Madison Public Schools
Advanced Placement Calculus BC Curriculum

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Course Overview

Description

AP Calculus BC focuses on conceptual understanding of limits, derivatives, integrals and series. This is implemented by presenting topics four different ways including graphically/visually, numerically, analytically, and verbally. AP Calculus BC offers opportunities for students to develop technical competence and a sense of utility of calculus. The course work encourages students to become logical thinkers, learning to write the solutions to problems in a connected, step-by-step manner with explanatory sentences. Consistent practice with using multiple representations of solutions allows students to develop a comprehensive understanding of each topic studied throughout the year.

Goals

As outlined by The College Board, this course aims for students to be able to:

- work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems.
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

In addition to the goals outlined by the College Board, this course also aims to:

- enable students to make sense of various types of problems and the reasonableness of their answers
- build student confidence with the various approaches to solving a problem and persevere in solving them
- encourage students to become abstract thinkers who make sense of quantities and their relationships in problem situations
- develop students' ability to cooperatively discuss, make conjectures and critique ideas of one another
- use, apply, and model mathematics to solve problems arising in everyday life, society, and the workplace
- consider the variety of available tools when solving a mathematical problem
- communicate mathematical ideas precisely and effectively to others
- determine a pattern or analyze structure within mathematical content to apply to related ideas
- use repeated reasoning to follow a multi-step process through to completion

Materials
Core: Rogawski, J., Cannon, R. J., & W.H. Freeman and Company. (2012). *Rogawski's Calculus for AP*. New York: W.H. Freeman. **Supplemental:** Various websites related to AP Calculus BC
Resources
Suggested activities and resources page
Benchmark Assessments
Benchmark Assessment are given for each unit with problems that focus on the main ideas and anchor standards of the course. Mid-year Full length AP Calculus AB Exam administered at some point in January/February
Modifications and Adaptations for Special Needs Learners (Gifted and Talented Students, English Language Learners, Special Education Students, At-Risk Students)

Scope and Sequence
(Pacing Guide)

Unit Number	Topic of Study	Duration (Weeks Taught)
1	Review of Limits and Continuity	2 weeks
2	Differentiation	4 weeks
3	Applications of Differentiation	3 weeks
4	Integration	2 weeks
5	Applications of Integration	2 weeks
6	Techniques of Integration	2 weeks
7	Further Applications of Integration and Taylor Polynomials	2 weeks
8	Introduction to Differential Equations	2 weeks
9	Infinite Series	4 weeks
10	Parametric Equations, Polar Coordinates, and Vector Functions	2 weeks
11	Projects and Activities to extend learning after the AP Calculus BC Exam	4 weeks

Unit 1 Overview

Unit Title: Review of Limits and Continuity

Unit Summary:
This unit motivates the study of limits by providing an intuitive approach to the concept of the limit. In this unit limits are initially studied particularly from a graphical and numerical standpoint. Students then move toward computing limits analytically using various algebraic techniques. Students will have the opportunity to work with limits at extremely large and extremely small values of x leading to the concept of limits at infinity. Students will acquire an intuitive feel for the notion of "approaching infinity" and how it relates to the end-behavior of a function. This unit introduces the terms "continuous" and "discontinuous" into the student's mathematical vocabulary. Students will explore graphically and algebraically where functions are and are not continuous and delve into the Intermediate-Value Theorem, an important outcome of continuity. Finally, students will have a chance to explore continuity as it pertains to trigonometric and inverse functions and then will prove and apply The Squeezing Theorem to evaluate trigonometric limits.

Suggested Pacing: 6 one-hour lessons, 1 one-hour review, 1 one-hour assessment

Learning Targets

Unit Essential Questions:

- What is the best method to use to find the limit of a function?
- What impact does continuity have on the limit of a function?
- How can we use limits to find instantaneous rates of change?
- How can limits be used to help describe the behavior of a function?

Unit Enduring Understandings:

- Limits can be determined using algebra, graphs and/or tables of data.
- Limits do not require continuity.
- Some graphs demonstrate asymptotic and unbounded behavior.
- Close values of the domain lead to close values of the range.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Limits and Continuity Assessment - Contains a variety of question types such as multiple choice and and free response questions, Section and Multi-section quizzes

Alternative Assessments:

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Use graphical, numerical and tabular methods to estimate limit values. Explore the circumstances under which limits fail to exist (ex. oscillating behavior asymptotic behavior and differing behavior from the left and the right). Distinguish between one-sided (left-hand and right-hand) limits and two-sided limits and explain what it means for such limits to exist.	Content: Definition of a limit, right-hand limit, left-hand limit, limits involving infinity, vertical asymptotes Skills: Evaluate limits using a graph or table.	Calculator Activity: Have students graph a rational function that exhibits a removable discontinuity (i.e. a hole). Explore both algebraically and graphically what happens as values get extremely close the value of x where has a hole. Graph examples of trigonometric functions that have oscillating behavior. AP Practice Multiple Choice Question - Limits Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Quiz: An Intuitive Approach to Limits, Computing Limits, and Limits at Infinity	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. RST.9-10.7 Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	1.5 hours
Use the properties of limits to evaluate limits of polynomial, rational, radical, and piecewise functions.	Content: Properties of limits, polynomial functions, rational functions, radical functions, and piecewise functions.	Group Work Activity: Two lists - one with a limit to evaluate and one with the solution. Students will work together to match the limit with its solution.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1.	1.5 hours

	Skills: Compute limits using various algebraic techniques.	AP Practice Multiple Choice Question - Limits of a Piecewise Function Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Quiz: An Intuitive Approach to Limits, Computing Limits, and Limits at Infinity	Act as a responsible and contributing citizen and employee A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.	
Understand limits at infinity and relate them to horizontal asymptotes of graphs. Be able to evaluate limits, at infinity possibly by using shortcuts for polynomial, rational, and/or algebraic functions. Describe asymptotic behavior in terms of limits involving infinity.	Content: Limits involving infinity, vertical and horizontal asymptotes Skills: Compute limits involving infinity. Find limits that involve euler's number and the natural logarithms.	Exploration Activity: Students will discuss what happens to the function, f(x) = 1/x as x gets extremely large in both the positive and negative direction. Calculus Lab: Investigating Limits at Infinity http://www.teacherspayteachers.com/Product/Limits-as-x-Approaches-Infinity-An-Investigation-861891	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. RST.11-12.9 Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	1.5 hours
Determine if a function is continuous at a point. Identify types of discontinuities.	Content: Continuity at a point, discontinuity, continuous functions, Intermediate Value Theorem, The Squeezing Theorem	Calculus Lab: The Intermediate Value Theorem AP Practice Multiple Choice Question - Continuity	CRP Utilize critical thinking to make sense of problems and persevere in solving them. CRP1	1.5 hours

Evaluate limits involving trigonometric functions.	Skills: Find values for which a function is discontinuous and determine whether the value is a removable discontinuity. Find an unknown value that will cause a function to be continuous everywhere. Locate discontinuities and find limits for functions involving trigonometry.	Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer. Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. Partner Activity: Pairs of students work together to put the step of The Squeezing Theorem proof (given on cards) in the correct order. Calculator Activity: Students graph *f(x) = (sin x)/x* on calculator to determine its limit as x approaches 0. Unit 1 Common Benchmark Assessment	Act as a responsible and contributing citizen and employee F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.	

