Madison Public Schools

AP Calculus AB

Grade 12

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Course Overview

Description

AP Calculus AB focuses on conceptual understanding of limits, derivatives and integrals. This is implemented by presenting topics four different ways including graphically/visually, numerically, analytically and verbally. AP Calculus AB offers opportunities for students to develop technical competence and a sense of utility of calculus. The course work encourages students to become logical thinkers, learning to write the solutions to problems in a connected, step-by-step manner with explanatory sentences. Consistent practice with using multiple representations of solutions allows students to develop a comprehensive understanding of each topic studied throughout the year.

Goals

As outlined by The College Board, this course aims for students to be able to:

- work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems.
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

In addition to the goals outlined by the College Board, this course also aims to:

- enable students to make sense of various types of problems and the reasonableness of their answers
- build student confidence with various approaches to solving a problem and persevere in solving them
- encourage students to become abstract thinkers who make sense of quantities and their relationships in problem situations
- develop students' ability to cooperatively discuss, make conjectures and critique ideas of one another
- use, apply, and model mathematics to solve problems arising in everyday life, society, and the workplace
- consider the variety of available tools when solving a mathematical problem
- communicate mathematical ideas precisely and effectively to others
- determine a pattern or analyze structure within mathematical content to apply to related ideas
- use repeated reasoning to follow a multi-step process through to completion

Materials

Core: Rogawski, J., Cannon, R. J., & W.H. Freeman and Company. (2012). *Rogawski's Calculus for AP*. New York: W.H. Freeman.
Supplemental: Various websites related to AP Calculus AB

Resources
Suggested activities and resources page
Benchmark Assessments
Benchmark Assessment are given for each unit with problems that focus on the main ideas and anchor standards of the course.
Modifications and Adaptations for Special Needs Learners (Gifted and Talented Students, English Language Learners, Special Education Students, At-Risk Students)

Scope and Sequence
(Pacing Guide)

Unit Number	Topic of Study	Duration (hours Taught)
1	Review of Prerequisite Skills/Introduction to Calculus	7 hrs
2	Limits and Continuity	15 hrs
3	Differentiation	24 hrs
4	Applications of Differentiation	18 hrs
5	Integration	18 hrs
6	Applications of Integration	6 hrs
7	Differential Equations	8 hrs
8	AP Calculus AB Exam Review	17 hrs
9	Projects and Activities to extend learning after the AP Calculus BC Exam	15 hrs

Unit 1 Overview

Unit Title: Review of Prerequisite Skills/Introduction to Calculus

Unit Summary:
This unit provides the opportunity to hone mathematical skills necessary for success in the AP Calculus AB course. Prior to returning to school the students are required to complete a packet of prerequisite skills and problems (with accompanying solutions). Skills that are highlighted in the packet include slope, parallel lines, function notation, domain and range, piecewise functions, exponents, logarithms, trigonometric functions, and interval notation. During the first four class meetings, the problems are reviewed and then tested. A solid grasp of algebra and precalculus is necessary for understanding and working through the topics that lie ahead in the AP Calculus AB course. The unit concludes with three class meetings dedicated to completing an introductory and discovery-based exercise highlighting five main topics of calculus, which include instantaneous rates of change, sketching graphs of functions, indefinite integral, definite integral and limits.

Suggested Pacing: 7 hours

Learning Targets

Unit Essential Questions:

- Why is it important to approach and solve problems using multiple representations (graphically, numerically, analytically and verbally)?
- What connections can be made amongst these representations?
- How are functions used in problem-solving when modeling real-world situations?

Unit Enduring Understandings:

- Math can be represented and communicated verbally, numerically, analytically and visually.
- A firm understanding of the critical elements of functions is essential to calculus.
- Calculus is the mathematics of change.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessments: Assessment for Unit 1

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Graph an interval on the number line given its notation. Denote an interval given its graph on the number line. Solve a polynomial or rational inequality and graph its solution. Solve an absolute value inequality and graph its solution. Determine the symmetry of a function and show algebraically whether it is odd, even, or neither. Find the slope of a line, the midpoint, and the distance between two points given a pair of coordinates. Find the equation of a line given two points or a slope and a point. Find the distance from a point to a line. Identify and graph a linear, quadratic, or cubic function. Understand and apply the relationships between slope and parallel or perpendicular lines.	Content: Polynomial functions, rational functions, radical functions, absolute value functions, piecewise functions, trigonometric functions, trigonometric identities, inequalities, slope, parallel and perpendicular lines, odd and even functions, domain and range, interval notation Skills: Graph and interpret many types of functions, including piecewise functions, using a graphing calculator. Interpret information given in a word problem and create a function to model it. Use a table to interpret data.	Diagnostic Tests: Algebra, Analytic Geometry, Functions, and Trigonometry Group Presentation: Students in the class work in groups to prepare answers to be presented to the class for the odd-numbered problems from the summer review packet. Prerequisite Skills Test	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9 Compare properties of two functions each represented in a different way	4 hours

