Madison Public Schools

Calculus

Grade 12

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Course Overview

Description

The Calculus curriculum focuses on the following three main areas of calculus - the study of limits, differentiation, and integration. General theory is developed and applications are made to real-world situations. Students are required to know, occasionally prove, and apply theorems based on the fundamental concepts of calculus. Calculus topics are presented four ways: geometrically, numerically, algebraically and verbally. Topics include the study of functions, finding derivatives by definition, finding derivatives by the rules and their application, defining integrals and their applications, differential equations and approximations.

Goals

This course aims to:

- enable students to make sense of various types of problems and the reasonableness of their answers
- build student confidence with the various approaches to solving a problem and persevere in solving them
- encourage students to become abstract thinkers who make sense of quantities and their relationships in problem situations
- develop students' ability to cooperatively discuss, make conjectures and critique ideas of one another
- use, apply, and model mathematics to solve problems arising in everyday life, society, and the workplace
- consider the variety of available tools when solving a mathematical problem
- communicate mathematical ideas precisely and effectively to others
- determine a pattern or analyze structure within mathematical content to apply to related ideas
- use repeated reasoning to follow a multi-step process through to completion

Materials

Core: Calculus Early Transcendental Functions fourth Ed
Supplemental: Khan Academy, Various websites related to Calculus

Resources

Suggested activities and resources page

Benchmark Assessments

Common Benchmark Assessment are given for each unit with common types of questions across the levels including problems that focus on the main ideas and anchor standards of the course.

Modifications and Adaptations for Special Needs Learners

(Gifted and Talented Students, English Language Learners, Special Education Students, At-Risk Students)

Scope and Sequence
(Pacing Guide)

Unit Number	Topic of Study	Duration (Lessons Taught)
1	Preparation for Calculus	17 lessons
2	Limits and Continuity	21 lessons
3	Differentiation	22 lessons
4	Applications of Differentiation	28 lessons
5	Integration	18 lessons
6	Applications of Integration	13 lessons
7	Differential Equations	8 lessons

Unit 1 Overview

Unit Title: Preparation for Calculus

Unit Summary:
This unit provides the opportunity to review how to find, graph and compare mathematical models for different data sets. The students will reflect on and think through problems involving linear models and rates of change as well as functions and their graphs. Mathematical modeling using the many different types of functions including linear, quadratic, cubic, rational and trigonometric is practiced and applied with the use of technology. The unit concludes with the development and applications involving inverse functions, logarithmic functions and exponential functions.

Suggested Pacing: 17 lessons

Learning Targets

Unit Essential Questions:
- Without a complete understanding of functions is it possible to understand calculus?
- How are functions used in solving problems involving real-world scenarios?

Unit Enduring Understandings:
- Analysis of important elements of functions is essential to calculus.
- Functions can be analyzed graphically and tabularly.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc...

Summative Assessment: Assessment for Unit 1

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJSLS)	Pacing
Sketch the graph of multiple types of functions including linear, quadratic, absolute value, cubic, rational and radical and find its intercepts. Test a graph for symmetry with respect to an axis and the origin. Find the points of intersection of two graphs.	Content: Graph of an equation, symmetry, intercepts, points of intersection, mathematical models Skills: Sketch the graph of an equation by point-plotting. Find intercepts. Test for symmetry with respect to each axis and the origin. Find the point(s) of intersection of the graphs of equations. Use a graphing calculator to graph an equation, test for symmetry, find intercepts and find point(s) of intersection.	Diagnostic Tests: Algebra, Analytic Geometry, Functions, and Trigonometry Graphing Calculator Partner Activity: Students will explore the graphing calculator to find its features to calculate its intercepts, zeros, values at a certain point and points of intersection. Calculus Lab: Finding Zeros of a Function http://www.jamesrahn.com/CalculusI/PDF/finding%20zeros%20of%20a%20function.pdf	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9 Compare properties of two functions each represented in a different way (algebraically,graphically, numerically in tables, or by verbal descriptions). F.BF.1 Write a function that describes a relationship between two quantities. 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.	1 lesson
Find the slope of a line passing through two points. Write an equation of a line given a point and a slope. Interpret slope as a ratio or as a rate in real-life application. Sketch the graph of a linear equation in slope intercept form.	Content: Slope, equations of lines, rates of change and ratios, graphing linear models, parallel and perpendicular lines Skills: Find slope given two points. Find the slope and y-intercept of a line.	Partner Activity: Have students complete a circuit activity of slope and equation of a line and parallel and perpendicular line problems.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by	2 lessons

Write equations of lines that are parallel or perpendicular to a given line.	Find and sketch an equation of a line that passes through a given point and has a given slope. Find and sketch an equation of a line that passes through two given points.		hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.BF.1 Write a function that describes a relationship between two quantities. RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words. SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.	
Use function notation to represent and evaluate a function. Find the domain and range of a function and relate it to the quantitative relationship it describes, if applicable. Sketch the graph of a function. Identify different types of transformations of functions. Classify and compare functions and recognize combinations of functions.	Content: Function notation, domain and range, piecewise functions, transformations of functions, odd and even functions, combinations of functions Skills: Evaluate a function at a given value. Find the domain and range of a function. Sketch graphs of functions. Determine whether y is a function of x given an equation or a graph. Sketch the graph of function and its parent function and describe the transformation. Determine whether a function is odd, even, or neither.	Calculus Lab: Transformation of Functions http://www.teacherspayteachers.com/Product/Transformations-of-Functions-860670 Quiz: Graphs,and Models, Linear Models and Rates of Change and Functions and their Graphs	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	2 lessons