Unit 2 Overview

Unit Title: Differentiation

Unit Summary:
This unit begins with developing close connections among the problems of finding the tangent line to a curve, determining the velocity of an object moving along a straight line and expressing the rate of change of one variable with respect to another. The derivative function is defined formally and the role of the derivative as a “slope-producing function” is fully explored. After the students have a firm grasp on the idea of the derivative representing slope they can move on to developing and using various techniques of differentiation including the power rule, the constant multiple rule, the sum and difference rules, the product and quotient rules and the chain rule. This unit emphasizes the different forms of derivative notation and the notations for higher order derivatives. This unit contains the derivatives for the six trigonometric functions and the students will have the opportunity to work out the proof for the derivative of sin *x*. This unit includes the method of implicit differentiation and emphasizing what it means to say that an equation defines a function implicitly. This unit concludes with the topic of related rates a area of calculus that provides skills and techniques for solving problems that exhibit areas where calculus can be used in a real-life situations.

Suggested Pacing: 14 one-hour lessons, 1 one-hour review, 1 one-hour assessment

Learning Targets

Unit Essential Questions:

- How can one use the concept of the limit to determine instantaneous rates of change?
- How does one use derivatives to analyze the behavior of functions?
- How is differentiability and continuity related?

● How does the idea of the derivative as a rate of change help us to understand the relationships between position, velocity and acceleration?
Unit Enduring Understandings: ● The derivative is an instantaneous rate of change. ● The derivative of position is velocity and the second derivative of position is acceleration. ● The average rate of change corresponds with slope of a secant line and the instantaneous rate of change corresponds with slope of a tangent line.
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...
Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Alternative Assessments: Find the Error - Analysis Task

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Explain how a derivative can be interpreted as an instantaneous rate of change. Interpret derivative as applied to position, velocity, and acceleration Find the derivative of a function using definition of a derivative function. Find the derivative of a function at a given point using definition of a derivative function. Determine whether a function is differentiable or not over a given interval. Understand the relationship between differentiability and continuity.	Content: Tangent lines, rectilinear motion, displacement, average velocity, instantaneous velocity, definition of a derivative function, differentiability Skills: Find instantaneous rate of change of y with respect to x at a specified value of x and at an arbitrary value of x. Use the limit definition to find a derivative. Determine whether a function is continuous and differentiable at a given value of x. Estimate derivative values given the graph of a function. Find the equation of a tangent line at a given value of x.	Open-ended Class Discussion: How can you apply the definition of a limit to find the slope of a function at a given point? Quiz Question: Suppose that the line $2x + 3y = 5$ is tangent to the graph of $y = f(x)$ at $x = 1$. Find the value of $f(1)$ and $f'(1)$. NCTM Illuminations: Interactive Calculus Tool - Tangent http://illuminations.nctm.org/Activity.aspx?id=3570 Use Calculus in Motion: Definition of a Derivative to motivate a class open-ended discussion of how a limit leads us to the definition of a derivative. http://www.calculusinmotion.com/gsp4.html AP Practice Multiple Choice Question - Definition of Derivative Post question up for class to see. Have the students work out their solutions on	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7	3 hours

		whiteboards and display their answer for me to see. Quiz: Tangent Lines, Velocity and General Rates of Change and The Derivative Function	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.	
Compute derivatives using the power rule, product rule, quotient rule, and chain rule. Use proper notations for first and second derivatives. Compute higher order derivatives.	Content: Power rule, product rule quotient rule, second and higher order derivatives, chain rule Skills: Find derivatives using the power rule, the product rule, the quotient rule and the chain rule.	I Have...Who Has Card with derivatives. Partner Activity: Have students complete a circuit activity of derivative problems. Quiz: Techniques of Differentiation, Product Rule and Quotient Rule Quiz: The Chain Rule	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. A.SSE.1 Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	4 hours
Determine derivatives of trigonometric functions.	Content: Derivatives of trigonometric functions Skills: Find first and second derivatives of trigonometric functions.	NCTM Illuminations: Interactive Calculus Tool - Derivatives http://illuminations.nctm.org/Activity.aspx?id=3570	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4	1 hour

			For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	
Use implicit differentiation to find dy/dx and d^2y/dx^2. Explain the meaning of an implicitly defined function.	Content: Implicit differentiation Skills: Use implicit differentiation to find derivatives of functions that are not explicitly defined. Use implicit differentiation to find the slope of line at a specified point.	Quiz question: The tangent line to the graph of $x + y + xy = 3$ at the point (1, 1) is _______. Quiz: Implicit Differentiation	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). WHST.11-12.1. Write arguments focused on discipline-specific content.	2 hours
Compute derivatives of logarithmic functions, exponential functions and inverse trigonometric functions.	Content: Derivatives of exponential functions, derivatives logarithmic functions, differentiability of inverse functions, one-to-one functions, increasing and decreasing functions, derivatives of inverse trigonometric functions Skills: Determine if a function is one-to-one based on the sign of its derivative. Find dy/dx for functions involving logarithms, exponential, and inverse trigonometry. Find derivatives using logarithmic differentiation. Find the derivative of a function's inverse.	Guided Class Discussion: Find the derivative of y = arcsin x. Give the hint that y = arcsin x can be rewritten sin y = x and then group students off and ask guiding questions to have the groups arrive at its derivative. Students will continue to work in their groups to find the other five remaining inverse trigonometric functions derivatives. AP Practice Multiple Choice Question - Derivative of an Inverse Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. Partner Activity: Have students complete a circuit activity of derivative problems involving logarithmic, exponential,	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7	2 hours

		and inverse trigonometric functions. Use Calculus in Motion: Inverse Function's Derivative to motivate an open-ended class discussion of how to find derivatives of inverse trigonometric functions. http://www.calculusinmotion.com/gsp4.html	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	
Find the rate at which one quantity is changing by relating the quantity to other quantities whose rates of change are known.	Content: Related rates problems, strategies for solving related rates problems Skills: Solve various related rates problems.	Use Calculus in Motion: Related Rates to work on various problems generated whole class discussion of how to approach each problem. http://www.calculusinmotion.com/gsp4.html Calculus Lab: Related Rates http://www.teacherspayteachers.com/Product/Related-Rates-An-Introduction-934273 Unit 2 Common Benchmark Assessment	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.BF.1 Write a function that describes a relationship between two quantities. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	2 hours