Find the domain and range of a function. Form and graph function equations in the form of a word problem.			(algebraically,graphically, numerically in tables, or by verbal descriptions). F.BF.1 Write a function that describes a relationship between two quantities.	
Discover and draw conclusions about how to calculate instantaneous rate of change. Sketch graphs of functions by creating a set of table values for each function and then describe whether the function is increasing, decreasing, or not changing at a given value of x. Relate area under a curve to the concept of the integral and explain this connection. Find an approximation for the definite integral by finding the sum of the area of the five trapezoids. Explore the concept of the limit using a table of values.	Content: Instantaneous rate of change, functions, integrals, limits Skills: Read and interpret tables of values. Interpret data from a graph.	Discovery-based Activity: Intro to Calculus (written and prepared by James Rahn) http://www.jamesrahn.com/CalculusI/PAGES/intro_to_calculus.htm	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.9 Compare properties of two functions each represented in a different way (algebraically,graphically, numerically in tables, or by verbal descriptions).	3 hours

Unit 2 Overview

Unit Title: Limits and Continuity

Unit Summary:
This unit motivates the study of limits by providing an intuitive approach to the concept of the limit. In this unit limits are initially studied particularly from a graphical and numerical standpoint. Students then move toward computing limits analytically using various algebraic techniques. Students will have the opportunity to work with limits at extremely large and extremely small values of x leading to the concept of limits at infinity. Students will acquire an intuitive feel for the notion of "approaching infinity" and how it relates to the end-behavior of a function. This unit introduces the terms "continuous" and "discontinuous" into the student's mathematical vocabulary. Students will explore graphically and algebraically where functions are and are not continuous and delve into the Intermediate-Value Theorem, an important outcome of continuity. Finally, students will have a chance to explore continuity as it pertains to trigonometric and inverse functions and then will prove and apply The Squeezing Theorem to evaluate trigonometric limits.

Suggested Pacing: 15 hours

Learning Targets

Unit Essential Questions:
- What is the best method to use to find the limit of a function?
- What impact does continuity have on the limit of a function?
- How can we use limits to find instantaneous rates of change?
- How can limits be used to help describe the behavior of a function?

Unit Enduring Understandings:
- Limits can be determined using algebra, graphs and/or tables of data.
- Limits do not require continuity.
- Some graphs demonstrate asymptotic and unbounded behavior.
- Close values of the domain lead to close values of the range.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessment: Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 2 assessment

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJCCCS CPIs, CCSS, NGSS)	Pacing
Use graphical, numerical and tabular methods to estimate limit values. Explore the circumstances under which limits fail to exist (ex. oscillating behavior asymptotic behavior and differing behavior from the left and the right). Distinguish between one-sided (left-hand and right-hand) limits and two-sided limits and explain what it means for such limits to exist.	Content: Definition of a limit, right-hand limit, left-hand limit, limits involving infinity, vertical asymptotes Skills: Evaluate limits using a graph or table.	Calculator Activity: Have students graph a rational function that exhibits a removable discontinuity (i.e. a hole). Explore both algebraically and graphically what happens as values get extremely close the value of x where has a hole. Graph examples of trigonometric functions that have oscillating behavior. AP Practice Multiple Choice Question - Limits Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Quiz: An Intuitive Approach to Limits, Computing Limits, and Limits at Infinity	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. RST.9-10.7 Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	2 hours
Use the properties of limits to evaluate limits of polynomial, rational, radical, and piecewise functions.	Content: Properties of limits, polynomial functions, rational functions, radical	Group Work Activity: Two lists - one with a limit to evaluate and one with the solution. Students will work	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.	2 hours

	functions, and piecewise functions. Skills: Compute limits using various algebraic techniques.	together to match the limit with its solution. AP Practice Multiple Choice Question - Limits of a Piecewise Function Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Quiz: An Intuitive Approach to Limits, Computing Limits, and Limits at Infinity	9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Understand limits at infinity and relate them to horizontal asymptotes of graphs. Be able to evaluate limits, at infinity possibly by using short cuts for polynomial, rational, and/or algebraic functions. Describe asymptotic behavior in terms of limits involving infinity.	Content: Limits involving infinity, vertical and horizontal asymptotes Skills: Compute limits involving infinity. Find limits that involve euler's number and the natural logarithms.	Exploration Activity: Students will discuss what happens to the function, $f(x) = 1/x$ as x gets extremely large in both the positive and negative direction. Calculus Lab: Investigating Limits at Infinity http://www.teacherspayteachers.com/Product/Limits-as-x-Approaches-Infinity-An-Investigation-861891	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. RST.11-12.9 Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	2 hours

			CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Determine if a function is continuous at a point. Identify types of discontinuities. Evaluate limits involving trigonometric functions.	Content: Continuity at a point, discontinuity, continuous functions, Intermediate Value Theorem, The Squeezing Theorem Skills: Find values for which a function is discontinuous and determine whether the value is a removable discontinuity. Find an unknown value that will cause a function to be continuous everywhere. Locate discontinuities and find limits for functions involving trigonometry.	Calculus Lab: The Intermediate Value Theorem AP Practice Multiple Choice Question - Continuity Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer. Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. Partner Activity: Pairs of students work together to put the step of The Squeezing Theorem proof (given on cards) in the correct order. Calculator Activity: Students graph $f(x) = (\sin x)/x$ on calculator to determine its limit as x approaches 0. Unit 2 Common Benchmark Assessment	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. CRP Utilize critical thinking to make sense of problems and persevere in solving them. CRP1 Act as a responsible and contributing citizen and employee 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.	5 hours

Unit 3 Overview

Unit Title: Differentiation

Unit Summary:
This unit begins with developing close connections among the problems of finding the tangent line to a curve, determining the velocity of an object moving along a straight line and expressing the rate of change of one variable with respect to another. The derivative function is defined formally and the role of the derivative as a "slope-producing function" is fully explored. After the students have a firm grasp on the idea of the derivative representing slope they can move on to developing and using various techniques of differentiation including the power rule, the constant multiple rule, the sum and difference rules, the product and quotient rules and the chain rule. This unit emphasizes the different forms of derivative notation and the notations for higher order derivatives. This unit contains the derivatives for the six trigonometric functions and the students will have the opportunity to work out the proof for the derivative of sin x. The notion of approximating a function locally by a linear function is discussed and then related to error propagation in applied problems. This unit concludes with the method of implicit differentiation and emphasizing what it means to say that an equation defines a function implicitly.