			CRP11 Use technology to enhance productivity.	
Fit a linear, quadratic or trigonometric model to a real-life data set.	Content: Linear model, quadratic model, trigonometric model Skills: Plot data and use the regression capabilities of the a graphing calculator to model the data.	Partner Calculator Activity: Plot data and use the regression capabilities of a graphing calculator to model the data.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	2 lessons
Verify that one function is the inverse function of another function. Determine whether a function has an inverse function. Develop properties of the six inverse trigonometric functions.	Content: Inverse functions, one-to-one, inverse trigonometric functions Skills: Match the graph of a function with graph of its inverse function. Show that two functions are inverses analytically and graphically. Use the horizontal line test to determine if a function is one-to-one. Find the inverse of a function. Describe the relationship between the graph of a function and the graph of its inverse. Evaluate inverse trigonometric functions without using a calculator. Solve equations involving inverse trigonometric functions.	Jigsaw lesson: Students break into six groups each assigned one of the six trigonometric functions. Each group will study and present to the class how to graph their trigonometric functions' parent graphs and its inverse. Quiz Question: Solve $arctan(2x-3) = \frac{\pi}{4}$. Open-ended Class Discussion: Keeping right triangle trigonometry in mind, how would you find cos y given y = arcsin x where $0 < y < \pi/2$?	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	3 lessons
Develop and use properties of exponential functions. Understand the definition of Euler's number (*e*). Understand the definition of the natural logarithmic function.	Content: Exponential functions, the number *e*, the natural logarithmic function, properties of logarithms Skills:	Writing Activity: In your own words, state the properties of the natural logarithmic function. Common Benchmark Assessment for Unit 1	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this	2 lessons

Develop and use properties of the natural logarithmic function.	Evaluate and simplify expressions involving exponents and logarithms. Solve exponential equations and logarithmic equations. Sketch graphs of exponential equations and logarithmic equations. Find inverses of exponential and logarithmic functions. Use properties of logarithms to condense and expand logarithmic expressions.		classroom and during structured learning experiences. F.LE.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.	

Unit 2 Overview

Unit Title: Limits and Continuity

Unit Summary:
This unit explores the limit process as a fundamental concept of calculus. Students will have the opportunity to preview calculus and to acquaint themselves with the tangent line problem and area problem as essential to calculus. In this unit limits are initially studied particularly from a graphical and numerical standpoint. Students then move toward computing limits analytically using various algebraic techniques and apply The Squeezing Theorem to evaluate trigonometric limits. This unit introduces the terms "continuous" and "discontinuous" into the student's mathematical vocabulary. Students will explore graphically and algebraically where functions are and are not continuous and delve into the Intermediate-Value Theorem, an important outcome of continuity. Finally, students will have the opportunity to work with limits at extremely large and extremely small values of x leading to the concept of limits at infinity. Students will acquire an intuitive feel for the notion of "approaching infinity" and how it relates to the end-behavior of a function.

Suggested Pacing: 21 lessons

Learning Targets

Unit Essential Questions:
- Why are limits essential to calculus?
- What is the best method to use to find the limit of a function?
- What impact does continuity have on the limit of a function?
- How can we use limits to find instantaneous rates of change?
- How can limits be used to help describe the behavior of a function?

Unit Enduring Understandings:
- Calculus is based upon limits.
- Limits can be determined using algebra, graphs and/or tables of data.
- Limits do not require continuity.
- Some graphs demonstrate asymptotic and unbounded behavior.

• Close values of the domain lead to close values of the range.

Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc...
Summative Assessment: Unit 2 Assessment
Alternative Assessment: Students will create a multimedia presentation to report on their findings from the "investigating limits at infinity" lab activity

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards (NJSLS)**	**Pacing**
Understand what calculus is and how it compares with precalculus. Explain how the tangent line problem and the area problem are basic to calculus.	Content: The tangent line problem, the area problem Skills: Find slopes of secant lines. Estimates slopes of tangent lines. Approximate areas under curves by finding the areas of rectangles.	Discovery-based Partner Activity: Intro to Calculus (written and prepared by James Rahn) http://www.jamesrahn.com/CalculusI/PAGES/intro_to_calculus.htm Quiz, quick-write, group discussion, or homework problem: Explain how the tangent line problem and the area problem are basic and fundamental to calculus.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.9 Compare properties of two functions each represented in a different way (algebraically,graphically, numerically in tables, or by verbal descriptions).	2 lessons
Use graphical, numerical and tabular methods to estimate limit values. Explore the circumstances under which limits fail to exist (ex. oscillating behavior asymptotic behavior and differing behavior from the left and the right).	Content: Definition of a limit Skills: Evaluate limits using a graph or table.	Calculator Activity: Have students graph a rational function that exhibits a removable discontinuity (i.e. a hole). Explore both algebraically and graphically what happens as values get extremely close the value of x where has a hole. Graph examples of trigonometric functions that have oscillating behavior.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to	4 lessons