Unit 3 Overview
Unit Title: Applications of Differentiation
Unit Summary: This unit begins with the notion of approximating a function locally by a linear function is discussed and then related to error propagation in applied problems. Students are introduced to the vocabulary used to describe high and low places known as extreme values on a function's graph and where to look for them. The unit continues with defining the graphical attributes of increasing, decreasing, and concavity for a function and provides sufficient conditions for determining these attributes from the signs of the first and second derivatives. An inflection point is defined and discussed in applications. The students become very familiar with the First and Second Derivative Test as fundamental calculus tools used to identify and classify locations of relative extrema. This unit defines absolute maximum, minimum and extremum for a function on an interval and surveys the possible existence and locations for absolute extrema. L'Hopital's rule is introduced as a important tool for evaluating limits of particularly difficult functions of a certain form. This unit provides the skills and techniques for solving optimization problems that exhibit areas where calculus can be used in a real-life situations. Students get a glimpse of the range of theorems upon which the tools of calculus were built, including the Extreme Value Theorem, Rolle's Theorem and the Mean Value Theorem. Finally, antiderivatives are studied as a lead in to unit 4 in which the students will complete the inverse problem of being given the derivative recover the original function.
Suggested Pacing: 9 one-hour lessons, 1 one-hour review, 1 one-hour assessment

Learning Targets
Unit Essential Questions: ● What information does the derivative tell us about a graph? ● How do rates of change present relate in real-life situations? ● How do companies use derivatives to maximize profit and minimize cost?
Unit Enduring Understandings: ● Derivatives have both theoretical and real-life applications. ● Finding maximum and minimum values are common real world problems (for example, profit, cost, material used, volume of a container, etc.) ● The first and second derivative tests can be used together with skills from prior math courses to accurately graph a wide variety of functions. ● Derivatives are an underlying concept supporting physical applications that are rooted in many fields.

Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...
Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Alternative Assessments: Card-Grouping Task

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJCCCS CPIs, CCSS, NGSS)	Pacing
Understand and explain the relationship between differentiability and local linearity. Compare the value of the differential, *dy*, with the actual change in y, Δy.	Content: Local linear approximation Skills: Use differentials to find percentage error. Use differentials to estimate error in calculations. Find the differential *dy*. Find formulas for *dy* and Δy.	Quiz question: Find an equation for the local linear approximation to $y = 5 - x^2$ at $x_o = 2$. Think-Pair-Share: Estimate the error in calculation using differentials.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1 Act as a responsible and contributing citizen and employee F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	1 hour

			F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	
Locate the largest and smallest function over a particular interval. Find methods for finding the high and low points on the graph of a function and discuss approaches for analyzing the graphs of functions.	Content: Absolute extrema, The Extreme Value Theorem, absolute extrema on infinite intervals, absolute extrema on open intervals, Critical points, relative maxima and minima, relative extrema, First Derivative Test, Second Derivative Test Skills: Find absolute maximum and minimum values of a function on a closed interval. Find absolute maximum and minimum values of a function on an open interval. Find absolute maximum and minimum values of a periodic on the interval $(-\infty, \infty)$. Find and explain what the critical point of a function are. Identify if a critical point is a stationary point. Use the first and second derivative test to locate relative extrema.	Small group activity: Students work in small groups to come up with illustrations of various scenarios. For example, drawn a diagram of function that has an absolute maximum but no absolute minimum on the interval $(-\infty, \infty)$. This will lead into an open ended whole class discussion of the Extreme Value Theorem. Calculator Exploration Activity: Finding Extreme Values http://www.jamesrahn.com/CalculusI/PDF/Exploration%20Finding%20Extreme%20Values.pdf Think-Write-Pair-Share: For the problem $f(x) = x^4 - 12x^3$ students will first find the critical points. Next they will jot down their ideas about how to determine if each critical point is a maxima or a minima. They will then turn to a partner to discuss and compare their ideas and come up with a strategy for completing the problem. Open-ended Class Discussion: How do we use the second derivative to sketch the graph of the original function? What information does the second derivative give us? AP Practice Multiple Choice Question - Relative maxima and minima Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	1 hour

Explore and discuss the important consequences of the Mean-Value Theorem.	Content: Rolle's Theorem, Mean-Value Theorem, Constant Difference Theorem Skills: For a given function, verify the hypotheses of Rolle's Theorem and find all the values in the interval that satisfy Rolle's Theorem. For a given function, verify the hypotheses of Mean-Value Theorem and find all the values in the interval that satisfy Mean-Value Theorem.	Think-Pair-Share: Find function such that its graph contains the point (1, 5) and such that for every value of x_o the tangent line to the graph of the function at x_o is parallel to the tangent line to graph of $y = x^2$ at x_o. Calculus Lab: An Introduction to the Mean-Value Theorem http://www.teacherspayteachers.com/Product/Mean-Value-Theorem-An-Introduction-934619 Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	1 hour
Develop mathematical tools for determining the exact shape of a graph and the specific locations of its key features.	Content: Increasing and decreasing function, concavity, inflection points Skills: Use sign analysis to identify the intervals on which a function is increasing or decreasing. Use sign analysis to find open intervals on which the function is concave up or concave down. Locate inflection points of a graph a function.	Class Open-Ended Discussion: How would you go about determining the intervals on which a function is increasing or decreasing? Quiz Question: If $f(x)$ has derivative $f'(x) = (x-4)^2 e^{-x/2}$, then find the intervals on which a function is increasing, decreasing, concave up or concave down. Students can create flashcards of the derivative graphs of the function graphs provided to them. Students can then get into groups where they can match function graphs to their derivative graphs.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. RST.11-12.9 Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	1 hour

Apply L'Hopital's Rule to compute limits of functions in which the function has an indeterminate form at the point of interest..	Content: Indeterminate forms 0/0 and ∞/∞, $0 \cdot \infty$, $\infty - \infty$, 0^0 and 1^∞, rate of growth Skills: State the conditions in which L'Hopital's Rule does not apply. Use L'Hopital's Rule to evaluate limits.	Class Activity: Complete an informal proof of L'Hopital's Rule by graphing the two functions (numerator and denominator) at once, and zoom in on their point of intersection. Since the functions are differentiable, they are locally linear, so the graphs appear to be lines. The ratio of the y-coordinates of two lines that intersect on the x-axis is the ratio of the slopes (i.e. their derivatives). Thus the limit of the ratio of the two functions is the ratio of their derivatives.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	1 hour
Explore and determine procedures for graphing rational functions and other types of functions. Describe the interaction between calculus and technology in curve sketching.	Content: Rational functions, properties of graphs - symmetries, x- and y-intercepts, relative extrema, intervals of increase and decrease, asymptotes, periodicity, concavity, inflection points, end behavior, oblique and curvilinear asymptotes, vertical tangents and cusps Skills: Analyze and sketch graphs of a function, labeling key feature of the graph including x- and y-intercepts, relative extrema, intervals of increase and decrease, and asymptotes, and inflection points.	Small Group Discussion: Given function $f(x) = 6x^{1/3} + 3x^{4/3}$, graph the function first by finding the symmetries, x- and y-intercepts, relative extrema, intervals of increase and decrease, asymptotes, concavity,inflection points end behavior, oblique and curvilinear asymptotes, vertical tangents and cusps. Then as a group notice something interesting happening at x = 1 and make note of your observations. Partner Activity: Match graphs of functions with their derivative graph. Calculus Lab: Curve Sketching http://www.jamesrahn.com/CalculusI/PDF/CURVESKETCHING.pdf Whole Class Card Game: Each person gets a card or two and they must find their partners. Students will not know if they have f, f ' or f ". Once they have found the complete set, they can then pick out the equation. http://www.teacherspayteachers.com/Product/Match-the-Graphs-of-f-f-and-f--884947	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	1.5 hours
Solve applied optimization problems.	Content: Finite closed intervals, revenue, profit, cost, marginal analysis, procedure for solving applied maximum and minimum problems Skills:	Take-home Writing Activity: Is it theoretically possible to have a box with 0 width? Fully explain and use outside resources if desired.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	1.5 hours