Suggested Pacing: 24 hours

Learning Targets

Unit Essential Questions:
- How can one use the concept of the limit to determine instantaneous rates of change?
- How does one use derivatives to analyze the behavior of functions?
- How is differentiability and continuity related?
- How does the idea of the derivative as a rate of change, help us to understand the relationships between position, velocity and acceleration?

Unit Enduring Understandings:
- The derivative is an instantaneous rate of change.
- The derivative of position is velocity and the second derivative of position is acceleration.
- The average rate of change corresponds with slope of a secant line and the instantaneous rate of change corresponds with slope of a tangent line.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

S**ummative Assessment:** Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 3 Common Benchmark Assessment - Part 1
Unit 3 Common Benchmark Assessment - Part 2

Alternative Assessments: Find the Error - Analysis Task

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Explain how a derivative can be interpreted as an instantaneous rate of change. Interpret derivative as applied to position, velocity, and acceleration Find the derivative of a function using definition of a derivative function. Find the derivative of a function at a given point using definition of a derivative function. Determine whether a function is differentiable or not over a given interval Understand the relationship between differentiability and continuity.	Content: Tangent lines, rectilinear motion, displacement, average velocity, instantaneous velocity, definition of a derivative function, differentiability Skills: Find instantaneous rate of change of y with respect to x at a specified value of x and at an arbitrary value of x. Use the limit definition to find a derivative. Determine whether a function is continuous and differentiable at a given value of x. Estimate derivative values given the graph of a function. Find the equation of a tangent line at a given value of x.	Open-ended Class Discussion: How can you apply the definition of a limit to find the slope of a function at a given point? Quiz Question: Suppose that the line $2x + 3y = 5$ is tangent to the graph of $y = f(x)$ at $x = 1$. Find the value of $f(1)$ and $f'(1)$. NCTM Illuminations: Interactive Calculus Tool - Tangent http://illuminations.nctm.org/Activity.aspx?id=3570 Use Calculus in Motion: Definition of a Derivative to motivate a class open-ended discussion of how a limit leads us to the definition of a derivative. http://www.calculusinmotion.com/gsp4.html AP Practice Multiple Choice Question - Definition of Derivative Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Quiz: Tangent Lines, Velocity and General Rates of Change and The Derivative Function	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.	4 hours
Compute derivatives using the power rule, product rule, quotient rule, and chain rule. Use proper notations for first and second derivatives. Compute higher order derivatives.	Content: Power rule, product rule quotient rule, second and higher order derivatives, chain rule Skills: Find derivatives using the power rule, the product rule, the quotient rule and the chain rule.	I Have…Who Has Card with derivatives. Partner Activity: Have students complete a circuit activity of derivative problems. Quiz: Techniques of Differentiation, Product Rule and Quotient Rule Quiz: The Chain Rule	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.RN.2	7 hours

			Rewrite expressions involving radicals and rational exponents using the properties of exponents. A.SSE.1 Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Determine derivatives of trigonometric functions.	Content: Derivatives of trigonometric functions Skills: Find first and second derivatives of trigonometric functions.	NCTM Illuminations: Interactive Calculus Tool - Derivatives http://illuminations.nctm.org/Activity.aspx?id=3570 Unit 3 Common Benchmark Assessment - Part 1	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	1 hour

			CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Understand and explain the relationship between differentiability and local linearity. Compare the value of the differential, *dy*, with the actual change in y, Δy.	Content: Local linear approximation Skills: Use differentials to find percentage error. Use differentials to estimate error in calculations. Find the differential *dy*. Find formulas for *dy* and Δy.	Quiz question: Find an equation for the local linear approximation to $y = 5 - x^2$ at $x_0 = 2$. Think-Pair-Share: Estimate the error in calculation using differentials.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	3 hours
Use implicit differentiation to find dy/dx and d^2y/dx^2. Explain the meaning of an implicitly defined function.	Content: Implicit differentiation Skills: Use implicit differentiation to find derivatives of functions that are not explicitly defined. Use implicit differentiation to find the slope of line at a specified point.	Quiz question: The tangent line to the graph of $x + y + xy = 3$ at the point (1, 1) is _____. Quiz: Implicit Differentiation	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f	3 hours

