Madison Public Schools
Algebra 2 Curriculum

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Course Overview

Description

Algebra 2 builds on concepts mastered in Algebra 1 and Geometry. Algebra 2 offers opportunities for students to develop as active problem solvers, critical thinkers and effective communicators. Consistent practice of algebraic skills enables students to make conjectures while working through challenging problems. Algebra 2 requires students to explain their thinking and analyze diverse problems, while also providing students with the chance to develop mathematical reasoning to work through everyday mathematical challenges. Each unit provides students opportunities to develop a deeper understanding of mathematics coupled with gaining procedural skill and fluency and application as outlined in the Common Core State Standards. Major topics include polynomial functions, radical functions, exponential and logarithmic functions, rational functions, sequences and series, trigonometric ratios and functions, probability, and data analysis and statistics. This course is offered in three levels that include Algebra 2 Essentials (E), Algebra 2 On Level (OL) and Algebra 2 Honors (H).

Goals

This course aims to:

- enable students to make sense of various types of problems and the reasonableness of their answers
- build student confidence with the various approaches to solving a problem and persevere in solving them
- encourage students to become abstract thinkers who make sense of quantities and their relationships in problem situations
- develop students' ability to cooperatively discuss, make conjectures and critique ideas of one another
- use, apply, and model mathematics to solve problems arising in everyday life, society, and the workplace
- consider the variety of available tools when solving a mathematical problem
- communicate mathematical ideas precisely and effectively to others
- determine a pattern or analyze structure within mathematical content to apply to related ideas
- use repeated reasoning to follow a multi-step process through to completion

Materials

Core: Big Ideas Math, Algebra 2
Supplemental: Khan Academy, Various websites related to Algebra 2

Resources

Suggested activities and resources page

Benchmark Assessments

Common Benchmark Assessment are given for each unit with common types of questions across the levels including problems that focus on the main ideas and anchor standards of the course.

Modifications and Adaptations for Special Needs Learners

(Gifted and Talented Students, English Language Learners, Students with Special Needs, At-Risk Students, and Students with 504 Plans)

Scope and Sequence (Pacing Guide)

Unit Number	Topic of Study	Duration (Weeks Taught)
1	Linear Functions	1 - 2 weeks
2	Quadratic Functions and Equations	4 - 6 weeks
3	Polynomial Functions	3 - 4 weeks
4	Radical Functions	3 - 4 weeks
5	Exponential and Logarithmic Functions	2 - 4 weeks
6	Rational Functions	4 - 6 weeks
7	Trigonometric Ratios and Functions	3 - 5 weeks
8	Sequences and Series	2 - 3 weeks
9	Probability	2 - 3 weeks
10	Data Analysis and Statistics	2 - 3 weeks

Unit 1 Overview

Unit Title: Linear Functions

Unit Summary:
In this unit students will recognize and graph parent functions and their transformations. The students will also use linear functions and systems of linear equations to model and analyze real-life situations. A firm understanding of the concepts of slope and x- and y-intercepts is essential to the topics covered in unit 1.

Suggested Pacing: 8 hours

Learning Targets

Unit Essential Questions:

- What are characteristics of some of the basic parent functions?
- How are equations and graphs related?
- What type of real life situation can be modeled with a system of linear equations?

Unit Enduring Understandings:

- The characteristics of linear functions and their representations are useful to solving real-world problems.
- The concept of transformations is applicable to all parent functions.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 1 Exam (Chapter 1 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Secret of the Hanging Baskets Performance Task

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Identify families of functions. (E/OL/H) Describe transformations of parent functions. (E/OL/H) Describe combinations of transformations. (E/OL/H) Write functions representing translations and reflections. (E/OL/H) Write functions representing stretches and shrinks. (OL/H) Write functions representing combinations of transformations. (E/OL/H)	Content: parent function transformation translation reflection vertical stretch vertical shrink Function Domain Range Slope Scatter plot Skills: Graph a parent function and its transformation. Describe transformation of given equation from parent function. Identify and correct the error described in the transformation of a parent function.	Vocabulary and Core Concept Check (Big Ideas) p. 8 and 16 Quiz question: Graph a function and its parent function and then describe the transformation. Partner Activity: Compare graphs of the eight parent functions and then classify the type of graph.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs. RST.9-10.7 Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	3.5 hours
Write equations of linear functions using points and slopes. (E/OL/H) Find lines of fit and lines of best fit. (E/OL/H)	Content: Line of fit Line of best fit Correlation coefficient Slope Slope-intercept form Point-slope form Scatter plot Skills: Write the equation of a line and interpret the slope. Explain how the correlation coefficient relates to a scatterplot. Describe and correct the error in interpreting slope in the context of the situation.	Vocabulary and Core Concept Check (Big Ideas) p. 26 Exploring Linear Data Activity from illuminations.nctm.org Journal Writing: Describe a situation in the real-world that can be modeled using a linear function.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	1 hour