		Quiz: A Preview of Calculus and Finding Limits Graphically and Numerically	another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	
Use the properties of limits to develop strategies to evaluate limits of polynomial, rational, radical, and trigonometric functions.	Content: Properties of limits, polynomial functions, rational functions, radical functions, trigonometric functions, The Squeezing Theorem Skills: Compute limits using various algebraic techniques.	Think-Write-Pair-Share: For the problem $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$ students will jot down their ideas before turning to a partner to discuss them. Then they will pair up and compare their ideas and come up with a strategy for completing the problem. Group Work Activity: Two lists - one with a limit to evaluate and one with the solution. Students will work together to match the limit with its solution. Calculator Activity: Students graph *f(x) = (sin x)/x* on calculator to determine its limit as x approaches 0. Partner Activity: Pairs of students work together to put the step of The Squeezing Theorem proof (given on cards) in the correct order. Quiz: Evaluating Limits Analytically	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.3a	5 lessons

			Factor a quadratic expression to reveal the zeros of the function it defines. F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. CRP4 Communicate clearly and effectively and with reason.	
Determine if a function is continuous at a point. Identify types of discontinuities. Distinguish between one-sided (left-hand and right-hand) limits and two-sided limits and explain what it means for such limits to exist.	Content: Continuity at a point, discontinuity, properties of continuity, continuous functions, right-hand limit, left-hand limit, Intermediate Value Theorem Skills: Find values for which a function is discontinuous and determine whether the value is a removable discontinuity. Find an unknown value that will cause a function to be continuous everywhere. Locate discontinuities and find limits for functions involving trigonometry.	Open-ended Group Discussion: What impact does continuity have on the limit of a function? Calculus Lab: The Intermediate Value Theorem http://www.teacherspayteachers.com/Product/Intermediate-Value-Theorem-An-Investigation-853944 Quiz: Continuity and One-sided Limits	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue. SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience. NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. NJSLSA.W4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a	5 lessons

			coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	
Understand limits at infinity and relate them to horizontal and vertical asymptotes of graphs. Evaluate limits, at infinity possibly by using shortcuts for polynomial, rational, and/or algebraic functions. Describe asymptotic behavior in terms of limits involving infinity.	Content: Limits involving infinity, vertical and horizontal asymptotes Skills: Compute limits involving infinity. Find vertical asymptotes of a function.	Exploration Activity: Students will discuss what happens to the function, f(x) = 1/x as x gets extremely large in both the positive and negative direction. Calculus Lab: Investigating Limits at Infinity. http://www.teacherspayteachers.com/Product/Limits-as-x-Approaches-Infinity-An-Investigation-861891 Common Benchmark Assessment for Unit 2	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. CRP4 Communicate clearly and effectively and with reason. CRP11 Use technology to enhance productivity.	3 lessons

Unit 3 Overview
Unit Title: Differentiation
Unit Summary: This unit begins with developing a close connection between finding the tangent line to a curve and the concept of the derivative. The derivative function is defined formally and the role of the derivative as a "slope-producing function" is fully explored. After the students have a firm grasp on the idea of the derivative representing slope they can move on to developing and using various techniques of differentiation including the power rule, the constant multiple rule, the sum and difference rules, the product and quotient rules and the chain rule. This unit emphasizes the different forms of derivative notation and the notations for higher order derivatives. The derivatives for the six trigonometric functions are introduced and the students will have the opportunity to work out the proof for the derivative of sin *x*. This unit also contains the derivatives for logarithmic functions, exponential functions. The method of implicit differentiation developed,

emphasizing what it means to say that an equation defines a function implicitly. This unit concludes with derivatives of inverse functions including inverse trigonometric functions.
Suggested Pacing: 22 lessons
Learning Targets
Unit Essential Questions: ● How can one use the concept of the limit to determine instantaneous rates of change? ● How does one use derivatives to analyze the behavior of functions? ● How is differentiability and continuity related? ● How does the idea of the derivative as a rate of change help us to understand the relationships between position, velocity and acceleration?
Unit Enduring Understandings: ● The derivative is an instantaneous rate of change. ● The derivative of position is velocity and the second derivative of position is acceleration. ● The average rate of change corresponds with slope of a secant line and the instantaneous rate of change corresponds with slope of a tangent line.
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc...
Summative Assessment: Unit 3 Assessment

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards (NJSLS)**	**Pacing**
Find the slope of a tangent line to a point on a curve Find the derivative of a function using the limit definition. Determine whether a function is differentiable or not over a given interval Understand the relationship between differentiability and continuity.	Content: Tangent lines, definition of a derivative function, differentiability, sharp turn Skills: Use the limit definition to find a derivative. Determine whether a function is continuous and differentiable at a given value of x. Estimate derivative values given the graph of a function. Find the equation of a tangent line at a given value of x.	Open-ended Class Discussion: How can you apply the definition of a limit to find the slope of a function at a given point? Quiz question: Suppose that the line 2x + 3y = 5 is tangent to the graph of y = f(x) at x = 1. Find the value of f(1) and $f'(1)$. NCTM Illuminations: Interactive Calculus Tool - Tangent http://illuminations.nctm.org/Activity.aspx?id=3570 Calculus Lab: Connecting Continuity with Differentiability	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2	3 lessons