			corresponding to the input x. The graph of f is the graph of the equation y = f(x). CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee WHST.11-12.1. Write arguments focused on discipline-specific content.	
Compute derivatives of logarithmic functions, exponential functions and inverse trigonometric functions.	Content: Derivatives of exponential functions, derivatives logarithmic functions, differentiability of inverse functions, one-to-one functions, increasing and decreasing functions, derivatives of inverse trigonometric functions Skills: Determine if a function is one-to-one based on the sign of its derivative. Find dy/dx for functions involving logarithms, exponential, and inverse trigonometry. Find derivatives using logarithmic differentiation. Find the derivative of a function's inverse.	Guided Class Discussion: Find the derivative of y = arcsin x. Give the hint that y = arcsin x can be rewritten sin y = x and then group students off and ask guiding questions to have the groups arrive at its derivative. Students will continue to work in their groups to find the other five remaining inverse trigonometric functions derivatives. AP Practice Multiple Choice Question - Derivative of an Inverse Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. Partner Activity: Have students complete a circuit activity of derivative problems involving logarithmic, exponential, and inverse trigonometric functions. Use Calculus in Motion: Inverse Function's Derivative to motivate an open-ended class discussion of how to find derivatives of inverse trigonometric functions. http://www.calculusinmotion.com/gsp4.html Unit 3 Common Benchmark Assessment - Part 2	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	3 hours

Unit 4 Overview

Unit Title: Applications of Differentiation

Unit Summary:
This unit begins with defining the graphical attributes of increasing, decreasing, and concavity for a function and provides sufficient conditions for determining these attributes from the signs of the first and second derivatives. An inflection point is defined and discussed in applications. Students are introduced to the vocabulary used to describe high and low places on a function's graph and where to look for them. The students become very familiar with the First and Second Derivative Test as fundamental calculus tools used to identify and classify locations of relative extrema. This unit defines absolute maximum, minimum and extremum for a function on an interval and surveys the possible existence and locations for absolute extrema. This unit provides the skills and techniques for solving optimization problems and related rates problems that exhibit areas where calculus can be used in a real-life situations. Students get a glimpse of the range of theorems upon which the tools of calculus were built, including the Extreme Value Theorem, Rolle's Theorem and the Mean Value Theorem. Finally, rectilinear motion is studied as an extension of the discussion from unit 3 of average and instantaneous velocity for objects moving along a straight line.

Suggested Pacing: 18 hours

Learning Targets

Unit Essential Questions:
- What information does the derivative tell us about a graph?
- How do rates of change present relate in real-life situations?
- How do companies use derivatives to maximize profit and minimize cost?

Unit Enduring Understandings:
- Derivatives have both theoretical and real-life applications.
- Finding maximum and minimum values are common real world problems (for example, profit, cost, material used, volume of a container, etc.)
- The first and second derivative tests can be used together with skills from prior math courses to accurately graph a wide variety of functions.
- Derivatives are an underlying concept supporting physical applications that are rooted in many fields.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessment: Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 4 Assessment - Part 1
Unit 4 Assessment - Part 2

Alternative Assessments: Card-Grouping Task

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Develop mathematical tools for determining the exact shape of a graph and the specific locations of its key features.	Content: Increasing and decreasing function, concavity, inflection points Skills: Use sign analysis to identify the intervals on which a function is increasing or decreasing. Use sign analysis to find open intervals on which the function is concave up or concave down. Locate inflection points of a graph a function.	Class Open-Ended Discussion: How would you go about determining the intervals on which a function is increasing or decreasing? Quiz Question: If $f(x)$ has derivative $f'(x) = (x-4)^2 e^{-x/2}$, then find the intervals on which a function is increasing, decreasing, concave up or concave down. Students can create flashcards of the derivative graphs of the function graphs provided to them. Students can then get into groups where they can match function graphs to their derivative graphs.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1 Act as a responsible and contributing citizen and employee	1 hour
Find methods for finding the high and low points on the graph of a function and discuss approaches for analyzing the graphs of functions.	Content: Critical points, relative maxima and minima, relative extrema, First Derivative Test, Second Derivative Test Skills: Find and explain what the critical point of a function are. Identify if a critical point is a stationary point. Use the first and second derivative test to locate relative extrema.	Think-Write-Pair-Share: For the problem $f(x) = x^4 - 12x^3$ students will first find the critical points. Next they will jot down their ideas about how to determine if each critical point is a maxima or a minima. They will then turn to a partner to discuss and compare their ideas and come up with a strategy for completing the problem. Open-ended Class Discussion: How do we use the second derivative to sketch the graph of the original function? What information does the second derivative give us? AP Practice Multiple Choice Question - Relative maxima and minima	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	1 hour

		Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see.	F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Explore and determine procedures for graphing rational functions and other types of functions. Describe the interaction between calculus and technology in curve sketching.	Content: Rational functions, properties of graphs - symmetries, x- and y-intercepts, relative extrema, intervals of increase and decrease, asymptotes, periodicity, concavity, inflection points, end behavior, oblique and curvilinear asymptotes, vertical tangents and cusps Skills: Analyze and sketch graphs of a function, labeling key feature of the graph including x- and y-intercepts, relative extrema, intervals of increase and decrease, and asymptotes, and inflection points.	Small Group Discussion: Given function $f(x) = 6x^{1/3} + 3x^{4/3}$, graph the function first by finding the symmetries, x- and y-intercepts, relative extrema, intervals of increase and decrease, asymptotes, concavity,inflection points end behavior, oblique and curvilinear asymptotes, vertical tangents and cusps. Then as a group notice something interesting happening at x = 1 and make note of your observations. Partner Activity: Match graphs of functions with their derivative graph. Calculus Lab: Curve Sketching http://www.jamesrahn.com/CalculusI/PDF/CURVESKETCHING.pdf Whole Class Card Game: Each person gets a card or two and they must find their partners. Students will not know if they have f, f ' or f ". Once they have found the complete set, they can then pick out the equation. http://www.teacherspayteachers.com/Product/Matchthe-Graphs-of-f-f-and-f--884947	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee RST.11-12.9 Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	2 hours
Locate the largest and smallest function over a particular interval.	Content: Absolute extrema, The Extreme Value Theorem, absolute extrema on infinite intervals, absolute extrema on open intervals	Small group activity: Students work in small groups to come up with illustrations of various scenarios. For example, drawn a diagram of	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2	2 hours

