverse trig functions. Skills: Find missing angle measures Graph the arcsine function Graph the arccosine function Graph the arctangent function Evaluate inverse trig functions using restricted ranges Sketch inverse trig graphs with basic transformations (stretch/compress) Use inverse trig functions to solve problems Find exact values of trig/trig-inverse compositions Decompose algebraic functions **[H]**	Journal/pair-share: How does an inverse trigonometric function compare to an algebraic inverse function. Support your answer with a sketch or example. Quiz question: explain why it is important to understand the restricted range of y = arccos(x) when evaluating $\cos^{-1}(-\frac{\sqrt{3}}{2})$ Real world example 5: Using an inverse trigonometric function, Movies, followed by guided practice www.connectED.mcgraw-hill.com , p. 285 and eTeacher's edition Additional example 5 Movies. Quiz question: write arccos(x) as an algebraic expression of x that does not involve trigonometric functions **[H]**	F.TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]** MP.1-MP.8	3 lessons
Apply the laws of sine and cosine to solve problems	Content:	Journal/quickwrite: Describe a situation in the ambiguous case where a	G.SRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.	3 lessons

	Oblique triangles Law of Sines Ambiguous case Law of Cosines Heron's Formula Skills: Solve oblique triangles by using the law of sines or cosines	given triangle has no solution. Explain how you can tell. Explain how you can tell if a triangle would have one solution? Two solutions? Provide an example to support your reasoning. Word Problem Practice McGraw Hill eTeacher's edition, p. 40, The Law of Sines and Cosines	G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. G.SRT.9 Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them MP.1-MP.8	
Understand and apply trigonometric identities	Content: Reciprocal identities Quotient identities Pythagorean identities Even/Odd identities Sum and Difference identities Reduction formula Double Angle identities Half angle identities Product-to Sum identities Sum-to-Product identities Skills: Use reciprocal and quotient identities to evaluate co-functions. Use Pythagorean identities to help find trig values Use identities to simplify trig expressions Use identities to find trig values without a calculator Use an identity to reduce a power Use identities to solve problems	Group activity: add the reciprocal, quotient, and pythagorean identities to the Unit Circle Poster Project. Small group activity: create a study guide of all trig identities. Provide a tip for remembering each. Quiz question: Which type of identity would you use to evaluate $(-\frac{\pi}{12})$. Why? Pair-share/quick write: How are trigonometric identities useful?	F.TF.9 Prove the addition and subtractions formulas for sine, cosine, and tangent and use them to solve problems	3 lessons
Verify trigonometric identities	Content: Verifying Identities Skills: Verify identities by rewriting in terms of sines and cosines Verify identities by combining fractions	TI-84 Trig Proofs activity: https://education.ti.com/en/84activitycentral/us/detail?id=D3BC4BD6CE484CED99EC8AE930645F68&t=941182F9B74346B9A3699EDF0A0282F9 Technology activity: Provide pairs of students with several different	F.TF.9 Prove the addition and subtractions formulas for sine, cosine, and tangent and use them to solve problems CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. CRP 11 Use technology to enhance productivity	2 lessons

	Use factoring with identities to simplify trig expressions Determine whether an equation is an identity Use graphing technology to verify an identity	identities. Each group should get a different set of problems. Each pair should both verify the identity using algebraic techniques and should use graphing technology such as the TI-84 or www.desmos.com to check their work. The teacher should then jigsaw students and do either a gallery walk or expert-group teaching.	8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs.	
Solve trig equations	Content: Trig equation Solution Skills: Solve trig equations over a restricted domain Isolate trig expressions to solve equations Use square roots to solve trig equations Factor to solve trig equations Use trig identities to solve equations Use trig equations to solve problems Use graphing technology to solve trig equations Find all solutions to a trig equation **(H)**	Small group activity: Assign each group a problem using a different trig identity. Have each group create a video recording showing how to apply the appropriate identity to solve the equation. Technology activity: students should solve an equation algebraically and verify their results using a graphing calculator. Sample quiz question: Use an identity to simplify the following equation and find all solutions: $\cos 2\theta - \cos \theta = 2$ on the interval $[0, 2\pi)$, **or for all real values (H)**.	F.TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]** MP.1-MP.8	3 lessons

Unit 5 Overview

Unit Title: Systems and Matrices

Unit Summary:
This unit students will use more advanced methods of solving multivariable linear systems, including matrices, determinants, Cramer's rule, and graphing technology. They will ultimately use techniques learned in this unit to solve optimization problems.

Suggested Pacing: 17 lessons

Learning Targets

Unit Essential Questions:

- How can matrix algebra simplify the process of solving systems of equations?
- What real world questions can be answered using linear algebra techniques?
- Why is equivalence important in mathematics?

Unit Enduring Understandings:

- Matrices can help solve systems of equations that are too large and/or difficult to do by hand.
- Optimization problems can have various numbers and types of solutions.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment:assessment for Unit 5
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: solving multivariable linear systems, using matrices, determinants, Cramer's rule, and graphing technology and solving optimization problems.