		http://www.teacherspayteachers.com/Product/Connecting-Continuity-with-Differentiability-863635 Quiz: The Derivative and the Tangent Line Problem	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	
Compute derivatives using the constant rule, power rule, constant multiple rule, and the sum and difference rules. Evaluate derivatives of sine, cosine and exponential functions. Use derivative to find rates of change.	Content: Constant rule, power rule, constant multiple rule, the sum and difference rules, average velocity, instantaneous velocity Skills: Find derivatives using the various techniques of differentiation. Find derivatives of sine, cosine and exponential functions. Find average rates of change over an interval and find instantaneous rates of change at a given point.	I Have...Who Has Card with derivatives. Partner Activity: Have students complete a circuit activity of derivative problems. Quiz: Basic Differentiation Rules and Rates of Change Writing Activity: If f'(x) = g'(x), does f(x) = g(x)? Explain.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. A.SSE.1 Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience. NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners,	2 lessons

			building on others' ideas and expressing their own clearly and persuasively. NJSLSA.W4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	
Determine derivatives using the product and quotient rule. Find derivatives using trigonometric functions. Compute higher order derivatives.	Content: Product rule, quotient rule, derivatives of trigonometric functions, , second and higher order derivatives Skills: Find derivatives using the product rule and the quotient rule. Find first and second derivatives of trigonometric functions.	I Have...Who Has Card with derivatives. NCTM Illuminations: Interactive Calculus Tool - Derivatives http://illuminations.nctm.org/Activity.aspx?id=3570 Partner Activity: Supply steps on cards to a fully worked out product rule derivative problem. Have the students order the steps and then work together to try to come up with the rule. Students discuss and share ideas with the class.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated case	3 lessons
Compute derivatives using the chain rule. Differentiate the natural logarithm function and the exponential function. Compute higher order derivatives.	Content: Chain rule, least powers, derivatives of exponential functions, derivatives logarithmic functions Skills: Find derivatives using the chain rule. Find dy/dx for functions involving logarithms, and exponentials.	Partner Calculator Activity: Work with a partner on Discovering the Chain Rule through the Graphing Calculator http://www.jamesrahn.com/CalculusI/PDF/DISCOVERING%20THE%20CHAIN%20RULE%20THROUGH%20THE%20GRAPHINGCALCULATORIn%20this.pdf Writing Activity: Find the first eight derivatives of y = sin x . What do you observe? Discuss finding as a class.	N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. A.SSE.1 Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	4 lessons

		Partner Activity: Have students complete a circuit activity of derivative problems involving logarithmic, exponential, and chain rule. Quiz: The Chain Rule	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs. CRP11 Use technology to enhance productivity.	
Explain the meaning of an implicitly defined function. Use implicit differentiation to find dy/dx and d^2y/dx^2.	Content: Implicit differentiation Skills: Use implicit differentiation to find derivatives of functions that are not explicitly defined. Use implicit differentiation to find the slope of line at a specified point.	Quiz question: The tangent line to the graph of x + y + xy = 3 at the point (1, 1) is ________. Quiz: Implicit Differentiation	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	4 lessons
Compute derivatives of inverse functions. Differentiate inverse trigonometric functions.	Content: Differentiability of inverse functions, derivatives of an inverse function, derivatives of inverse trigonometric functions Skills: Find dy/dx for functions involving inverse trigonometry. Find the derivative of a function's inverse.	Guided Class Discussion: Find the derivative of y = arcsin x. Give the hint that y = arcsin x can be rewritten sin y = x and then group students off and ask guiding questions to have the groups arrive at its derivative. Students will continue to work in their groups to find the other five remaining inverse trigonometric functions derivatives. Use Calculus in Motion: Inverse Function's Derivative to motivate a class open-ended discussion of how to find derivatives of inverse trigonometric functions. http://www.calculusinmotion.com/gsp4.html Common Benchmark Assessment for Unit 3	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing	3 lessons

			key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	

Unit 4 Overview
Unit Title: Applications of Differentiation
Unit Summary: This unit focuses on using calculus to analyze graphs of functions. This unit explores the graphical attributes of increasing, decreasing, and concavity for a function and provides sufficient conditions for determining these attributes from the signs of the first and second derivatives. An inflection point is defined and discussed in applications. Students are introduced to the vocabulary used to describe high and low places on a function's graph and where to look for them. The students become very familiar with the First and Second Derivative Test as fundamental calculus tools used to identify and classify locations of relative extrema. This unit defines absolute maximum, minimum and extremum for a function on an interval and surveys the possible existence and locations for absolute extrema. Students will have the opportunity to build skills and techniques for solving optimization problems and related rates problems that exhibit areas where calculus can be used in real-life situations. Students get a glimpse of the range of theorems upon which the tools of calculus were built, including the Extreme Value Theorem, Rolle's Theorem and the Mean Value Theorem. Finally, tangent line approximation and differentials are studied and applied in real-life settings.
Suggested Pacing: 28 lessons
Learning Targets
Unit Essential Questions: ● What information does the derivative tell us about a graph? ● How do rates of change relate in real-life situations? ● How do companies use derivatives to maximize profit and minimize cost?
Unit Enduring Understandings: ● Derivatives have both theoretical and real-life applications. ● The first and second derivative tests can be used together with skills from prior math courses to accurately graph a wide variety of functions. ● Derivatives are an underlying concept supporting physical applications that are rooted in many fields.
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc...