	Skills: Find absolute maximum and minimum values of a function on a closed interval. Find absolute maximum and minimum values of a function on an open interval. Find absolute maximum and minimum values of a periodic on the interval $(-\infty,\infty)$.	function that has an absolute maximum but no absolute minimum on the interval $(-\infty,\infty)$. This will lead into an open ended whole class discussion of the Extreme Value Theorem. Calculator Exploration Activity: Finding Extreme Values http://www.jamesrahn.com/CalculusI/PDF/Exploration%20Finding%20Extreme%20Values.pdf Unit 4 Common Benchmark Assessment - Part 1	Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Find the rate at which one quantity is changing by relating the quantity to other quantities whose rates of change are known.	Content: Related rates problems, strategies for solving related rates problems Skills: Solve various related rates problems.	Use Calculus in Motion: Related Rates to work on various problems generated whole class discussion of how to approach each problem. http://www.calculusinmotion.com/gsp4.html Calculus Lab: Related Rates http://www.teacherspayteachers.com/Product/Related-Rates-An-Introduction-934273	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.BF.1 Write a function that describes a relationship between two quantities. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	3 hours

Solve applied optimization problems.	Content: Finite closed intervals, revenue, profit, cost, marginal analysis, procedure for solving applied maximum and minimum problems Skills: Solve various types of optimization problems.	Take-home Writing Activity: Is it theoretically possible to have a box with 0 width? Fully explain and use outside resources if desired. Use Calculus in Motion: Optimization to work on various problems generated whole class discussion of how to approach each problem. http://www.calculusinmotion.com/gsp4.html Partner Optimization Activity: http://www.jamesrahn.com/CalculusI/optimization.htm	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.BF.1 Write a function that describes a relationship between two quantities. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	2 hours
Explore and discuss the important consequences of the Mean-Value Theorem.	Content: Rolle's Theorem, Mean-Value Theorem, Constant Difference Theorem Skills: For a given function, verify the hypotheses of Rolle's Theorem and find all the values in the interval that satisfy Rolle's Theorem. For a given function, verify the hypotheses of Mean-Value Theorem and find all the values in the interval that satisfy Mean-Value Theorem.	Think-Pair-Share: Find function such that its graph contains the point (1, 5) and such that for every value of x_0 the tangent line to the graph of the function at x_0 is parallel to the tangent line to graph of $y = x^2$ at x_0. Calculus Lab: An Introduction to the Mean-Value Theorem http://www.teacherspayteachers.com/Product/MeanValue-Theorem-An-Introduction-934619 Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. CRP8.	2 hours

			Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Explore rectilinear motion and define the notion of “acceleration” mathematically. Use the tools of calculus developed in this unit to analyze rectilinear motion in depth.	Content: Velocity, speed, acceleration, position versus time curve Skills: Given a position function of a particle moving along an s-axis find the velocity, speed and acceleration functions. Given a position versus time curve describe how the position of the particle changes with time. Use a position versus time curve, a velocity versus time curve and an acceleration versus time curve to determine when a particle is speeding up or slowing down.	Calculus Lab: Studying Rectilinear Motion on the Graphing Calculator http://www.jamesrahn.com/CalculusI/PDF/rectilinear%20motion%20on%20the%20calculator.pdf Unit 4 Common Benchmark Assessment - Part 2	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	2 hours

Unit 5 Overview

Unit Title: Integration

Unit Summary:
This unit begins with introducing the rectangle method and the antiderivative method for finding area between the graph of a function and the x-axis. These two approaches provide an intuitive introduction to the concepts that form the foundation for the Fundamental Theorem of Calculus. This unit introduces some basic antidifferentiation formulas and interprets the process of antidifferentiation geometrically in terms of integral curves and slope fields. The students move on to more complicated indefinite integrals in which the u-substitution method is necessary. Students will refamiliarize themselves with sigma notation and use it to define the area between the graph of a function and an interval on the x-axis in terms of a limit. This unit describes the definite integral in terms of Riemann sums and emphasizes that a definite integral is simply a number defined by a particular limiting process. The students are provided with the techniques for computing definite integrals by using Part I of the Fundamental Theorem of Calculus. An essential outcome of this unit is for students to interpret an integral as an "accumulated change." As the unit concludes, students will have the opportunity to apply integration to the study of rectilinear motion and extend their knowledge of the u-substitution method to apply to evaluating definite integrals.

Suggested Pacing: 18 hours

Learning Targets

Unit Essential Questions:
- How can we approximate area under a curve?
- Given the rate of change of a quantity, how do we find the original quantity?
- How can definite integrals be used to solve real-world problems?