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Solve systems of equations by hand	Content: Substitution method Elimination method Consistent system Inconsistent system Independent system Dependent system Skills: Solve a 2-variable linear equation by substitution Solve a 2-variable linear system by elimination Solve a multi-variable linear system by hand Identify a system as consistent and independent, consistent and dependent, or inconsistent	Quickwrite: Given each of the following systems use slopes to classify as consistent and independent, consistent and dependent, or inconsistent. (Provide at least one of each type of system.) Quiz question: Why is the following system consistent and independent? Solve by the method of your choice.	A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	2 lessons
Perform operations on matrices	Content: Matrix Element Row Column Dimensions Square matrix Equal matrices Scalar multiplication Addition/subtraction of matrices Multiplication of Matrices Commutative Property of Matrix addition Associative Property of Matrix addition Scalar Distributive properties Skills: Identify dimensions of a matrix Add and subtract matrices Multiply matrices Perform scalar multiplication Solve matrix equations.	Quiz question: Additional examples #4 and 5 McGraw Hill eTeacher's edition p. P25 Pair activity: Read together: Real World example #6 p. P26. Do Additional example #6 eTeacher's edition and compare results with another group of students. Matrix operations worksheets: http://cdn.kutasoftware.com/Worksheets/Alg2/All%20Matrix%20Operations.pdf	N.VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network N.VM.7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions. N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	2 lessons

	Use matrices to model and solve real world problems.			
Multiply multivariable systems and compute determinants	Content: Matrix multiplication Associative property Left Distributive property Right Distributive property Identity matrix Square matrix Inverse of a square matrix 2x2 determinant 3x3 determinant **(H)** Skills: Multiply matrices Identify a matrix as invertible or not Find the inverse of a square matrix Calculate the determinant of a 2x2 square matrix Calculate the determinant of a 3x3 square matrix **(H)**	Journal/Pair-share: write two matrices with different dimensions that can not be multiplied. Explain why they can not be multiplied. Guided Practice 7A and 7B, eStudent edition p. 382 Additional Example #7, eTeachers edition. p. 382 **(H)**	N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions. N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	3 lessons
Perform row operations on multivariable systems	Content: Equivalent systems Row operations Gaussian elimination Augmented matrices Row-Echelon Form Gauss Jordan Elimination Skills: Write a system of equations as an augmented matrix Perform row operations on a system. Use Gaussian Elimination to solve a system. Use Gauss-Jordan elimination to solve a system. Solve real world problems involving linear systems. Use technology to solve multivariable linear systems	Technology activity: Reduce It! Students write augmented matrices to solve multivariable systems. All materials available at: https://education.ti.com/en/84activitycentral/us/detail?id=9A25366EC1364BEF867B1F4B9505F6A0&t=A1CC892BE658479AAAAE9052B8A9DE94 Journal/quickwrite: When solving a system of linear equations by elimination, what must be true in order to eliminate one of the variables? Provide an example to illustrate. Word Problem Practice 6-1 www.connectED.mcgraw-hill.com Worksheets p. 8, TE p. 373	A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable. N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	2 lessons
Solve linear systems using inverses and Cramer's Rule	Content: Square system Coefficient matrix Cramer's rule	Journal/quickwrite: Explain how to use Cramer's rule to solve a system. Provide an example of why Cramer's rule does not apply and the determinant of the coefficient matrix is 0.	A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3x3 or greater) CRP 11	3 lessons

	Skills: Solve a 2x2 system using an inverse matrix by hand Solve a multivariable system using an inverse matrix and graphing technology Use Cramer's rule solve a 2x2 linear system Use Cramer's rule solve a 3x3 linear system **(H)**	Cramer's Rule worksheets: http://cdn.kutasoftware.com/Worksheets/Alg2/Systems%20of%20Three%20Equations%20Cramers%20Rule.pdf Word problem practice www.connectED.mcgraw-hill.com , worksheets p. 19 TE p. 393 **[H]**	Use technology to enhance productivity **[H]** 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs.	
Rewrite rational expressions as the sum of partial fractions	Content: Partial fraction Decomposition Improper rational expression Non-repeated linear factors Repeated Linear factors Prime quadratic factors Skills: Decompose a fraction into separate more simple rational expressions	Guided Practice 2A, 2B, 3A and 3B www.connectED.mcgraw-hill.com p. 399 Quiz question: Find the partial fraction decomposition of $\frac{x^3-x}{(x^2-3)^2}$. Demonstrate how you would check your work by adding the fractions in your answer. Practice 6-4, McGraw Hill eTeacher's edition, worksheets p. 24 Writing summary: have students summarize how to perform partial fraction decomposition, including an example of each type of denominator. Have students peer-edit and add/edit their own work.		2 lessons
Solve optimization problems using linear programming	Content: Optimization Linear programming Objective function Constraints Feasible region Vertices Convex set Skills: Write objective functions Write equations for linear constraints Sketch a feasible region Find vertices of a feasible region Solve optimization problems	Technology activity: TI-84 activity where students must use linear programming to determine the optimum production level to determine profit. All materials can be found at: https://education.ti.com/en/84activitycentral/us/detail?id=91AD2F6C82E7422596A29A74C4756393&t=A1CC892BE658479AAAAE9052B8A9DE94 Project: Place students in pairs. Students are to write and graph a system of inequality constraints and objective function for a problem they make up. Students may research data online to find a realistic situation to optimize. They should then use linear programming to determine the optimal solution to their problem and display	A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. CRP 11	2 lessons

	Determine if a problem is infeasible Find multiple optimal values Determine optimal solutions of an unbounded region	their work in a poster and/or presentation.	Use technology to enhance productivity 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs. MP.1-MP.8	

Decode messages using matrices	Content: Cryptography Encoding key Invertible matrix Encoding Skills: Encode a message Decode a message Use inverses to decode **(H)**	TI-84 activity: All Systems Go! All materials can be found at https://education.ti.com/en/84activitycentral/us/detail?id=1EFB29ACD854438593E6F25206393643&t=A1CC892BE658479AAAAE9052B8A9DE94 Graphing Technology Lab: 6-3 Matrices and Cryptography, www.connectED.mcgraw-hill.com , p. 395	A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. CRP 11 Use technology to enhance productivity 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs. MP.1-MP.8	1 lesson

Unit 6 Overview

Unit Title: Conic Sections and Parametric Equations

Unit Summary:
In this unit students will study quadratic relations that are not necessarily functions. They will write equations, identify special geometric properties of conic sections, and graph them. They will then further extend the study to parametrically defined equations and apply them to the motion of projectiles.