	Solve various types of optimization problems.	Use Calculus in Motion: Optimization to work on various problems generated whole class discussion of how to approach each problem. http://www.calculusinmotion.com/gsp4.html Partner Optimization Activity: http://www.jamesrahn.com/CalculusI/optimization.htm	F.BF.1 Write a function that describes a relationship between two quantities. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. 9.3.ST-SM.2 Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.	
Develop and use fundamental formulas and properties of antidifferentiation.	Content: Antiderivatives, indefinite integral, integrand, integration formulas, properties of the indefinite integral, integral curves, initial value problems, initial condition Skills: Evaluate indefinite integrals using the power formula for integration. Evaluate trigonometric integrals. Evaluate integrals first by rewriting the integrand appropriately.	Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. Quiz Question: Evaluate the integral $\int \frac{1}{1+sinx} dx$ by multiplying the numerator and the denominator by an appropriate expression. Think-Write-Pair-Share: Given a set of functions students, ask students "What function has the given function as derivative?" Class will use this activity to come up with the power formula for integration. Exit Ticket: Write the definition of the power formula for integration in your own words.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	1 hour

Unit 4 Overview
Unit Title: Integration
Unit Summary: This unit begins with introducing the rectangle method and the antiderivative method for finding area between the graph of a function and the x-axis. These two approaches provide an intuitive introduction to the concepts that form the foundation for the Fundamental Theorem of Calculus. This unit introduces some basic antidifferentiation formulas and interprets the process of antidifferentiation geometrically in terms of integral curves and slope fields. The students move on to more complicated indefinite integrals in which the u-substitution method is necessary. Students will re-familiarize themselves with sigma notation and use it to define the area between the graph of a function and an interval on the x-axis in terms of a limit. This unit describes the definite integral in terms of Riemann sums and emphasizes that a definite integral is simply a number defined by a particular limiting process. The students are provided with the techniques for computing definite integrals by using Part I of the Fundamental Theorem of Calculus. An essential outcome of this unit is for students to interpret an integral as an "accumulated change." As the unit concludes, students will have the opportunity to apply integration to the study of rectilinear motion and extend their knowledge of the u-substitution method to apply to evaluating definite integrals.
Suggested Pacing: 8 one-hour lessons, 1 one-hour review, 1 one-hour assessment
Learning Targets
Unit Essential Questions: ● How can we approximate area under a curve? ● Given the rate of change of a quantity, how do we find the original quantity? ● How can definite integrals be used to solve real-world problems?
Unit Enduring Understandings: ● Differentiation and integration are inverse operations. ● The definite integral can take on many interpretations, including precise area under a curve, distance traveled, net change in temperature, etc.
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...
Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Alternative Assessments:

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJCCCS CPIs, CCSS, NGSS)	Pacing
Calculate the areas of plane regions with curvilinear boundaries.	Content: The Area Problem, rectangle method for finding areas, trapezoidal approximation, antiderivative method for finding areas Skills: Estimate the area between the graph of function f on a given interval using the rectangle method. Estimate the area between the graph of function f on a given interval using trapezoidal approximation. Use simple area formulas from geometry to find the area function $A(x)$ that gives the area between the graph of a specified function f on a given interval.	Discovery Learning Partner Activity: The students are given a graph with a pre-drawn curve on the graph paper and are asked to develop a strategy to calculate the area between the curve and the x - axis using geometry. Calculus Lab: Trapezoidal Method for Definite Integrals http://www.jamesrahn.com/CalculusI/PDF/trapezoidal.pdf Use Calculus in Motion: Definition of Integration to motivate a class open-ended discussion of how a the area problems leads us to the definition of an integral. http://www.calculusinmotion.com/gsp4.html	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.	1 hour
Evaluate definite integrals using limits and properties of definite integrals.	Content: Riemann sum, definite integral, properties of definite integrals Skills: Understand and be able to express a definite integral as a limit of Riemann sums . Sketch the region whose area is given by a definite integral and then use geometric formulas to evaluate the integral. Use the properties of definite integrals to evaluate a definite integral.	Calculus Lab: Developing an Understanding of the Definite Integral http://www.jamesrahn.com/CalculusI/PDF/Developing%20an%20Understanding%20of%20the%20Definite%20Integral.pdf Use Calculus in Motion: Riemann sums to generate a whole class discussion of how to Riemann sums connect to the definite integral. http://www.calculusinmotion.com/gsp4.html	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	1 hour

			F.BF.1 Write a function that describes a relationship between two quantities. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible. RST.9-10.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context.	
Evaluate definite integrals using the Fundamental Theorem of Calculus - Part 1. Use the Fundamental Theorem of Calculus- Part 2 to evaluate definite integrals using antiderivatives. Find the average value of a function over a closed interval. Study and understand the uses of the Mean-Value Theorem for Integrals.	Content: The Fundamental Theorem of Calculus - Part 1, net area, total area, dummy variables, the Mean-Value Theorem for Integrals, The Fundamental Theorem of Calculus - Part 2, integrating rates of change Skills: Use The Fundamental Theorem of Calculus - Part 1 to evaluate definite integrals. Evaluate definite integrals involving absolute value. Find area under a curve over a specified interval. Sketch a curve and find the total area between the curve and a specified interval on the x-axis. Use The Fundamental Theorem of Calculus - Part 2 to find a derivative. Find values of x in a specified interval that satisfy the Mean-Value Theorem for Integrals and explain what the values represent.	Quiz Question: Find the total area between the graph of $y = 2x + 2$ and the interval [-4, 2]. Think-Write-Pair-Share: How do you evaluate integrals involving absolute value? Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. AP Practice Multiple Choice Question - Definite Integrals Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Calculus Lab: Discovering the Average Value of a Function http://www.jamesrahn.com/CalculusI/PDF/Discovering%20the%20Average%20Value%20of%20A%20Function.pdf Calculus Lab: Understanding The Fundamental Theorem of Calculus - Part 2 http://www.jamesrahn.com/CalculusI/PDF/2ndfundamentaltheorem2.pdf Quiz: The Definite Integral and The Fundamental Theorem of Calculus	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities.	2 hours
Study rectilinear motion using the tools of integration.	Content: Position, velocity, displacement, distance traveled, velocity versus time curve, uniformly	Quiz Question: Let v(t) denote the velocity function of a particle that is moving along an s-axis with	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.	0.5 hour