Unit Enduring Understandings:
- Differentiation and integration are inverse operations.
- The definite integral can take on many interpretations, including precise area under a curve, distance traveled, net change in temperature, etc.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessment: Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 5 Assessment

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Calculate the areas of plane regions with curvilinear boundaries.	Content: The Area Problem, rectangle method for finding areas, trapezoidal approximation, antiderivative method for finding areas	Discovery Learning Partner Activity: The students are given a graph with a pre-drawn curve on the graph paper and are asked to develop a strategy to calculate the area between	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2	1 hour

	Skills: Estimate the area between the graph of function f on a given interval using the rectangle method. Estimate the area between the graph of function f on a given interval using trapezoidal approximation. Use simple area formulas from geometry to find the area function $A(x)$ that gives the area between the graph of a specified function f on a given interval.	the curve and the x - axis using geometry. Calculus Lab: Trapezoidal Method for Definite Integrals http://www.jamesrahn.com/CalculusI/PDF/trapezoidal.pdf Use Calculus in Motion: Definition of Integration to motivate a class open-ended discussion of how a the area problems leads us to the definition of an integral. http://www.calculusinmotion.com/gsp4.html	Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Develop and use fundamental formulas and properties of antidifferentiation. .	Content: Antiderivatives, indefinite integral, integrand, integration formulas, properties of the indefinite integral, integral curves, initial value problems, initial condition Skills: Evaluate indefinite integrals using the power formula for integration. Evaluate trigonometric integrals. Evaluate integrals first by rewriting the integrand appropriately.	Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. Quiz Question: Evaluate the integral $\int \frac{1}{1+\sin x}\,dx$ by multiplying the numerator and the denominator by an appropriate expression. Think-Write-Pair-Share: Given a set of functions students, ask students "What function has the given function as derivative?" Class will use this activity to come up with the power formula for integration. Exit Ticket:	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1.	2 hours

		Write the definition of the power formula for integration in your own words.	Act as a responsible and contributing citizen and employee RST.9-10.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context.	
Study and use the technique of substitution to transform complicated integration problems into simpler ones before integrating.	Content: u-substitution, guidelines for u-substitution Skills: Evaluate integrals using appropriate substitutions. Evaluate integrals first by modifying the form of the integrand and using appropriate substitutions.	AP Practice Multiple Choice Question - Integration by Substitution Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Writing Activity: Explain the connection between the chain rule for differentiation and the method of u-substitution for integration. Quiz: The Area Problem, The Indefinite Integral, and Integration by Substitution	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1 Interpret expressions that represent a quantity in terms of its context. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	2 hours
Use sigma notation to express lengthy sums in a compact form. Use the rectangle method to give an exact mathematical definition of the “area under a curve.”	Content: Sigma notation, properties of sums, summation formulas, a definition of area, net signed area Skills: Evaluate sums expressed in sigma notation. Use the limit definition for area under a curve to find the area between the graph of a function on a specified interval.	Quiz Question: Find the left endpoint approximation for the area between the curve $y = x_2$ and the interval [1, 3] using $n = 4$ equal subdivisions on the interval.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	1 hour

Evaluate definite integrals using limits and properties of definite integrals.	Content: Riemann sum, definite integral, properties of definite integrals Skills: Understand and be able to express a definite integral as a limit of Riemann sums . Sketch the region whose area is given by a definite integral and then use geometric formulas to evaluate the integral. Use the properties of definite integrals to evaluate a definite integral.	Calculus Lab: Developing an Understanding of the Definite Integral http://www.jamesrahn.com/CalculusI/PDF/Developing%20an%20Understanding%20for%20the%20Definite%20Integral.pdf Use Calculus in Motion: Riemann sums to generate a whole class discussion of how to Riemann sums connect to the definite integral. http://www.calculusinmotion.com/gsp4.html	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	3 hours
Evaluate definite integrals using the Fundamental Theorem of Calculus - Part 1. Use the Fundamental Theorem of Calculus- Part 2 to evaluate definite integrals using antiderivatives. Find the average value of a function over a closed interval. Study and understand the uses of the Mean-Value Theorem for Integrals.	Content: The Fundamental Theorem of Calculus - Part 1, net area, total area, dummy variables, the Mean-Value Theorem for Integrals, The Fundamental Theorem of Calculus - Part 2, integrating rates of change Skills: Use The Fundamental Theorem of Calculus - Part 1 to evaluate definite integrals. Evaluate definite integrals involving absolute value. Find area under a curve over a specified interval. Sketch a curve and find the total area between the curve and a specified interval on the x-axis. Use The Fundamental Theorem of Calculus - Part 2 to find a derivative. Find values of x in a specified interval that satisfy the Mean-Value Theorem for Integrals and explain what the values represent.	Quiz Question: Find the total area between the graph of $y = 2x + 2$ and the interval [-4, 2]. Think-Write-Pair-Share: How do you evaluate integrals involving absolute value? Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home. AP Practice Multiple Choice Question - Definite Integrals Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Calculus Lab: Discovering the Average Value of a Function http://www.jamesrahn.com/CalculusI/PDF/Discovering%20the%20Average%20Value%20of%20A%20Function.pdf Calculus Lab: Understanding The Fundamental Theorem of Calculus - Part 2 http://www.jamesrahn.com/CalculusI/PDF/2ndfundamentaltheorem2.pdf	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	3 hours