Suggested Pacing: 14 lessons

Learning Targets

Unit Essential Questions:

- How do conic sections help us describe the physical world?
- What situations can be modeled by parabolas, ellipses, circles, and hyperbolas?
- How is defining equations parametrically helpful

Unit Enduring Understandings:

- Important geometric properties are key to understanding how and why certain conic sections look and behave the way they do.
- Algebraic properties of quadratic relationships are responsible for defining shape in various conic sections.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment: assessment for Unit 6
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: quadratic relations that are not necessarily functions, writing equations, identifying special geometric properties of conic sections, and graphing them, and parametrically defined equations that apply to the motion of projectiles.

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Analyze and graph parabolas	Content: Parabola Focus Directrix Axis of symmetry Vertex Standard forms of equations for Parabolas Tangent **(H)** Skills: Identify an equation as the graph of a parabola Identify characteristics from the Standard form Graph a parabola given standard form Write a quadratic relation in standard parabola form. Write the equation of a parabola given its geometric characteristics Find a tangent line at a point **[H]**	Locus discovery activity: Paper folding a parabola. This works great with wax, tracing or patty paper: http://mathequalslove.blogspot.com/2014/01/wax-paper-parabolas.html Technology activity: TI-84, Exploring the Parabola. All materials can be found at https://education.ti.com/en/84activitycentral/us/detail?id=67BFF2D4312F400FAC07A2EFA44B93A0&t=9D4B66DD534F4D7A9C6F11FF72497E86 Quiz question: Identify the vertex, focus, axis of symmetry and directrix then graph the parabola $(y-3)^2=-8(x+1)$ Journal/quickwrite: Compare $(y-3)^2=-8(x+1)$ and $(x-3)^2=-8(y+1)$. Describe the similarities and differences. Project: Find the equation of a parabola that models a real world situation. Graph using www.desmos.com and write a paragraph comparing the graph to its physical characteristics. You can even superimpose it on a picture. Quiz question: Write an equation for the tangent line to $y = x^2 - 2$ at the point (2,2). Sketch it! **[H]**	G.GPE.2 Derive the equation of a parabola given a focus and directrix. G.MG.1 Use geometric shapes, their measures, and their properties to describe objects. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]**	2 lessons
Analyze and graph ellipses and circles	Content: Ellipse Foci Major Axis Center Minor axis Vertices Eccentricity Standard equation of an Ellipse Skills: Complete the square to write the equation of an	Technology activity: Properties of an ellipse. All materials can be found at https://education.ti.com/en/84activitycentral/us/detail?id=C420B2F595CB4508AA80B6025DAD26A8&t=9D4B66DD534F4D7A9C6F11FF72497E86 Partner activity: Find the equation for an ellipse with vertices at (3, -4) and (3, 6) with foci at (3, 4) and (3, -2). Sketch a graph first.	G.GPE.1 Derive the equation of a circle given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.MG.1 Use geometric shapes, their measures, and their properties to describe objects.	3 lessons

	ellipse or circle in Standard form Identify characteristics of ellipses (foci, vertices, eccentricity) Graph circles and ellipses Identify conic sections as a particular type Use eccentricity to solve problems.	Quiz question: Write each conic section in standard form. Identify all key characteristics and graph (Additional example 5 p. 437 McGraw Hill eTeacher's edition)		
Analyze and graph hyperbolas	Content: Standard form of a hyperbola Transverse axis Conjugate axis Asymptotes of a hyperbola Eccentricity of a hyperbola Skills: Write the equation of a hyperbola in Standard form. Identify characteristics of hyperbolas (foci, vertices, Graph hyperbolas Use hyperbolas to model real world situations Solve systems of nonlinear equations **[H]**	Project: Find the equation of a parabola that models a real world situation. Graph using www.desmos.com and write a paragraph comparing the graph to its physical characteristics. You can even superimpose it on a picture. Sample situations can be found at http://www.pleacher.com/mp/mlessons/calculus/apphyper.html Word Problem Practice 7-3 Hyperbolas, www.connectED.mcgraw-hill.com Worksheets p. 19, TE p. 451 Classifying conics and summary activity: http://cdn.kutasoftware.com/Worksheets/Alg2/Classifying%20Conic%20Sections.pdf Graphing Technology Lab, Extend 7-4 pp. 462-463 **[H]**	G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.MG.1 Use geometric shapes, their measures, and their properties to describe objects. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]**	3 lessons
Identify and graph rotations of conic sections **[H]**	Content: Rotation of Axes of Conics **[H]** Equation of a rotated conic **[H]** Skills: Find the angle of rotation of a conic **[H]** Eliminate the xy-term in a rotated conic **[H]** Write an equation of a conic in standard form **[H]**	Practice 7-4 Rotations of Conic Sections www.connectED.mcgraw-hill.com Worksheets p. 23 Quiz question: **[H]** Identify each type of conic section, explain how you know, then rewrite in Standard form and graph: (1) $2x^2+y^2-2x+5xy+12-0$ (2) $4x^2+4y^2-4x+8=0$ (3) $2x^2+2y^2-6y+4xy-10=0$	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects. **[H]**	2 lessons