Summative Assessment:
Assessment for Unit 4 - Part 1
Assessment for Unit 4 - Part 2

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJSLS)	Pacing
Find methods for finding the high and low points on the graph of a function. Locate the largest and smallest function over a particular interval.	Content: Critical numbers, relative maxima and minima, relative extrema, absolute maxima and minima, absolute extrema, The Extreme Value Theorem Skills: Find and explain what the critical numbers of a function are. Determine whether a function has a relative maximum, relative minimum, absolute maximum, absolute minimum, or none of these at a critical number. Locate absolute extrema of a function on a closed interval.	Writing Activity: Explain why the function y = tan x has a maximum on $[0, \pi/4]$ but not on $[0, \pi]$. Students can create flashcards of the derivative graphs of the function graphs provided to them. Students can then get into groups where they can match function graphs to their derivative graphs. Calculator Exploration Activity: Finding Extreme Values http://www.jamesrahn.com/CalculusI/PDF/Exploration%20Finding%20Extreme%20Values.pdf	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	2 lessons
Explore and discuss the important consequences of the Mean-Value Theorem.	Content: Rolle's Theorem, Mean-Value Theorem, Constant Difference Theorem Skills: For a given function, verify the hypotheses of Rolle's Theorem and find all the values in the interval that satisfy Rolle's Theorem. For a given function, verify the hypotheses of Mean-Value Theorem and find all the values in the interval that satisfy Mean-Value Theorem.	Think-Pair-Share: Find a function such that its graph contains the point (1, 5) and such that for every value of x_0 the tangent line to the graph of the function at x_0 is parallel to the tangent line to the graph of $y = x^2$ at x_0. Calculus Lab: An Introduction to the Mean-Value Theorem http://www.teacherspayteachers.com/Product/Mean-Value-Theorem-An-Introduction-934619 Writing Activity: Let *f* be continuous on [a, b] and differentiable on (a, b). If there exists c in (a, b) such that *f′(c)* = 0, does it follow that f(a) = f(b)? Explain your reasoning. Quiz: Extrema on an Interval and Rolle's Theorem and the Mean-Value Theorem	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	2 lessons
Determine intervals on which a function is increasing or decreasing.	Content: Increasing and decreasing function, relative extrema, First Derivative Test	Class Open-Ended Discussion: How would you go about determining the intervals on	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.	3 lessons

Apply the First Derivative Test to find relative extrema of a function.	Skills: Use sign analysis to identify the intervals on which a function is increasing or decreasing. Use the first derivative test to locate relative extrema.	which a function is increasing or decreasing? Quiz Question: If $f(x)$ has derivative $f'(x) = (x-4)^2 e^{-x/2}$, then find the intervals on which a function is increasing or decreasing. Students can create flashcards of the derivative graphs of the function graphs provided to them. Students can then get into groups where they can match function graphs to their derivative graphs. Think-Write-Pair-Share: For the problem $f(x) = x^4 - 12x^3$ students will first find the critical points. Next they will jot down their ideas about how to determine if each critical point is a maxima or a minima. They will then turn to a partner to discuss and compare their ideas and come up with a strategy for completing the problem. Section Project: Rainbows (Calculus Early Transcendental Functions, p. 229) Quiz: Increasing and Decreasing Functions and the First Derivative Test	9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
Determine intervals on which a function is concave up or concave down. Determine locations where the graph of a function changes concavity. Apply the Second Derivative Test to find relative extrema of a function.	Content: Second Derivative Test, concavity, inflection points Skills: Use sign analysis to find open intervals on which the function is concave up or concave down. Locate inflection points of a graph a function. Use the second derivative test to locate relative extrema. Sketch graphs of functions having as set of given characteristics.	Open-ended Class Discussion: How do we use the second derivative to sketch the graph of the original function? What information does the second derivative give us? Quiz Question: If $f(x)$ has derivative $f'(x) = (x-4)^2 e^{-x/2}$, then find the intervals on which a function is concave up or concave down. Writing Activity: Consider a function *f* such that *f'* is increasing. Sketch a possible graph of *f* for (a) f' < 0 and (b) f' > 0. Quiz: Concavity and the Second Derivative Test	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5	3 lessons

			Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
Understand limits at infinity and relate them to horizontal asymptotes of graphs. Describe asymptotic behavior in terms of limits involving infinity.	Content: Limits involving infinity, vertical and horizontal asymptotes, infinite limits at infinity Skills: Compute limits involving infinity. Match a function with its graph by analyzing the horizontal asymptotes.	Exploration Activity: Students will discuss what happens to the function, f(x) = 1/x as x gets extremely large in both the positive and negative direction. Calculus Lab: Investigating Limits at Infinity http://www.teacherspayteachers.com/Product/Limits-as-x-Approaches-Infinity-An-Investigation-861891 Quiz: Limits at Infinity	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible. SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience. NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. NJSLSA.W4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. CRP11 Use technology to enhance productivity.	3 lessons