		Quiz: The Definite Integral and The Fundamental Theorem of Calculus		
Study rectilinear motion using the tools of integration.	Content: Position, velocity, displacement, distance traveled, velocity versus time curve, uniformly accelerated motion, free-fall model Skills: Find the position function of a particle given velocity and a given condition. Find the position function of a particle given acceleration and two given conditions. Find the displacement and distance traveled by a particle given the velocity function over a given time interval. Derive the formula for uniformly accelerated motion and use it to solve real-world applications problems.	Quiz Question: Let v(t) denote the velocity function of a particle that is moving along an s-axis with constant acceleration a = -2. If v(1) = 4, find v(t). Small Group Activity: Rectilinear Motion http://college.cengage.com/mathematics/larson/calculus_analytic/7e/instructors/downloads/apthemes/2258_08.pdf	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1 Write a function that describes a relationship between two quantities. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	2 hours
Evaluate definite integrals using the technique of substitution.	Content: u-substitution Skills: Evaluate definite integrals using appropriate substitutions. Evaluate definite integrals first by modifying the form of the integrand and using appropriate substitutions.	AP Practice Multiple Choice Question - Integration by Substitution for Definite Integrals Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Unit 5 Common Benchmark Assessment	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.1 Interpret expressions that represent a quantity in terms of its context. A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	1 hour

Unit 6 Overview

Unit Title: Applications of Integration

Unit Summary:
This unit provides the students the opportunity to build on the connections they made in the previous unit between the notation for Riemann sums and that of the corresponding integral. With a few modifications the students extend the application of definite integrals from the area of a region under a curve to the area of a region between two curves. When integrating with respect to x the students will interpret the definition for area as an integral in which the integrand is the top curve minus the bottom curve. When integrating with respect to y an alternative formula is considered in the integrand is the right curve minus the left curve. This unit provides the techniques and skills for finding volumes of solids of revolution. Students will learn when it is appropriate to use the disk method or the washer method. This unit expands the concept of finding volumes of solids by slicing to include finding volumes of solids with known cross section.

Suggested Pacing: 6 hours

Learning Targets

Unit Essential Questions:
- How can an integral be used to solve problems in various fields including geometry, science, manufacturing and engineering?
- How can integrals be applied to finding bounded areas and generated volume?

Unit Enduring Understandings:
- Integrals have many uses including measuring the area between curves and finding volumes of solids.
- Using Calculus to determine areas and volumes of regions bounded by curves is utilized by in many fields including engineering (for example mechanical engineers use it for dam and water tower designs and construction.)

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessment: Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 6 Assessment

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Find the area between two curves using integration. Find the area of a region between intersecting curves using integration.	Content: Area of a region between two curves, area of a region between intersecting curves, representative rectangle, reversing the roles of x and y	Partner Activity: Approximating the Area of the Nike Symbol http://www.teacherspayteachers.com/Product/Approximating-the-Area-of-theNike-Symbol-963427 I Have... Who Has... Cards	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences.	2 hours

	Skills: Set up a definite integral that gives the area of a region given in a diagram. Find the area of a region by integrating with respect to x. Find the area of a region by integrating with respect to y. Sketch a region bounded by the graphs of functions and find the area of the region.	- Bounded Area http://www.teacherspayteachers.com/Product/I-Have-Who-Has-Cards-Bounded-Area-1037849 Think-Write-Pair-Share: For the problem: Find the area bounded by the graphs $x = 3-y^2$ and $x = y+1$ students will find the functions first. Next they will jot down their strategies for finding the area of the bounded region. They will then turn to a partner to discuss and compare their ideas and come up with a strategy for completing the problem. Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home.	F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Find the volumes of three-dimensional solids. Apply the understanding of integration to solve problems related to various fields of study.	Content: Volumes by slicing, solids of revolution, volumes by disks perpendicular to the x-axis, volumes by washers perpendicular to the x-axis, volumes by disks and washers perpendicular to the y-axis, solids of known cross section Skills: Find the volume of a solid of revolution using the disk method. Find the volume of a solid of revolution using the washer method. Find the volume of a solid with known cross section.	Partner Activity: Exercises Finding Volume http://www.jamesrahn.com/CalculusI/PDF/volumes.pdf Whole Class Activity: Volumes of Solids with Known Cross Sections - An Exploration http://www.teacherspayteachers.com/Product/Volumes-of-Solids-with-Known-Cross-Sections-An-Exploration-865090 Unit 6 Common Benchmark Assessment	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.BF.1 Write a function that describes a relationship between two quantities. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee 8.1.12.C.1	3 hours

			Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community. CRP11 Use technology to enhance productivity.	

Unit 7 Overview

Unit Title: Differential Equations

Unit Summary:
This unit provides the solution techniques and applications for first-order differential equations. Students will have the opportunity to study slope fields which provide a graphical approach to learning about the solutions of a differential equation. This unit concludes with showing the student how to use first-order differential equations to create mathematical models of physical phenomena as well as develop basic properties of the exponential growth and decay models.

Suggested Pacing: 8 hours

Learning Targets

Unit Essential Questions:
- How are differential equations used to model real world problems?
- How can differential equations be analyzed analytically, graphically, and numerically to make predictions?