	accelerated motion, free-fall model Skills: Find the position function of a particle given velocity and a given condition. Find the position function of a particle given acceleration and two given conditions. Find the displacement and distance traveled by a particle given the velocity function over a given time interval. Derive the formula for uniformly accelerated motion and use it to solve real-world applications problems.	constant acceleration a = -2. If v(1) = 4, find v(t). Small Group Activity: Rectilinear Motion http://college.cengage.com/mathematics/larson/calculus_analytic/7e/instructors/downloads/apthemes/2258_08.pdf	CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities.	
Study and use the technique of substitution to transform complicated integration problems into simpler ones before integrating.	Content: u-substitution, guidelines for u-substitution Skills: Evaluate integrals using appropriate substitutions. Evaluate integrals first by modifying the form of the integrand and using appropriate substitutions.	AP Practice Multiple Choice Question - Integration by Substitution Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Writing Activity: Explain the connection between the chain rule for differentiation and the method of u-substitution for integration. Quiz: The Area Problem, The Indefinite Integral, and Integration by Substitution	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee A.SSE.1 Interpret expressions that represent a quantity in terms of its context. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.	1 hours
Use sigma notation to express lengthy sums in a compact form. Use the rectangle method to give an exact mathematical definition of the "area under a curve."	Content: Sigma notation, properties of sums, summation formulas, a definition of area, net signed area Skills: Evaluate sums expressed in sigma notation. Use the limit definition for area under a curve to find the area between the graph of a function on a specified interval.	Quiz Question: Find the left endpoint approximation for the area between the curve $y = x^2$ and the interval [1, 3] using $n = 4$ equal subdivisions on the interval.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour

Evaluate definite and indefinite integrals involving inverse trigonometric functions and exponential functions.	Content: Transcendental functions Skills: Evaluate definite and indefinite integrals involving transcendental functions that go beyond using the power rule.	Class Challenge: Prove: $\int arcsint\, dt = \sqrt{1-t^2} + tarcsint$	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	0.5 hour
Model exponential growth and decay using differential equations.	Content: Exponential growth and decay models, population growth, doubling-time, half-life, radioactive decay, carbon dating Skills: Solve applied problems using exponential growth and decay models On the graph of a slope field, sketch the graph of a solution that satisfies a given initial condition.	Quiz Question: Describe that the values of C and k represent in the exponential growth and decay model, $y = Ce^{kt}$. Calculus Lab: Social Diffusion http://www.jamesrahn.com/CalculusI/PDF/social%20diffusion.pdf Unit 4 Common Benchmark Assessment	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.1 Write a function that describes a relationship between two quantities. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	1 hour

Unit 5 Overview
Unit Title: Applications of Integration
Unit Summary: This unit provides the students the opportunity to build on the connections they made in the previous unit between the notation for Riemann sums and that of the corresponding integral. With a few modifications the students extend the application of definite integrals from the area of a region under a curve to the area of a region between two curves. When integrating with respect to x the students will interpret the definition for area as an integral in which the integrand is the top curve minus the bottom curve. When integrating with respect to y an alternative formula is considered in the integrand is the right curve minus the left curve. This unit provides the techniques and skills for finding volumes of solids of revolution. Students will learn when it is appropriate to use the disk method or the washer method. This unit expands the concept of finding volumes of solids by slicing to include finding volumes of solids with known cross section.
Suggested Pacing: 4 one-hour lessons, 1 one-hour review, 1 one-hour assessment
Learning Targets
Unit Essential Questions:

• How can an integral be used to solve problems in various fields including geometry, science, manufacturing and engineering? • How can integrals be applied to finding bounded areas and generated volume?
Unit Enduring Understandings: • Integrals have many uses including measuring the area between curves and finding volumes of solids. • Using Calculus to determine areas and volumes of regions bounded by curves is utilized by in many fields including engineering (for example mechanical engineers use it for dam and water tower designs and construction.)
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...
Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Alternative Assessments:

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Find the area between two curves using integration. Find the area of a region between intersecting curves using integration.	Content: Area of a region between two curves, area of a region between intersecting curves, representative rectangle, reversing the roles of x and y Skills: Set up a definite integral that gives the area of a region given in a diagram. Find the area of a region by integrating with respect to x. Find the area of a region by integrating with respect to y. Sketch a region bounded by the graphs of functions and find the area of the region.	Partner Activity: Approximating the Area of the Nike Symbol http://www.teacherspayteachers.com/Product/Approximating-the-Area-of-the-Nike-Symbol-963427 I Have... Who Has... Cards - Bounded Area http://www.teacherspayteachers.com/Product/I-Have-Who-Has-Cards-Bounded-Area-1037849 Think-Write-Pair-Share: For the problem: Find the area bounded by the graphs $x = 3 - y^2$ and $x = y + 1$ students will the functions first. Next they will jot down their strategies for finding the area of the bounded region. They will then turn to a partner to discuss and compare their ideas and come up with a strategy for completing the problem. Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	2 hours

Find the volumes of three-dimensional solids. Apply the understanding of integration to solve problems related to various fields of study.	Content: Volumes by slicing, solids of revolution, volumes by disks perpendicular to the x-axis, volumes by washers perpendicular to the x-axis, volumes by disks and washers perpendicular to the y-axis, solids of known cross section Skills: Find the volume of a solid of revolution using the disk method. Find the volume of a solid of revolution using the washer method. Find the volume of a solid with known cross section.	Partner Activity: Exercises Finding Volume http://www.jamesrahn.com/CalculusI/PDF/volumes.pdf Whole Class Activity: Volumes of Solids with Known Cross Sections - An Exploration http://www.teacherspayteachers.com/Product/Volumes-of-Solids-with-Known-Cross-Sections-An-Exploration-865090 Unit 5 Common Benchmark Assessment	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities. 8.1.12.C.1 Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community. CRP11 Use technology to enhance productivity.	2 hours

Unit 6 Overview
Unit Title: Techniques of Integration
Unit Summary: In this unit students will work with techniques for finding antiderivatives that include integration by parts and partial fraction decomposition. Integration by parts requires the students to recognize derivatives produced by the product rule and they will be provided strategies about the best ways to choose u and dv. When using the method of partial fractions the students will learn to recognize that if the integrand is in the form of an algebraic fraction and the integral cannot be evaluated by other methods, the fraction needs to be expressed in partial fractions before integration occur. For this topic students are only required to be able to deal with non-repeated linear factors in the denominator. The students will study improper integrals in which infinity might appear in the limits of integration or the function itself could also go to infinity. The students must be able to

work with both unbounded domains, and unbounded functions and they should also realize improper integrals are the limit of definite integrals, and be careful in their treatment of the Fundamental Theorem of Calculus during evaluations. Lastly, the students will compute numerical approximations to a definite integral. It is not always possible to find a formula for the antiderivative of the integrand so in these instances the students need to integrate numerically. The three numerical techniques discussed in the text are the Trapezoidal Rule, the Midpoint Rule, and Simpson's Rule. Along with the left- and right-side approximations, students should become familiar with using trapezoids and midpoint rectangles to approximate the value of a definite integral.