Explore and determine procedures for sketching graphs of functions. Describe the interaction between calculus and technology in curve sketching.	Content: Properties of graphs - symmetries, x- and y-intercepts, relative extrema, intervals of increase and decrease, asymptotes, periodicity, concavity, inflection points, end behavior, oblique and curvilinear asymptotes, vertical tangents and cusps Skills: Graph rational functions and finding and labeling key feature of the graph. Graph functions and identify the locations of all critical points and inflection points.	Small Group Discussion: Given function $f(x) = 6x^{1/3} + 3x^{4/3}$, graph the function first by finding the symmetries, x- and y-intercepts, relative extrema, intervals of increase and decrease, asymptotes, concavity,inflection points end behavior, oblique and curvilinear asymptotes, vertical tangents and cusps. Then as a group notice something interesting happening a x = 1 and make note of your observations. Partner Activity: Match graphs of functions with their derivative graphs. Calculus Lab: Curve Sketching http://www.jamesrahn.com/CalculusI/PDF/CURVESKETCHING.pdf Whole Class Card Game: Each person gets a card or two and they must find their partners. Students will not know if they have f, f ' or f ". Once they have found the complete set, they can then pick out the equation. http://www.teacherspayteachers.com/Product/Match-the-Graphs-of-f-f-and-f--884947 Partner Quiz: Curve Sketching Common Benchmark Assessment for Unit 4 - Part 1	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	3 lessons
Find the rate at which one quantity is changing by relating the quantity to other quantities whose rates of change are known.	Content: Related rates problems, strategies for solving related rates problems Skills: Solve various related rates problems.	Use Calculus in Motion: Related Rates to work on various problems generated whole class discussion of how to approach each problem. http://www.calculusinmotion.com/gsp4.html Calculus Lab: Related Rates http://www.teacherspayteachers.com/Product/Related-Rates-An-Introduction-934273	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.BF.1 Write a function that describes a relationship between two quantities. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	3 lessons

Solve applied optimization problems.	Content: Finite closed intervals, revenue, profit, cost, marginal analysis, procedure for solving applied maximum and minimum problems Skills: Solve various types of optimization problems.	Take-home Writing Activity: Is it theoretically possible to have a box with 0 width? Fully explain and use outside resources if desired. Use Calculus in Motion: Optimization to work on various problems generated whole class discussion of how to approach each problem. http://www.calculusinmotion.com/gsp4.html Partner Optimization Activity: http://www.jamesrahn.com/CalculusI/optimization.htm Section Project: Connecticut River (Calculus Early Transcendental Functions, p. 270) Quiz: Related Rates and Optimization	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.BF.1 Write a function that describes a relationship between two quantities. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	3 lessons
Understand the concept of tangent line approximation. Compare the value of the differential, *dy*, with the actual change in y, Δy. Estimate propagated error using a differential.	Content: Local linear approximation Skills: Use differentials to find percentage error. Use differentials to estimate error in calculations. Find the differential *dy*. Find formulas for *dy* and Δy.	Quiz question: Find an equation for the local linear approximation to $y = 5 - x^2$ at $x_o = 2$. Think-Pair-Share: Estimate the error in calculation using differentials. Common Benchmark Assessment for Unit 4 - Part 2	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key	3 lessons

			features of the graph, by hand in simple cases and using technology for more complicated cases.	

Unit 5 Overview

Unit Title: Integration

Unit Summary:
This unit provides the language and techniques for finding antiderivatives. Students will solve initial condition problems and problems involving vertical motion. The students are provided with the tools to find the area of a plane region by using the rectangle method. Students will learn sigma notation and use it to define the area between the graph of a function and an interval on the x-axis in terms of a limit. This unit describes the definite integral in terms of Riemann sums and emphasizes that a definite integral is simply a number defined by a particular limiting process.
In this unit, students will work with basic antidifferentiation formulas and interpret the process of antidifferentiation geometrically in terms of an area of a known region. The students move on to more complicated indefinite integrals in which the u-substitution method is necessary. The students are provided with the techniques for computing definite integrals by using Part I of the Fundamental Theorem of Calculus. An essential outcome of this unit is for students to interpret an integral as an "accumulated change."

Suggested Pacing: 18 lessons

Learning Targets

Unit Essential Questions:
- How can we approximate area under a curve?
- Given the rate of change of a quantity, how do we find the original quantity?
- How can definite integrals be used to solve real-world problems?

Unit Enduring Understandings:
- Differentiation and integration are inverse operations.
- The definite integral can take on many interpretations, including precise area under a curve, distance traveled, net change in temperature, etc.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc...

Summative Assessment: Unit 5 Common Benchmark Assessment

Objectives (Students will be able to…)	Essential Content/Skills	Suggested Assessments	Standards (NJSLS))	Pacing
Develop and use fundamental formulas and properties of antidifferentiation.	Content: Antiderivatives, indefinite integral, integrand, integration formulas, properties of the indefinite integral, integral curves, initial value problems, initial condition Skills: Evaluate indefinite integrals using the power formula for integration. Evaluate trigonometric integrals. Evaluate integrals first by rewriting the integrand appropriately.	Quiz Question: Evaluate the integral $\int \frac{1}{1+sinx}dx$ by multiplying the numerator and the denominator by an appropriate expression. Think-Write-Pair-Share: Given a set of functions students, ask students "What function has the given function as derivative?" Class will use this activity to come up with the power formula for integration. Exit Ticket: Write the definition of the power formula for integration in your own words. Quiz: Antiderivatives and Indefinite Integration	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	3 lessons
Use sigma notation to write and evaluate a sum. Approximate the area of a plane region using the rectangle method and trapezoidal approximation. Use sigma notation to express lengthy sums in a compact form.	Content: The Area Problem, rectangle method for finding areas, trapezoidal approximation, antiderivative method for finding areas Skills: Estimate the area between the graph of function f on a given interval using the rectangle method. Estimate the area between the graph of function f on a given interval using trapezoidal approximation. Use the limit definition for area under a curve to find the area between the graph of a function on a specified interval.	Discovery Learning Partner Activity: The students are given a graph with a pre-drawn curve on the graph paper and are asked to develop a strategy to calculate the area between the curve and the x - axis using geometry. Calculus Lab: Trapezoidal Method for Definite Integrals http://www.jamesrahn.com/CalculusI/PDF/trapezoidal.pdf Use Calculus in Motion: Definition of Integration to motivate a class open-ended discussion of how a the area problems leads us to the definition of an integral. http://www.calculusinmotion.com/gsp4.html Quiz: Area	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.LE.5	3 lessons