Use parametric equations to express position as a function of time	Content: Parametric equation Parameter Orientation Parametric curve Rectangular form Domain restrictions Angular parameter Skills: Graph parametric equations Sketch curves with parametric equations Re-write parametric equations in rectangular form Re-write parametric equations in rectangular form given an angular parameter Write parametric equations from graphs	Word Problem practice www.connectED.mcgraw-hill.com , worksheets p. 29 TE p. 470 Quick Write/Pair-share: How do parametric equations help you see the "whole picture"? Include an example to illustrate. NCTM Illuminations Mars Orbit activity: http://illuminations.nctm.org/Lesson.aspx?id=3980 Quiz question: Write $x = t^2+2$ and $y = 2t$ in rectangular form. Technology activity: Sketch the parametric equations $x = t^2+2$ and $y = 2t$ using the TI-84 in parametric mode.	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. CRP 11 Use technology to enhance productivity 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs.	2 lessons
Solve problems related to the motion of projectiles	Content: Projectile motion Horizontal distance Vertical position Skills: Use parametric equations to simulate projectile motion Find horizontal positions Find vertical positions Write parametric equations modelling projectile motion	Real world example 6 McGraw Hill eStudent edition p. 468 and Additional Example #6 eTeacher's edition. (Optional) TI-NSpire activity: https://education.ti.com/en/us/activity/detail?id=2EE9CB185F8346B395081717DD86A661	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. MP.1-MP.8	2 lessons

Unit 7 Overview

Unit Title: Vectors

Unit Summary:
This unit introduces the concept of vector quantities to students and guides them through the process of performing mathematical operations using vectors. A focus on geometrical representation and clear mathematical notation is emphasized with the ultimate goal of using vectors to solve problems in the physical world.

Suggested Pacing: 13 lessons

Learning Targets

Unit Essential Questions:

- How can you represent physical quantities that you cannot see?
- How does the use of vectors help solve problems in the real world?

Unit Enduring Understandings:

- Two distinct properties of vectors, direction and magnitude, are important in solving problems.
- Clearly labelled diagrams are helpful when solving problems represented by vector quantities.
- Mathematical operations can be performed on vector quantities for use in various situations.
- Clear and specific mathematical notation is used when working with vectors.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment: assessment for Unit 7
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: vector quantities, performing mathematical operations using vectors, and using vectors to solve problems in the physical world.

Alternative assessment: Students will create a presentation/video tutorial on solving a real world problems using vectors.

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Represent vectors graphically and perform mathematical operations on them in order to solve problems.	Content: Scalar Vector Initial point Terminal point Standard position Direction Magnitude Bearings Parallel vectors Equivalent vectors Opposite vectors Resultant vector Skills: Sketch vectors given magnitude and direction (angle) Sketch equal vectors Sketch opposite vectors Add/subtract vectors using the triangle method Add/subtract vectors using the parallelogram method Find a resulting vector's vertical and horizontal magnitudes Use the Law of Cosines to solve problems involving vectors in Navigation Solve vector problems involving force. Find resultants of more than two vectors. **[H]**	Partner activity: Provide each group with a pair of vectors. Make enough different sets of vectors for each group to start with one pair. Provide students with a ruler and protractor. Students use either the triangle or parallelogram method to find the resultant vector for their given vectors then find its magnitude and direction relative to the horizontal. Record each pair of vectors with answers on assessment sheet or notebook. Rotate until all pairs of vectors have been added. Alternate partner activity: Same as above except provide 3 different vectors to all students. Provide a different vector operation problem to each pair of students (e.g., 3u - 2w), have students draw and calculate the resulting vector. Rotate and record. Practice 8-1, Introduction to Vectors, Worksheet Chapter 8, p. 7 www.connectED.mcgraw-hill.com Word Problem Practice 8-1, Introduction to Vectors, Worksheet Chapter 8, p. 8 www.connectED.mcgraw-hill.com Quiz question: Sketch two vectors then add twice, once using the triangle method and once using the rectangular method. McGraw Hill eTeacher edition p. 491: Differentiated Instruction Extension. **[H]**	N-VM.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point form the coordinates of a terminal point. N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.4.c Understand vector subtraction v - w as v + (-w), where -2 is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N.VM.5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]** NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. NJSLSA.W4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	2 lessons

			RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	
Represent and perform operations on vectors using component form in the coordinate plane.	Content: Component form Magnitude in the coordinate plane Vector operations with component form Unit vector Linear combination of vectors Direction angles Skills: Sketch a vector given component form. Represent a vector in component form given initial and terminal points Calculate magnitude Add vectors in component form Subtract vectors in component form Perform scalar multiplication on component form Find a unit vector Express a vector sum in a*i*+b*j* form. Find component form given a direction (angle) and magnitude Solve vector problems using component form **[H]**	Journal/quickwrite: Provide an example where vectors can be used to model a real world situation. Sketch the vectors. Pair-share & whole-class debrief: Why might a unit vector be helpful? McGraw Hill eStudent edition Guided Practice 6A, 6B, 7A, 7B Quiz question: Find the component form of the vector with magnitude 7 and direction angle 60°. Quiz question: What must be true about the direction of a vector which is multiplied by the scalar -4? Pair problem solving: A farmer and a neighbor are removing a large boulder from a field. They are pulling on two ropes attached to the boulder that make a 35° angle with each other. If the farmer is pulling with a force of 105 newtons and the neighbor is pulling with a force of 95 newtons, what is the magnitude and direction of the force on the boulder? **[H]**	N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]**	3 lessons
Calculate and solve problems involving dot products of vectors.	Content: Dot product Orthogonal vectors Properties of dot products Vector projection Work Skills: Calculate dot product	Quickspeak: Given two vectors, explain how you can determine whether or not they are orthogonal. Provide examples verbally. For description of the quickspeak and other strategies visit: http://www.pps.k12.or.us/schools/gray/files/bmadison/AVID_Instruction_Strategies.pdf	N.VM.5.b Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for c > 0) or against v (for c < 0).	3 lessons

	Determine if two vectors are perpendicular Find magnitude using dot product Find the angle of between two vectors Find the projection of one vector onto another Use a vector to calculate force and/or work	Quiz question: Given two vectors in a*i*+b*j* form, calculate the angle between them. Quiz questions: Quiz 1, p. 33 www.connectED.mcgraw-hill.com Real world examples 6 and 7 McGraw Hill eStudent edition pp. 504-505, followed by Guided practice #6: Sledding, and #7: Cleaning		
Represent vectors and perform vector operations in three-dimensional space.	Content: Z-axis Ordered triple Midpoint in space Distance in space 3D vector Component form Vector operations in 3D Vector-Matrix transformations **[H]** Skills: Plot points in 3-space Represent a vector in 3-space Find the midpoint of a 3-dimensional vector Plot vectors in 3 space Find the magnitude of a vector in 3 space Find a unit vector in 3 space Perform vector operations in 3 space	Journal/quickwrite: Compare vectors in three-dimensional space and the two-dimensional coordinate plane. How are they the same? How are they different? Do any challenges arise given operations in 3 space? Provide an example to support your thoughts. Practice 8-4, Vectors in Three-Dimensional Space, worksheets Chapter 8 p. 23 www.connectED.mcgraw-hill.com . Enrichment 8-4 Basis Vectors in Three-Dimensional Space, Chapter 8 Enrichment p. 24 www.connectED.mcgraw-hill.com . **[H]** Graphing technology Lab: Vector Transformations with Matrices, McGraw HIll eStudent edition p. 517 **[H]**	N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. CRP 11 Use technology to enhance productivity **[H]** 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs.	3 lessons
Use vector products in space to solve problems.	Content: Orthogonal vectors Cross product Area Parallelepiped Volume Triple scalar product Skills: Calculate dot product in 3D	Journal/quickwrite: Why is the cross product only defined for three-dimensional space? Quiz question: (1) Sketch a parallelogram with adjacent sides defined by u = -3i - 4j + 2k and v = 5i - 4j - k. (2) Find the area of the parallelogram using a cross product. Word problem practice: Dot and Cross Products of	N.VM.5 Multiply a vector by a scalar MP.1-MP.8 CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them.	2 lessons

	Calculate a cross product of two vectors. Find area of a parallelogram in space using cross product Find volume using triple scalar product	Vectors in Space, worksheets Chapter 8 p. 29, www.connectED.mcgraw-hill.com .		

Unit 8 Overview

Unit Title: Polar Coordinates and Complex Numbers

Unit Summary:
In this unit students will extend their understanding of trigonometry and the rectangular coordinate system to a new system for naming coordinates on the plane. They will write equations and graph curves defined on the polar coordinate system and will solve problems where modelling in polar coordinates is preferred over rectangular coordinates.

Suggested Pacing: 11 lessons

Learning Targets

Unit Essential Questions:

- Why might a mathematician or scientist use the polar coordinate system over the rectangular coordinate system?
- Why is it helpful to have more than one coordinate system?

Unit Enduring Understandings:

- Polar coordinates are an alternate way of representing points on a curved surface.
- Some curves that are non-functions in the rectangular system can be represented as the function of an angle in the polar coordinate system.
- There are times when it is easier to work with rectangular coordinates and times when polar coordinates make mathematical calculations easier.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment: assessment for Unit 8
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: polar coordinates, writing equations and graphing curves defined on the polar coordinate system and solving problems where modelling in polar coordinates is preferred over rectangular coordinates.