Suggested Pacing: 4 one-hour lessons, 1 one-hour review, 1 one-hour assessment

Learning Targets

Unit Essential Questions:
- Why are some integrals impossible to compute analytically?
- Why are some integration methods more effective than others?
- Why do some improper integrals converge and others diverge?

Unit Enduring Understandings:
- The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
- Integration is useful tool in solving problems related to area, velocity, acceleration, volume, and area of a solid of revolution.
- The definite integral can be used to find exact area by using the limit of Riemann sums.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes

Alternative Assessments:

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Evaluate indefinite and definite integrals using integration by parts.	Content: Integration by Parts Formula, LIATE Skills: Integrate by parts more than once. Use LIATE method for choosing u and dv.	Investigation Activity: Integration by Parts https://education.ti.com/en/timathnspired/us/detail?id=4A6988634254A45889059A7859242251E&t=35CC215D8281470BB97CEA2C1806A9A4 Think-Write-Pair-Share: Prove that $\int xb^x dx = b^x\left(\frac{x}{\ln b} - \frac{1}{\ln^2 b}\right) + C$.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	1 hour

			F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	
Apply the Method of Partial Fractions to integrate rational functions where the numerator and the denominator are polynomials.	Content: Method of Partial Fractions, long division Skills: Find the constants for fractions with linear factors in the denominator. Use long division to change the form of the lintegrand before using the Method of Partial Fractions.	Partner Activity: Evaluate $\int \frac{\sqrt{x}}{x-1} dx$. Closure Question: Sample AP Practice Multiple Choice Question https://docs.google.com/document/d/1N3UW7yD5v1sn3lnFZPE6GdLfgzgexmwH36aby9Aa1No/edit?usp=sharing	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour
Use improper integrals to find the areas of unbounded regions.	Content: Divergence, convergence, improper integral, L'hopital's rule, p-series, Comparison Test for Improper Integrals Skills: Determine whether the improper integral converges. Evaluate improper integrals using limits. Evaluate improper integrals with infinite integration limits and infinite discontinuities. Use the Comparison Test for improper integrals that are not possible evaluate to determine whether the improper integral converges or not by comparing it to an integral that we can evaluate.	Class Discussion: How can you use L'Hopital's Rule to calculate $\int_0^\infty xe^{-x}\, dx$? Then evaluate the integral. Closure Question: Sample AP Practice Free Response: https://docs.google.com/document/d/1R_PzMoarhZhUL69T5hm-4s9-vHQiRYFZpf4nXCDXYXw/edit?usp=sharing	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	1 hour
Use numerical integration to approximate a definite integral.	Content: Trapezoidal rule, midpoint rule Skills: Approximate a definite integral using a left sum, a right sum, a midpoint sum and the Trapezoidal Rule using a small number of intervals	Class Activity: Numerical Integration https://www.cs.uic.edu/~hogand/cs107/numerical-integration/ Trapezoidal Rule Investigation: https://www.projectmaths.ie/documents/LCActivities/AreaOfASemiCircleUsingTrapezoidalRule.pdf	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.4	1 hour

	Obtain function values needed to use in the formulas from a table or a graph rather than an equation.	Unit 6 Common Benchmark Assessment	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	

Unit 7 Overview

Unit Title: Further Applications of Integration and Taylor Polynomials

Unit Summary:
The students will learn two essential topics in Unit 7. The students start off with deriving the formula for computing the arc length of a graph. This unit provides the students with the opportunity to discuss how the arc length of a continuous function f(x) over [a, b] can be approximated by partitioning the interval [a, b] and adding up the lengths of the line segments that connect the points. The students will use the Pythagorean Theorem to find the length of each line segment and then apply the Mean Value Theorem to get the arc length formula. The students will then delve into topic of Taylor polynomials, the special case of Maclaurin polynomials, and the establishment of the error bound. The students will learn the techniques for manipulating Taylor polynomials by completing problems and will apply the error bound established in this unit.

Suggested Pacing: 4 one-hour lessons, 1 one-hour review, 1 one-hour assessment

Learning Targets

Unit Essential Questions:
- How does a Taylor Series represent a function?
- How can integrals be applied to finding arc length?

Unit Enduring Understandings:
- A function can be represented by an associated power series over the interval of convergence for the power series.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes

Alternative Assessments:

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Find the arc length of a function between to points.	Content: Formula for arc length Skills: Express the arc length of a curve as an integral. Use the formula for arc length to Find the area of a region by calculate the arc length of a function over a given interval. Understand the proof for the formula for arc length.	Closure Question: Sample AP Practice Multiple Choice Question https://docs.google.com/document/d/1N3UW7yD5v1sn3lnFZPE6GdLfgzgexmwH36aby9Aa1No/edit?usp=sharing Group Work: Sample AP Practice Free Response: https://docs.google.com/document/d/1R_PzMoarhZhUL69T5hm-4s9-vHQiRYFZpf4nXCDXYXw/edit?usp=sharing	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour
Create a Taylor polynomial from give numerical derivatives. Identify numerical derivatives from a given Taylor or Maclaurin polynomial. Show that the error involved in an approximation of a function value is below a given amount.	Content: Taylor polynomial centered at x = a, Maclaurin Polynomials, Error Bound Skills: State and prove Taylor's Theorem that for a function f using Integration by Parts and the Fundamental Theorem of Calculus. State and prove the error bound for the Taylor polynomial.	Partner Activity: Sample AP Practice Free Response: https://docs.google.com/document/d/1R_PzMoarhZhUL69T5hm-4s9-vHQiRYFZpf4nXCDXYXw/edit?usp=sharing Unit 7 Common Benchmark Assessment	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	3 hours

Unit 8 Overview
Unit Title: Introduction to Differential Equations
Unit Summary: This unit provides the solution techniques and applications for first-order differential equations. The students will be introduce to what a differential equation is, and emphasizes that a solution is a function, which has a domain. Students will have the opportunity to study slope fields which provide a graphical approach to learning about the solutions of a differential equation. This unit includes showing the student how to use first-order differential equations to create mathematical models of physical phenomena as well as develop basic properties of the exponential growth and decay models. The unit concludes with logistic differential equation. The logistic differential equation is frequently used in problems involving population, where the population growth is limited by the capacity of the environment and competition for resources. Please note that students are not required to solve a logistic differential equation but they are expected to know the shape and behavior of the solution.
Suggested Pacing: 5 one-hour lessons, 1 one-hour review, 1 one-hour assessment
Learning Targets
Unit Essential Questions: ● How are differential equations used to model real world problems? ● How can differential equations be analyzed analytically, graphically, and numerically to make predictions?
Unit Enduring Understandings: ● Real life problems can be solved by solving differential equations. ● Solutions to differential equations can be represented graphically, numerically and algebraically. ● Many of the fundamental laws of the physical and social sciences involve rates of change and can therefore be modeled using differential equations. ● Antidifferentiation is an underlying concept involved in solving separable differential equations.
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...
Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Alternative Assessments:

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJCCCS CPIs, CCSS, NGSS)	Pacing
Solve basic types of differential equations.	Content:	Writing Activity:	CRP8.	1.5 hours

Use differential equations to model and solve applied problems.	Differential equation, initial-value problems, first-order separable equations, separation of variables Skills: Confirm solutions to differential equations. Find the general solution of a differential equation using separation of variables. Solve initial value problems.	In your own words, describe how to recognize and solve differential equation that can be solved by separation of variables. AP Practice Open-ended Question - Separation of Variables Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Exit Ticket: In your own words, describe the difference between the general solution of a differential equation and a particular solution.	Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.1 Write a function that describes a relationship between two quantities. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	
Use slope fields to approximate solutions of differential equations.	Content: Functions of two variables, slope fields Skills: Construct basic slope fields for differential equations. On the graph of a slope field, sketch the graph of a solution that satisfies a given initial condition.	Partner Activity: Given two sets of cards, one with diagrams of slope fields and the other with the corresponding differential equation. Students match the slope field with its corresponding differential equation. AP Practice Multiple Choice Question - Slope Fields Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Calculus Lab: Creating Basic Slope Fields http://www.jamesrahn.com/CalculusI/PDF/creating%20Basic%20Slope%20Fields.pdf Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home.	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	1.5 hours
Determine the carrying capacity in a logistic growth setting. Find the maximum growth rate in a logistic growth setting.	Content: Logistic Growth Equation, carrying capacity, growth constant Skills: Find the limit of the population over a long period of time. Understand that the limit of the population over a long period of time is the carrying capacity regardless of the initial population.	Class Discussion: National Math and Science Initiative: Logistic Growth Function (APSI Resource Book) Small Group Activity: Sample AP Practice Free Response: https://docs.google.com/document/d/1R_PzMoarhZhUL69T5hm-4s9-vHQiRYFZpf4nXCDXYXw/edit?usp=sharing Unit 8 Common Benchmark Assessment	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	2 hours

	Determine when the population is growing the fastest. Understand that the population is growing the fastest when the population is half of the carrying capacity.		F.BF.1 Write a function that describes a relationship between two quantities. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	

Unit 9 Overview

Unit Title: Infinite Series

Unit Summary:
This unit begins with the students learning about sequences and what it means for a sequence to converge. The unit then moves on to tackle the subject of convergence of infinite series with attention to two special kinds of series: telescoping and geometric. This unit introduces and has the students working with series with positive terms, leading to the integral test, the p-series test, and both the direct comparison test and the limit comparison test. The unit includes learning about the alternating series test with the corresponding error bound and the ratio test. This unit provides the techniques and skills for completing problems with power series, the radius of convergence, and the interval of convergence. The students will study the topic of using power series to solve differential equations and gives them practice in formal manipulation of series. This unit concludes with Taylor series in general, and Maclaurin series in particular, and provides more practice in producing new Taylor series by formal manipulation of known series.

Suggested Pacing: 10 one-hour lessons, 1 one-hour review, 1 one-hour assessment

Learning Targets

Unit Essential Questions:
- How are transcendental functions described using infinite series?
- What is an infinite series and does it have a sum?

Unit Enduring Understandings:
- The sum of an infinite number of real numbers may converge. (EU 4.1)
- Limits play a key role in infinite series.
- Power series are important tools for approximating and defining functions.
- A function can be represented by an associated power series over the interval of convergence for the power series. (EU 4.2)

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc…

Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes

Alternative Assessments:

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJCCCS CPIs, CCSS, NGSS)	Pacing
Determine convergence or divergence of a sequence.	Content: Sequence, terms, index, general term, limit of a sequence, recursive sequence, Geometric sequences, limit laws of sequences, bounded sequences Skills: Prove convergence of a sequence. Calculate terms of a sequence. Determine the limit of a sequence.	Class Discussion: National Math and Science Initiative: Sequences (APSI Resource Book) Partner Activity: Sample AP Practice Free Response: https://docs.google.com/document/d/1R_PzMoarhZhUL69T5hm-4s9-vHQiRYFZpf4nXCDXYXw/edit?usp=sharing	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	1.5 hours
Find sums of an Infinite Series.	Content: Infinite series, partial sums, convergence of an infinite series, sum of a geometric series, divergence test Skills: Prove the divergence test. Prove divergence of a series. Find the formula for the general term of an infinite series.	Partner Exploration: Two Geometric Series (Foerster: Exploration 12-2a)	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	1.5 hours

Determine the convergence or divergence of a positive infinite series of constants using the Integral Test, the *p*-Series Test and the Limit Comparison Test.	Content: Positive series, Dichotomy for Positive Series, Integral test, Convergence of *p*-series, Limit comparison Test Skills: Use the Integral Test to determine if the infinite series is convergent. Use the Comparison Test to determine if the infinite series is convergent. Use the Limit Comparison Test to determine the convergence or divergence of an infinite series.	Partner Activity: Sample AP Practice Free Response: https://docs.google.com/document/d/1R_PzMoarhZhUL69T5hm-4s9-vHQiRYFZpf4nXCDXYXw/edit?usp=sharing Closure Question: Sample AP Practice Multiple Choice Question https://docs.google.com/document/d/1N3UW7yD5v1sn3lnFZPE6GdLfgzgexmwH36aby9Aa1No/edit?usp=sharing Convergence of a Series - National Math and Science Initiative: https://www.scott.k12.ky.us/userfiles/1659/my%20files/sss%20handouts/16%20convergence%20of%20series.pdf?id=64454	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour
Determine if a series converges absolutely or conditionally.	Content: Absolute convergence, conditional convergence, Leibniz Test for alternating series Skills: Show that a series converges conditionally. Show that a series converges absolutely..	Class Discussion Question: https://docs.google.com/document/d/1R_PzMoarhZhUL69T5hm-4s9-vHQiRYFZpf4nXCDXYXw/edit?usp=sharing	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour
Determine the convergence or divergence of a infinite series using the Ratio and Root Tests.	Content: Ratio Test, Root Test Skills: Apply the Ratio Test to determine the convergence or divergence of a series or state	Class Discussion: Can the Ratio Test be used to show convergence if the series is only conditionally convergent? Class Activity:	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1.	1 hour

	that the Ratio Test is inconclusive. Apply the Root Test to determine the convergence or divergence of a series or state that the Ratio Test is inconclusive.	Proof of the Root Test.	Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	
Write a power series representing a given function.	Content: Power series, Radius of Convergence Power Series Solutions of differential equations Skills: Find an interval of convergence for a Power Series. Find a power series for a function by equating derivatives.	Partner Exploration: Two Geometric Series (Foerster: Exploration 12-3, 12-4a & 12-5a) AP Calculus Practice: https://www.slideshare.net/jaflint718/ap-calculus-bc-series-frq-solutions AP Calculus BC Calculus 2 Convergence of Power Series Tri Fold and Practice: https://www.teacherspayteachers.com/Product/AP-Calculus-BC-Calculus-2-Convergence-of-Power-Series-Tri-Fold-and-Practice-3077341	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	2 hours
Find the Taylor Series for a function.	Content: Taylor series expansion, Maclaurin series, error bounds Skills: Find the Maclaurin series and an interval on which the expansion is valid. Find the interval of convergence for a Taylor series.	Exploration: Constructing Taylor Series (APSI Resource Book) Class Activity: Derive the formula for Taylor Series. Class Discussion: National Math and Science Initiative: Taylor Series (APSI Resource Book)	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret	2 hours

			statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.BF.1 Write a function that describes a relationship between two quantities.	