			Interpret the parameters in a linear or exponential function in terms of a context.	
Evaluate definite integrals using limits and properties of definite integrals.	Content: Riemann sum, definite integral, properties of definite integrals Skills: Understand and be able to express a definite integral as a limit of Riemann sums. Sketch the region whose area is given by a definite integral and then use geometric formulas to evaluate the integral. Use the properties of definite integrals to evaluate a definite integral.	Calculus Lab: Developing an Understanding of the Definite Integral http://www.jamesrahn.com/CalculusI/PDF/Developing%20an%20Understanding%20for%20the%20Definite%20Integral.pdf Use Calculus in Motion: Riemann sums to generate a whole class discussion of how Riemann sums connect to the definite integral. http://www.calculusinmotion.com/gsp4.html	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities.	3 lessons
Evaluate definite integrals using the Fundamental Theorem of Calculus - Part 1. Use the Fundamental Theorem of Calculus- Part 2 to evaluate definite integrals using antiderivatives. Find the average value of a function over a closed interval. Study and understand the uses of the Mean-Value Theorem for Integrals.	Content: The Fundamental Theorem of Calculus - Part 1, net area, total area, dummy variables, the Mean-Value Theorem for Integrals, The Fundamental Theorem of Calculus - Part 2 Skills: Use The Fundamental Theorem of Calculus - Part 1 to evaluate definite integrals. Evaluate definite integrals involving absolute value. Find area under a curve over a specified interval. Sketch a curve and find the total area between the curve and a specified interval on the x-axis. Use The Fundamental Theorem of Calculus - Part 2 to find a derivative. Find values of x in a specified interval that satisfy the Mean-Value Theorem for Integrals and explain what the values represent.	Quiz Question: Find the total area between the graph of $y = 2x + 2$ and the interval [-4, 2]. Think-Write-Pair-Share: How do you evaluate integrals involving absolute value? AP Practice Multiple Choice Question - Definite Integrals Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer. Calculus Lab: Discovering the Average Value of a Function http://www.jamesrahn.com/CalculusI/PDF/Discovering%20the%20Average%20Value%20of%20A%20Function.pdf Calculus Lab: Understanding The Fundamental Theorem of Calculus - Part 2 http://www.jamesrahn.com/CalculusI/PDF/2ndfundamentaltheorem2.pdf Section Project: Demonstrating the Fundamental Theorem	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process,	3 lessons

		(Calculus Early Transcendental Functions, p. 330) Quiz: Riemann Sums and Definite Integrals and The Fundamental Theorem of Calculus	phenomenon, or concept, resolving conflicting information when possible. SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.	
Study and use the technique of substitution to transform complicated integration problems into simpler ones before integrating.	Content: u-substitution, change of variables, general power rule for integration Skills: Evaluate integrals using appropriate substitutions. Evaluate integrals first by modifying the form of the integrand and using appropriate substitutions. Evaluate definite integrals using appropriate substitutions.	Partner Exploration: Recognizing Patterns (Calculus Early Transcendental Functions, p. 331) AP Practice Multiple Choice Question - Integration by Substitution Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Writing Activity: Explain the connection between the chain rule for differentiation and the method of u-substitution for integration. Quiz: Integration by Substitution Unit 5 Common Benchmark Assessment	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1 Interpret expressions that represent a quantity in terms of its context. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.	4 lessons

Unit 6 Overview
Unit Title: Application of Integration
Unit Summary: This unit provides the students the opportunity to build on the connections they made in the previous unit between the notation for Riemann sums and that of the corresponding integral. With a few modifications the students extend the application of definite integrals from the area of a region under a curve to the area of a region between two curves. When integrating with respect to x the students will interpret the definition for area as an integral in which the integrand is the top curve minus the bottom curve. When integrating with respect to y an alternative formula is considered in the integrand is the right curve minus the left curve. This unit provides the techniques and skills for finding volumes of solids of revolution. Students will learn when it is appropriate to use the disk method or the washer method. This unit expands the concept of finding volumes of solids by slicing to include finding volumes of solids with known cross section. Finally, students will wrap up the study of finding volumes of solids with an alternative method called the shell method because it uses cylindrical shells.
Suggested Pacing: 13 lessons
Learning Targets
Unit Essential Questions: • How can an integral be used to solve problems in various fields including geometry, science and engineering? • How can integrals be applied to finding bounded areas and generated volume?
Unit Enduring Understandings: • Integrals have many uses including measuring the area between curves and finding volumes of solids. • Using Calculus to determine areas and volumes of regions bounded by curves is utilized by in many fields including engineering (for example mechanical engineers use it for dam and water tower designs and construction.)
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc...
Summative Assessment: Unit 6 Assessment
Alternative Assessment: Students will choose and apply an appropriate method to calculate the volume of a three-dimensional solid and present their findings to their peers