Unit Enduring Understandings:
- Real life problems can be solved by solving differential equations.
- Solutions to differential equations can be represented graphically, numerically and algebraically.
- Many of the fundamental laws of the physical and social sciences involve rates of change and can therefore be modeled using differential equations.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessment: Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 7 Assessment

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Solve basic types of differential equations.	Content: Differential equation, initial-value problems, first-order separable	Writing Activity: In your own words, describe how to recognize and solve differential equation that	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences.	2 hours

Use differential equations to model and solve applied problems.	equations, separation of variables Skills: Confirm solutions to differential equations. Find the general solution of a differential equation using separation of variables. Solve initial value problems.	can be solved by separation of variables. AP Practice Open-ended Question - Separation of Variables Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Exit Ticket: In your own words, describe the difference between the general solution of a differential equation and a particular solution.	9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.1 Write a function that describes a relationship between two quantities. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	
Use slope fields to approximate solutions of differential equations.	Content: Functions of two variables, slope fields Skills: Construct basic slope fields for differential equations. On the graph of a slope field, sketch the graph of a solution that satisfies a given initial condition.	Partner Activity: Given two sets of cards, one with diagrams of slope fields and the other with the corresponding differential equation. Students match the slope field with its corresponding differential equation. AP Practice Multiple Choice Question - Slope Fields Post question up for class to see. Have the students work out their solutions on whiteboards and display their answer for me to see. Calculus Lab: Creating Basic Slope Fields http://www.jamesrahn.com/CalculusI/PDF/creating%20Basic%20Slope%20Fields.pdf Long Term Assignment: Students are assigned two open-ended problems in AP format to complete at home.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	2 hours

Model exponential growth and decay using differential equations.	Content: Exponential growth and decay models, population growth, doubling-time, half-life, radioactive decay, carbon dating Skills: Solve applied problems using exponential growth and decay models On the graph of a slope field, sketch the graph of a solution that satisfies a given initial condition.	Quiz Question: Describe that the values of C and k represent in the exponential growth and decay model, $y = Ce^{kt}$. Calculus Lab: Social Diffusion http://www.jamesrahn.com/CalculusI/PDF/social%20diffusion.pdf Unit 7 Common Benchmark Assessment	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.BF.1 Write a function that describes a relationship between two quantities. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	2 hours

Unit 8 Overview

Unit Title: AP Calculus AB Exam Review

Unit Summary:
This unit provides the practice and preparation essential to getting students ready for the AP Calculus AB Exam. Students will have the opportunity to complete ample amounts of practice exercises including AP level multiple choice and open-ended problems. This unit will completely familiarize the students with the format of the test as well as test taking strategies to increase their chance at success on the exam.

Suggested Pacing: 17 hours

Learning Targets

Unit Essential Questions:
- How do we apply our knowledge of calculus to prepare for the AP exam?
- How can we communicate our mathematical thinking clearly?
- How is a graphing calculator used as a problem solving tool?

Unit Enduring Understandings:
- Math can be communicated verbally, analytically, numerically graphically, and using technology.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessment: Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 8 Assessment

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Solve multiple choice questions at the level of difficulty of The College Board Advanced Placement Calculus AB Examination. Solve free response questions at the level of difficulty of The College Board Advanced Placement Calculus AB Examination.	Content: Multiple-choice, free-response Skills: Complete multiple choice problems testing proficiency in a wide variety of topics. Complete free-response problems demonstrating the ability to solve problems involving a more extended chain of reasoning.	Coursera: Preparing for the AP Calculus AB Exam https://www.coursera.org/course/apcalc Previous Years AP Tests: http://www.jamesrahn.com/homepages/ap_tests.htm https://apstudent.collegeboard.org/apcourse/ap-calculus-ab/exam-practice Small Group Activity: Groups work on a selection of problems from AP Practice Books.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	17 hours

		Study Guide: http://www.elainetron.com/apcalc/apcalc.pdf Unit 8 Common Benchmark Assessment (Full length AP Calculus AB Exam taken over several class meetings)	F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.BF.1 Write a function that describes a relationship between two quantities. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. F.BF.4 Find inverse functions. F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	

			CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	

Unit 9 Overview

Unit Title: Projects and Activities to extend learning after the AP Calculus AB Exam

Unit Summary:
This unit provides the opportunity for students to work with other classmates to collaborate on a year-end final project. Students will have the chance to build on the content and skills attained throughout the course and apply them in a presentation-based project.

Suggested Pacing: 15 hours

Learning Targets

Unit Essential Questions:

- How do we demonstrate proficiency of calculus?
- How do we evaluate, solve and present a complex real world calculus problem?

Unit Enduring Understandings:

- Math can be represented and communicated verbally, numerically, analytically and visually.

Evidence of Learning

Formative Assessment: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc..

Summative Assessment: Contains a variety of question types such as multiple choice and free response questions, Section and Multi-section quizzes
Unit 9 Assessment

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Demonstrate and apply mastery of various calculus concepts to investigate an original idea, learn new problems, or present on a topic not previously studied.	Content: Final project Skills: Read and understand complex real world project. Work in cooperative teams to complete project. Meet scheduled deadlines. Present project and solution to class. Demonstrate thinking and analytical competency.	Group Presentation: Techniques of Integration - Break into groups of three or four students and prepare one of the units for a two-day presentation to the class. Ideas for After the AP Exam: http://www.jamesrahn.com/workshops/htm/ideas_for_after_the_ap.htm http://samjshah.com/2008/05/13/calculus-projects/ Attracting Mathematicians in High School: What Can be Taught After the AP Calculus Exam? http://sigmaa.maa.org/tahsm/PostAPIdeas/apcalcny4.pdf	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. Standards will depend on project completed. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP1. Act as a responsible and contributing citizen and employee	15 hours