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Use polar coordinates to model points on a plane	Content: Polar coordinates Pole Polar axis Directed angle Multiple representations Polar equation Polar graph Skills: Plot points in polar coordinates Write multiple representations of points in polar coordinates Graph polar equations in the form r = or θ = Find the distance between two points on the polar grid.	Pairs activity: Polar battleship. Provide each pair with polar graph paper. Have students plot 5 "ships" on his/her graph, one 2-point, two 3-point, 2-4 point, and 1 5-point ships. Points on each ship must be at adjacent "corners" of the graph paper and the ships can either be straight (along same theta but different r) or curved (along same r but different thetas.) Students take turns trying to "sink" each other's ships by calling out polar coordinates. Quiz question: given (-2, $(-2, \frac{\pi}{6})$, (1) Plot the point. (2) Name 3 different polar representations for this point.	A.SSE.2 Use the structure of an expression to identify ways to rewrite it. N.Q.2 Define appropriate quantities for the purpose of descriptive modelling	2 lessons
Graph polar equations and identify classical graphs	Content: Polar curve $\theta = \frac{\pi}{2}$ *line* symmetry Polar Axis symmetry Symmetry with respect to the Pole Zeros Maximum r-values Circle Limcon Cardioid Rose Lemniscate Spirals Skills: Graph a polar curve using a table Graph a polar curve using graphing technology Identify symmetry of a polar graph Use symmetry to help graph a polar curve Find zeros of a polar equation.	Journal/quickwrite: Describe an easy way to determine the symmetry of a polar curve. Provide an example to support your reasoning. Quiz question: Identify the type of curve given by the equation. Determine its symmetry, zeros, and maximum r-values then graph the curve. (1) $r^2 = 8 \sin 2\theta$ (2) $r = 2\theta$ *This activity can be differentiated by level by providing a chart on p. 546 or not. The teacher may also select other curves as needed. Graphing Technology Lab: 9-2 Investigate Graphs of Polar Equations. McGraw Hill eStudent edition p. 541 www.connectEd.mcgraw-ill.com Graphs of Polar Equations, Chapter 9 Quiz 1 p. 31, www.connectEd.mcgraw-ill.com	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]** CRP 11 Use technology to enhance productivity 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs.	3 lessons

	Find maximum r-values of a polar equation. Use symmetry, zeros, and maximum r-values to graph polar curves. Identify and graph classical polar curves Find the arc length of a curve **[H]**	Chapter 9 Connect to AP Calculus Arc Length. McGraw Hill eStudent edition pp. 586-587. www.connectED.mcgraw-hill.com **[H]**		
Convert between polar and rectangular coordinates	Content: Polar form Rectangular form Conversion formulas $x = r\cos(\theta)$ $y = r\sin(\theta)$ $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\frac{y}{x}$ Skills: Convert points from rectangular to polar form Convert points from polar to rectangular form Convert polar equations to rectangular form Convert rectangular equations to polar form	Journal/Quickwrite: Give an example of a polar equation whose graph is not a function in rectangular coordinates but is a function $r(\theta)$. Is the vertical line test valid for equations graphed in polar coordinates? Why/why not? Quiz question: Identify the graph of each rectangular equation. Then convert to polar form. Finally, graph. (1) $(x+2)^2+y^2=4$ (2) $2xy = 4$ Guided Practice 5A-5C, McGraw Hill eStudent edition p. 556	F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	3 lessons
Represent complex numbers in both rectangular and polar form.	Content: Complex plane Real axis Imaginary axis Absolute value of a complex number Modulus Polar (or trigonometric) form of a complex number Product of complex numbers in polar form Quotient of complex numbers in polar form DeMoivre's Theorem **[H]** Roots of a complex number **[H]** nth Roots of Unity **[H]** Skills: Graph a complex number Calculate the absolute value of a complex number Rewrite complex numbers in polar (or trig) form Find the modulus (r) of a complex number and the argument (θ).	Real-world example: Electricity: If a circuit has a voltage, E, of 100 volts and an impedance, Z, of 4 - 3j ohms, find the current I in the circuit in rectangular form using the formula E = I • Z. McGraw Hill eTeacher's edition Additional Example #5 p. 573 Quiz question: Graph on the complex plane then express in polar form: 6+2i Journal/quickwrite: How are polar and complex numbers useful in real-life situations? Complex numbers and DeMoivre's Theorem practice 9-3 www.connectED.mcgraw-hill.com , Chapter 9 worksheets p. 28 Quiz question: find the fifth roots of -2 - 2i using DeMoivre's Theorem. **[H]**	N.CN.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them. **[H]**	3 lessons

	Convert the polar form of a complex number to standard complex form Find the product of two complex numbers using polar form Find the quotient of two complex numbers using polar form. Express the product of two complex numbers in rectangular form. Express powers of complex numbers in rectangular form Find pth roots of a complex number	Pair-share: Find the fifth roots of unity and explain what they represent geometrically. **[H]** DeMoivre's Theorem from a Geometrical Perspective activities: http://www.mathed.soe.vt.edu/Undergraduates/EulersIdentity/DeMoivresTheorem.pdf **[H]**		

Unit 9 Overview

Unit Title: Sequences and Series

Unit Summary:
In this unit students will extend upon their knowledge of sequences and series from Algebra 2 in a more formal way. They will represent finite and infinite series using clear and accurate mathematical notation and will use them to solve problems involving higher level mathematics including the binomial theorem and power series.

Suggested Pacing: 8 lessons

Learning Targets

Unit Essential Questions:

- How can identifying patterns help you solve problems?
- What types of real world scenarios can finite and infinite series represent?

Unit Enduring Understandings:

- Clear, specific mathematical notation is important in order to communicate mathematics effectively.
- Patterns in the partial sums of an infinite series can help determine convergence or divergence of the series.
- Series can be used to approximate and simplify complicated mathematical formulas.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment:assessment for Unit 9
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: sequences and series, representing finite and infinite series using clear and accurate mathematical notation and using them to solve problems involving higher level mathematics including the binomial theorem and power series.