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJSLS)	Pacing
Find the area between two curves using integration. Find the area of a region between intersecting curves using integration.	Content: Area of a region between two curves, area of a region between intersecting curves, representative rectangle, reversing the roles of x and y Skills: Set up a definite integral that gives the area of a region given in a diagram. Find the area of a region by integrating with respect to x. Find the area of a region by integrating with respect to y. Sketch a region bounded by the graphs of functions and find the area of the region.	Partner Activity: Approximating the Area of the Nike Symbol http://www.teacherspayteachers.com/Product/Approximating-the-Area-of-the-Nike-Symbol-963427 I Have... Who Has... Cards - Bounded Area http://www.teacherspayteachers.com/Product/I-Have-Who-Has-Cards-Bounded-Area-1037849 Think-Write-Pair-Share: For the problem: Find the area bounded by the graphs $x = 3 - y^2$ and $x = y + 1$ students will the functions first. Next they will jot down their strategies for finding the area of the bounded region. They will then turn to a partner to discuss and compare their ideas and come up with a strategy for completing the problem. Quiz: Areas of Regions Between Two Curves	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	3 lessons
Find the volumes of three-dimensional solids using the disk method, washer method and known cross sections.	Content: Volumes by slicing, solids of revolution, volumes by disks perpendicular to the x-axis, volumes by washers perpendicular to the x-axis, volumes by disks and washers perpendicular to the y-axis, solids of known cross section Skills: Find the volume of a solid of revolution using the disk method. Find the volume of a solid of revolution using the washer method. Find the volume of a solid with known cross section.	Visualizing Volumes of a Solid Project Partner Activity: Exercises Finding Volume http://www.jamesrahn.com/CalculusI/PDF/volumes.pdf Whole Class Activity: Volumes of Solids with Known Cross Sections - An Exploration http://www.teacherspayteachers.com/Product/Volumes-of-Solids-with-Known-Cross-Sections-An-Exploration-865090 Quiz: The Disk Method, Washer Method and Known Cross Sections	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	3 lessons

			F.BF.1 Write a function that describes a relationship between two quantities.	
Find the volumes of three-dimensional solids using the shell method. Compare the uses of the disk method and the shell method.	Content: The Shell Method Skills: Find the volume of a solid formed by revolving a region about the y-axis using the shell method.	Section Project: Saturn (Calculus Early Transcendental Functions, p. 475) Quiz: The Shell Method Unit 6 Common Benchmark Assessment	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities. CRP4 Communicate clearly and effectively and with reason.	3 lessons

Unit 7 Overview (Optional)

Unit Title: Differential Equations

Unit Summary:
This unit provides the solution techniques and applications for first-order differential equations. Students will have the opportunity to study slope fields which provide a graphical approach to learning about the solutions of a differential equation. This unit concludes with showing the student how to use first-order differential equations to create mathematical models of physical phenomena as well as develop basic properties of the exponential growth and decay models.

Suggested Pacing: 8 lessons

Learning Targets

Unit Essential Questions:

- How are differential equations used to model real world problems?

- How can differential equations be analyzed analytically, graphically, and numerically to make predictions?

Unit Enduring Understandings:
- Real life problems can be solved by solving differential equations.
- Solutions to differential equations can be represented graphically, numerically and algebraically.
- Many of the fundamental laws of the physical and social sciences involve rates of change and can therefore be modeled using differential equations.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, etc…

Summative Assessment: Unit 7 Assessment

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards (NJSLS)**	**Pacing**
Solve basic types of differential equations. Use differential equations to model and solve applied problems.	Content: Differential equation, initial-value problems, first-order separable equations, separation of variables Skills: Confirm solutions to differential equations. Find the general solution of a differential equation using separation of variables. Solve initial value problems.	Writing Activity: In your own words, describe how to recognize and solve differential equation that can be solved by separation of variables. Exit Ticket: In your own words, describe the difference between the general solution of a differential equation and a particular solution.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.1 Write a function that describes a relationship between two quantities. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.	2 lessons
Use slope fields to approximate solutions of differential equations.	Content: Functions of two variables, slope fields Skills: Construct basic slope fields for differential equations.	Partner Activity: Given two sets of cards, one with diagrams of slope fields and the other with the corresponding differential equation. Students match the slope	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2	2 lessons

	On the graph of a slope field, sketch the graph of a solution that satisfies a given initial condition.	field with its corresponding differential equation. Calculus Lab: Creating Basic Slope Fields http://www.jamesrahn.com/CalculusI/PDF/creating%20Basic%20Slope%20Fields.pdf	Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	
Model exponential growth and decay using differential equations.	Content: Exponential growth and decay models, population growth, doubling-time, half-life, radioactive decay, carbon dating Skills: Solve applied problems using exponential growth and decay models On the graph of a slope field, sketch the graph of a solution that satisfies a given initial condition.	Quiz Question: Describe that the values of C and k represent in the exponential growth and decay model, $y = Ce^{kt}$. Calculus Lab: Social Diffusion http://www.jamesrahn.com/CalculusI/PDF/social%20diffusion.pdf Unit 7 Common Benchmark Assessment	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.1 Write a function that describes a relationship between two quantities. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	2 lessons