Unit 10 Overview
Unit Title: Parametric Equations, Polar Coordinates, and Vector Functions
Unit Summary: In this unit the students will work with the calculus of curves defined in ways other than the graph of a function y = f(x). The unit begins with the students learning the definition of a parametric equation and how to plot points with a parametric curve. The students will interpret a parametric curve as giving the location of a particle, (x(t), y(t)), thought of as a function of time, and will find the tangent line in terms of the derivatives of the coordinate functions. This unit introduces the concept of speed of a particle, and makes the connection between speed and the length of the curve. The students will learn the concept of polar coordinates and the calculus involved with polar coordinates. This unit includes the concept of vectors in the plane and reviews the basic terminology, notation dealing with two-dimensional vectors and the magnitude of a vector. The unit concludes with the students working with the velocity vector as the derivative of the position vector and the derivative of the curve given in parametric form.
Suggested Pacing: 7 one-hour lessons, 1 one-hour review, 1 one-hour assessment
Learning Targets
Unit Essential Questions: ● What are the advantages to parametric and polar representation over Cartesian representation? ● How are slope and area measured in a polar coordinate system? ● How is motion is using parametric equations and vectors?
Unit Enduring Understandings: ● The methods of differential and integral calculus can be extended to multiple representations and coordinate systems including parametric, polar, and vector forms.
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit Assessment - Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Alternative Assessments:

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Model planar curves using parametric equations or a vector-valued function. Find the slope and write the equation of a line tangent to a planar curve described by parametric equations or vector-valued function.	Content: Parametric equations, parametric curve, parametrization, slope of the tangent line Skills: Express parametric equations in function form. Graph parametric curves. Find parametric equations for a function.	Partner Activity: National Math and Science Initiative: Parametric Equations and Vector Functions (APSI Resource Book) Closure Question: Sample AP Practice Multiple Choice Question https://docs.google.com/document/d/1N3UW7yD5v1sn3lnFZPE6GdLfgzgexmwH36aby9Aa1No/edit?usp=sharing	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour
Find the length of a planar curve and find the speed of a particle moving along a planar curve described by parametric equations or vector valued function.	Content: Arc length, speed along a parameterized path, surface area Skills: Find the length of a path over a given interval. Determine the speed s at a time t. Find the volume of a solid with known cross section.	Partner Activity: Gateway Arch Length https://www.ck12.org/tebook/CK-12-Texas-Instruments-Calculus-Teachers-Edition/section/6.3/	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two	1 hour

			quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	
Review the polar coordinate system.	Content: Polar coordinates, rectangular coordinates, radial coordinate, angular coordinate Skills: Convert from rectangular coordinates to polar coordinates. Convert from polar coordinates to rectangular coordinates. Convert an equation in rectangular coordinates to equation in polar coordinates. Convert an equation in polar coordinates to equation in rectangular coordinates.	Partner work: National Math and Science Initiative: Polar Coordinates (APSI Resource Book)	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour
Find the area enclosed by polar curves. Find the first and second derivatives for a planar curve described by a polar equation. Find the slope and write the equation of a line tangent to a planar curve described by a polar equation.	Content: Area in polar coordinates, area between curves, arc length Skills: Compute the area of a shaded region as an integral in polar coordinates. Find the area of an intersection of two polar curves. Calculate the total length of a curve using an integral in polar coordinates.	Partner activity:: National Math and Science Initiative: Polar Coordinates (APSI Resource Book) Closure Question: Sample AP Practice Multiple Choice Question https://docs.google.com/document/d/1N3UW7yD5v1sn3lnFZPE6GdLfgzgexmwH36aby9Aa1No/edit?usp=sharing	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	1 hour

	Convert an equation in polar coordinates to equation in rectangular coordinates.		F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	
Review the basics of planar vectors.	Content: Components of a vector, vector operations using components, basic properties of vector algebra Skills: Find components of a vector. Compute operations with vectors. Sketch a vector. Calculate linear combinations with vectors.	Closure Questions: Find the components of the vector PQ, P = (1, -4) Q = (3, 5).	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	1 hour
Differentiate and integrate vector valued functions. Represent motion along a planar curve using parametric or vector-valued functions and solve problems involving position, distance traveled, velocity, speed, and acceleration using differentiation and integration.	Content: Limit of a vector-valued function, vector-valued integration Skills: Evaluate limits of vector-valued functions. Compute derivatives of vector-valued functions.	Partner activity: National Math and Science Initiative: Parametric Functions and Vector Functions (APSI Resource Book) Closure Question: Sample AP Practice Multiple Choice Question https://docs.google.com/document/d/1N3UW7yD5v1s	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee F.IF.2	2 hours

	Evaluate integrals of vector-valued functions.	n3lnFZPE6GdLfgzgexmwH36aby9Aa1No/edit?usp=sharing	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities.	

Unit 11 Overview
Unit Title: Projects and Activities to extend learning after the AP Calculus BC Exam
Unit Summary: This unit provides the opportunity for students to work with other classmates to collaborate on a year-end final project. Students will have the chance to build on the content and skills attained throughout the course and apply them in a presentation-based project.
Learning Targets
Unit Essential Questions: ● How do we demonstrate proficiency of calculus? ● How do we evaluate, solve and present a complex real world calculus problem?
Unit Enduring Understandings: ● Math can be represented and communicated verbally, numerically, analytically and visually
Evidence of Learning
Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...
Summative Assessments: Completed projects
Alternative Assessments:

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)
Demonstrate and apply mastery of various calculus concepts to investigate an original idea, learn new problems, present on a topic not previously studied or present previously study content in creative and interesting ways .	Content: Final project Skills: Read and understand complex real world project. Work in cooperative teams to complete project. Meet scheduled deadlines. Present project and solution to class. Demonstrate thinking and analytical competency.	Group Presentation: Techniques of Integration - Break into groups of three or four students and prepare one of the units for a two-day presentation to the class. Ideas for After the AP Exam: https://samjshah.com/2008/05/13/calculus-projects/ Attracting Mathematicians in High School: What Can be Taught After the AP Calculus Exam? https://sites.google.com/site/sigmaatahsm/home/post-ap-calculus	CRP8. Apply critical thinking and problem-solving strategies during structured learning experiences. CRP1. Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. Standards will depend on project completed. CRP4 Communicate clearly and effectively and with reason. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.