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Use sigma notation to represent and evaluate sums of series	Content: Sigma notation Finite sequence nth Partial Sum Infinite Series Converge Diverge Skills: Write terms of a sequence Determine convergence of a sequence Evaluate partial sums	Writing assessment: Write formulas for two sequences, one that converges and one that diverges. Explain. Guided Practice 6A-6C p. 594 McGraw Hill eStudent edition www.connectED.mcgraw-hill.com	F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. CPR 4 Communicate clearly and effectively and with reason	1 lesson
Evaluate sums of arithmetic and geometric series	Content: Arithmetic Sequence Common difference nth term Arithmetic series Geometric sequence Geometric series Sum of an infinite geometric series Skills: Evaluate nth terms Find the sum or an arithmetic series Find the partial sum of a geometric series Determine convergence/divergence of an infinite geometric series Find the sum of an infinite geometric series	Quiz question: Write each sum in sigma notation. Does the sum of the infinite series exist? If so, find it, if not, explain why not: (1) 24 + 18 + 13.5 (2) 0.33 + 0.66 + 1.32 Journal/quickwrite: Explain how you can tell if an infinite geometric series converges or diverges. Give an example of a convergent infinite geometric series in Sigma notation then find its sum. Journal/quickwrite: Are infinite arithmetic series convergent or divergent? Why?	F.LE.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. CPR 4 Communicate clearly and effectively and with reason	1 lesson
Use Mathematical Induction to prove summation formulas **[H]**	Content:**[H]** Mathematical induction Base case Inductive hypothesis Inductive step Skills: **[H]** Prove summation formulas by mathematical induction	Quiz question: Prove that the sum of all odds is n^2. That is, Prove by induction that $1 + 3 + 5 + \ldots + (2n-1) = n^2$ is true for all positive integers. **[H]** Journal/quickwrite: Summarize the steps to prove something by mathematical induction. How is it possible that this is sufficient as a proof? **[H]** Exercises 28-33 p. 625 McGraw Hill eStudent edition	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them **[H]**	2 lessons

Use the binomial theorem to expand a binomial and calculate probability	Content: Binomial coefficient Pascal's triangle Binomial theorem Skills: Calculate binomial coefficients Use Pascal's triangle to expand a binomial Find indicated terms in a binomial expansion Use binomial coefficients to calculate probability Write a binomial expansion using sigma notation	Quiz questions: (1) Expand $(2t + 3u)^3$ using the binomial theorem (2) Represent the expansion of $(3x - 5y)^{17}$ in sigma notation the find the 8th term. Word Problem Practice: Chapter 10 worksheets p. 29: The Binomial Theorem.	A.APR.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	2 lessons
Represent functions as infinite series **(H)**	Content: **[H]** Power series Euler's formula Exponential form of a complex number Skills: **[H]** Write power series to represent a rational function Write power series to represent e^x and other exponential series Write power series to represent sine and cosine Use power series to approximate values of transcendental functions Write a complex number in exponential form.	Partner activity/foldable: Have students write Power series to represent $y = 1/x$, $y = e^x$, $y = \sin(x)$, and $y = \cos(x)$. Next to each, choose an input value and use a calculator to evaluate. Then, have students calculate a series of partial sums to illustrate that the partial sums converge to the given series. Allow students to use enough terms so that the sums are within a given tolerance. **[H]** Enrichment 10-6 www.connectED.mcgraw-hill.com Chapte 10 p. 35. Alternating Series **[H]** Quickwrite/Journal: describe how using additional terms in the approximating series for e^x affects the outcome. **[H]** Quiz question: Write $1 + i$ in exponential form. **[H]**	F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **[H]**	2 lessons

Unit 10 Overview

Unit Title: Limits and Derivatives

Unit Summary:
This unit introduces students to the staples underlying all topics in calculus: limits, tangent lines, derivatives, and area under a curve. They will begin to learn formal notation, will evaluate limits graphically and algebraically, and will use slope to evaluate instantaneous rates of change. Students will then investigate the limit definition of the derivative and as time and understanding permits, students will be introduced to some basic derivative rules. They will also use sums to calculate areas under curves.

Suggested Pacing: 15 lessons

Learning Targets

Unit Essential Questions:
- Why do we measure rates of change in mathematics?
- What real world examples require understanding rates?
- How can we represent change graphically?

Unit Enduring Understandings:
- Observing patterns is key to understanding how functions behave.
- Calculus is often used to describe change in one quantity relative to another quantity.
- Calculus relates algebra and geometry in an intricate way.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc ...

Summative Assessment: Unit 10 assessment
This assessment will contain a variety of multiple choice, open-ended, and/or short answer questions that will be common across all Precalculus/Honors Precalculus classes with flexibility for each teacher and level course to differentiate with additional questions, as appropriate. The assessment will contain questions that address the following concepts: limits, tangent lines, derivatives, and area under a curve (evaluating limits graphically and algebraically, using slope to evaluate instantaneous rates of change, the limit definition of the derivative, basic derivative rules, and calculating areas under curves).