			F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	
Visualize solutions of systems of linear equations in three variables. (E/OL/H) Describe the meaning of the solution to a system of equations in three variables. (OL/H) Solve systems of linear equations in three variables algebraically. (E/OL/H) Solve real-life problems involving three unknowns. (OL/H)	Content: Linear equation in three variables System of three linear equations Solution of a system of three linear equations Ordered triples System of two linear equations Skills: Solve system of linear equations using elimination. Solve system of linear equations using substitution.	Writing activity: Explain when it might be more convenient to use the elimination method than the substitution method to solve a linear system. Give an example to support your claim. Performance Task: Secret of the Hanging Baskets (Big Ideas p. 37) Partner Activity: Each student writes a system of equations in three variables and their partner must solve it. Ch. 1 Standards Assessment (Big Ideas) p. 42-43 Common Benchmark Assessment for Unit 1	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. 8.1.12.A.3 Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue. CRP4 Communicate clearly and effectively and with reason.	1.5 hours

Unit 2 Overview

Unit Title: Quadratic Functions and Equations

Unit Summary:
In this unit students identify transformations and characteristics of quadratic functions. The students are provided opportunities to model real-life situations using quadratic functions. Students will become fluent in solving quadratic equations in various ways including by graphing, using square roots, factoring, completing the square and using the quadratic formula. Students will be introduced to and learn to work with the complex number system within the context of solving quadratic equations. This unit concludes with solving nonlinear systems graphically and algebraically as well as solving quadratic inequalities.

Suggested Pacing: 20 hours

Learning Targets

Unit Essential Questions:

- How are quadratic functions used to model and interpret real-life situations?
- How is any quadratic function related to the parent quadratic function?
- How are roots, x-intercepts, solutions and zeros alike? Different?
- What are the different ways to solve quadratic equations and when is each appropriate?

Unit Enduring Understandings:

- The graph of a quadratic function is always a parabola.
- The graph of any quadratic function is a transformation of the graph of the parent quadratic function.
- For any quadratic function in standard form, the values of a, b, and c provide key information about its graph.
- You can solve a quadratic equation in standard form in more than one way.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 2 Exam - Part 1 (Chapter 2 in Big Ideas), Unit 2 Exam - Part 2 (Chapter 3 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Accident Reconstruction Performance Task

Objectives **(Students will be able to…)**	**Essential Content/Skills**	**Suggested Assessments**	**Standards** **(NJCCCS CPIs, CCSS, NGSS)**	**Pacing**
Describe transformations of quadratic functions. (E/OL/H) Write transformations of quadratic functions. (E/OL/H)	Content: quadratic function parabola vertex of a parabola vertex form Skills: Describe/graph transformations of a quadratic function. Write a rule described by the transformations of a graph Match a function with its graph	Vocabulary and Core Concept Check (Big Ideas p.52) Work with a partner: match quadratic functions to graphs and explain your reasoning; use a graphing calculator to verify that your answer is correct. Have students go to Desmos online graphing tool to graph a parent function. Edit the parent function making one transformation at a time and observe and make note of the way the function changes. https://www.desmos.com/calculator	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	2 hours
Explore properties of parabolas. (E/OL/H) Find maximum and minimum values of quadratic functions. (E/OL/H) Graph quadratic functions using x-intercepts. (E/OL/H) Solve real-life problems. (OL/H)	Content: axis of symmetry standard form minimum value maximum value intercept form Skills: Graph a quadratic function and identify vertex, axis of symmetry. Find the minimum or maximum value of a function. Describe the domain and range of a function. Describe where a quadratic graph increases and decreases. Use a given quadratic function to answer questions related to a real-life situation.	Calculator Activity: Use the graphing calculator to graph a parabola and find its minimum, or maximums and x-intercepts, if any. Vocabulary and Core Concept Check(Big Ideas p.61) Partner Activity: Graph a quadratic function using specific values; identify the vertex; fold graph on vertical line to illustrate symmetry; determine equation of vertical line. Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.IF.9 Compare properties of two functions each represented in a different way	1.5 hours

			(algebraically, graphically, numerically in tables, or by verbal descriptions). A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Explore the focus and the directrix of a parabola. (E/OL/H) Derive the equation given a focus and directrix. (OL/H) Write equations of parabolas. (E/OL/H) Solve real-life problems. (OL/H)	Content: focus directrix Skills: Identify the focus, directix and axis of symmetry Write the equation of a given graph of a parabola Write the equation of a parabola given its characteristics.	Wax paper folding activity Vocabulary and Core Concept Check(Big Ideas p.72)	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. G.GPE.2 Derive the equation of a parabola given a focus and directrix.	1.5 hours
Write equations of quadratic functions using vertices, points and x-intercepts. (OL/H) Write quadratic equations to model data sets. (OL/H)	Content: vertex form intercept form Skills: Write an equation of a parabola given a point on the parabola and the vertex. Write an equation of a parabola given a point and x-intercepts of the parabola.	Vocabulary and Core Concept Check (Big Ideas p.80) Work with partner: using given data use a calculator to create a scatter plot of the data; use quadratic regression to find the quadratic model; graph the quadratic function to verify that it fits the data; answer specific questions using the calculator model. Ch. 2 Standards Assessment (Big Ideas) p. 88 - 89 Common Benchmark Assessment for Unit 2 Part A Performance Task: Accident Reconstruction (Big Ideas p. 83)	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the	1.5 hours