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJSLS)	**Pacing**
Estimate limits from a graph	Content: Limit notation One sided limit Two sided limits Limit independence of function value Limit at a point Unbounded behavior Limits that do not exist Limits at infinity Skills: Use limit notation Estimate a limit at a point Estimate limits whose function value doesn't exist Estimate one sided limits to estimate limits at infinity Estimate limits that become infinite at a point Estimate limits of functions with oscillating behavior	Group activity: Provide each group of students with several different functions along with questions such as "find $\lim_{x\to 2}\frac{x-2}{x^2-4}$" and others. Make sure there is a mix of different types of limits. Provide another set of answers. Have students use a graph/table to estimate each limit and match questions with answers. Repeat with limits at infinity. Problems can be taken from McGraw Hill Chapter 12 Assessment or Practice worksheet p. 7 Quiz question: Give an example of a limit that does not exist. Explain and justify by providing a graph and/or table. Word problem practice 12-1: McGraw-Hill eTeacher edition, Worksheets p. 8 Chapter 12.	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them	3 lessons
Evaluate limits algebraically	Content: Limit at a point Limit sum property Limit difference property Limit scalar multiple property Limit product property Limit quotient property Limit power/root property Indeterminate form Skills: Use limit properties Calculate limits by direct substitution Evaluate limits by removing factorable discontinuities Rationalize denominators to evaluate limits Evaluate limits at infinity Evaluate limits of reciprocal functions at infinity Evaluate limits of rational functions that tend toward infinity (end behavior)	Limits by direct evaluation worksheet: http://cdn.kutasoftware.com/Worksheets/Calc/01%20-%20Limits%20by%20Direct%20Evaluation.pdf Journal/quickwrite: Try to evaluate $\lim_{x\to 2}\frac{x-2}{x^2-4}$ by plugging in the function value 2. Explain what went wrong and how you can use algebra to simplify and find the limit. Support your answer with a graph or table. Practice worksheet 12-2 www.connectED.mcgraw-hill.com Chapter 12 worksheets p. 13 Quiz question: If R(x) is a rational function, is it sometimes, always, or never true that $\lim_{x\to c} R(x) = R(c)$? Justify your reasoning by providing specific examples. Enrichment 12-2 www.connectED.mcgraw-h	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them	3 lessons

	The Squeeze Theorem **[H]**	ill.com Chapter 12 worksheets p. 15 **[H]**		
Use limits to calculate the slope of a tangent line (a.k.a. instantaneous velocity) at a point	Content: Tangent line Slope at a point Instantaneous rate of change Average velocity Instantaneous velocity Skills: Evaluate the difference quotient Find the slope of a tangent line Find the slope of a graph at any point Find average velocity Find instantaneous velocity	Graphing Technology Lab: 12-3 The Slope of a Curve, p. 757 www.connectED.mcgraw-hill.com Quiz question: As part of a physics experiment, a ball is thrown upward. The height of the ball is $s(t) = -16t^2 + 95t + 15$, where t is seconds and the height is in feet. What is the ball's average velocity between t = 1 and t = 2? What is the ball's instantaneous velocity at t = 2 seconds? How do these values compare Mid-Chapter Quiz 12-1 to 12-3 www.connectED.mcgraw-hill.com p. 765	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them CRP11 Use technology to enhance productivity 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal, and or social needs.	2 lessons
Find the derivative of a function and use it to solve problems.	Content: Definition of the derivative Differentiation Differential operator Power rule Constant Rule Constant Multiple Rule Sum/Difference Rule Product Rule **[H]** Quotient Rule **[H]** Extreme value theorem **[H]** Skills: Find the derivative at any point using the definition Use derivative rules to find derivatives **[H]** Solve problems involving derivatives	Partner activity: give students several functions and ask them to find the derivative using the definition. Then use rules to find the derivatives. Compare answers. Questions can come from http://cdn.kutasoftware.com/Worksheets/Calc/03%20-%20Definition%20of%20the%20Derivative.pdf Journal/quickwrite: Why is it important to know the definition of the derivative when more simple rules exist for evaluating them? Practice 12-4 Chapter 12 worksheets p. 24 www.connectED.mcgraw-hill.com Additional Example 5, McGraw Hill eTeacher's edition p. 769. The height of a person jumping off a trampoline can be defined by $h(t) = 4 + 5t - 2t^2$ on the interval [0,3], where time is given in seconds. Find the maximum and minimum heights of the jump. Use the Extreme Value Theorem to justify that there must be both a maximum and minimum. **[H]**	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them **[H]**	4 lessons
Estimate the area under a curve. **[H]**	Content: **[H]** Area under a curve Definite Integral	Connect to AP Calculus, Riemann Sum Chapter 10 www.connectED.mcgraw-hill.com p. 650 **[H]**	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them **[H]**	2 lessons

	Riemann sum Integration Skills: **[H]** Use rectangles to approximate the area under a curve Evaluate an integral using summation formulas	Guided Practice p. 779 **[H]** Journal/quickwrite: How do left and right endpoint approximation methods compare? Will one always give an overestimate or underestimate? Justify with a graph. **[H]** Practice 12-5 Area Under a Curve and Integration www.connectED.mcgraw-hill.com Chapter 12 worksheets p. 29 **[H]**		
Use the Fundamental Theorem of Calculus to solve problems. **[H]**	Content: Antiderivative Antiderivative Rules Fundamental Theorem of Calculus Skills: Find antiderivatives Find indefinite integrals Calculate area under a curve Evaluate definite integrals	Journal/quickwrite: Derivatives give expressions for the slope of a tangent line. Anti-derivatives give expressions to help evaluate the area under a curve. Explain how these two concepts are related. Provide a diagram to support your reasoning. **[H]** Quiz question: Graph $3x^2$ on the interval [1,2]. Estimate the area under the curve using RAM. Next, Find an antiderivative for $5x^4$ and evaluate at $x = 1$ and $x = 2$. How can this help you find the area under $3x^2$ between 1 and 2? **[H]** McGraw-Hill eTeacher's edition Quiz 4 p. 38 Chapter 12 resources **[H]**	CRP 8 Utilize critical thinking to make sense of problems and persevere in solving them **[H]** MP.1-MP.8 **[H]**	1 lesson