			context. Emphasize linear and exponential models. RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	
Solve quadratic equations by graphing. (E/OL/H) Solve quadratic equations algebraically. (E/OL/H) Solve real-life problems. (OL/H)	Content: quadratic equation in one variable root of an equation zero of a function Skills: Solve an equation by graphing. Solve an equation using square roots. Solve an equation by factoring. Find the zero(s) of the function.	Vocabulary and Core Concept Check (Big Ideas p.99) Work with a partner matching a quadratic function with its graph; explain your reasoning; determine the number of x-intercepts of each graph; use results to find the real solutions of the equations algebraically.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	2 hours
Define and use the imaginary unit *i*. (E/OL/H) Add, subtract, and multiply complex numbers. (E/OL/H) Find complex solutions and zeros. (E/OL/H)	Content: imaginary unit *i* complex number imaginary number pure imaginary number Skills: Find the square root of a number. Add, subtract or multiply complex numbers. Write an expression as a complex number in standard form.	Vocabulary and Core Concept Check (Big Ideas p.108) Create a flowchart which illustrates the expansion of the system of numbers (include imaginary numbers and all subsets of real numbers.)	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.CN.1 Know there is a complex number i such that $i^2 = -1$ and every complex number has the form $a + bi$ with a and b real. N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. N.CN.7 Solve quadratic equations with real coefficients that have complex solutions. A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic	1.5 hours

			formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	
Solve quadratic equations using square roots. (E/OL/H) Solve quadratic equations by completing the square. (OL/H) Write quadratic functions in vertex form. (OL/H)	Content: Completing the square Skills: Solve equation by using square roots. Solve equations by completing the square. Solve equations by determining whether to use factoring, square roots or completing the square.	Vocabulary and Core Concept Check (Big Ideas p.116) Work with a partner: use algebra tiles to complete the square of a given expression. Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.CN.7 Solve quadratic equations with real coefficients that have complex solutions. A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	1.5 hours
Derive the quadratic formula. (OL/H) Solve quadratic equations using the Quadratic Formula. (E/OL/H) Analyze the discriminant to determine the number and type of solutions. (OL/H) Solve real-life problems. (OL/H)	Content: Quadratic Formula discriminant Skills: Solve equations using the Quadratic Formula. Find the discriminant of a quadratic equation and describe number/type of solutions of equation.	Vocabulary and Core Concept Check(Big Ideas p.127) Work with a partner:; use completing the square to derive the quadratic formula Chutes and Ladders - Quadratic Equations Review http://www.learnnc.org/lp/pages/2981	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.CN.7 Solve quadratic equations with real coefficients that have complex solutions. A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	1.5 hours
Solve systems of nonlinear equations. (OL/H) Solve quadratic equations by graphing, including using the graphing calculator. (E/OL/H)	Content: System of nonlinear equations Skills: Solve nonlinear system graphically.	Vocabulary and Core Concept Check (Big Ideas p.136) Work with partner: use graphing calculator to determine the solution(s) of nonlinear quadratic systems by utilizing the	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom	1.5 hours

Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). (OL/H)	Solve nonlinear system using substitution. Determine if an ordered pair is a solution of nonlinear system. Solve nonlinear system by elimination.	intersect function of the calculator	and during structured learning experiences. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs. RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	
Graph quadratic inequalities in two variables. (E/OL/H) Solve quadratic inequalities in one variable. (E/OL/H)	Content: Quadratic inequality in two variables Quadratic inequality in one variable Skills: Graph a quadratic inequality. Graph a system of quadratic inequalities. Solve a quadratic inequality algebraically.	Vocabulary and Core Concept Check (Big Ideas) p. 144 Activity: use a calculator to graph a quadratic function; explain how you can use the graph to solve an inequality involving the same function. Common Benchmark Assessment Unit 2 Part B Performance Task: Algebra in Genetics: The Hardy-Weinberg Law (Big Ideas p. 147) Ch. 3 Standards Assessment (Big Ideas) p. 152 - 153	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	1.5 hours

Unit 3 Overview

Unit Title: Polynomial Functions

Unit Summary:
This unit provides students the opportunity to do operations with polynomial functions and use various methods to solve polynomial equations. Students will also analyze graphs of polynomial functions and model real life situations.

Suggested Pacing: 15 hours

Learning Targets

Unit Essential Questions:

- What does the degree of a polynomial tell you about its related polynomial function?
- For a polynomial function, how are factors, zeros and x-intercepts related?
- For a polynomial function, how are factors and roots related?

Unit Enduring Understandings:

- When you look at the algebraic form of a polynomial function you can know something about its graph. Likewise, you can look at its graph and know something about its algebraic form.
- Knowing the zeros of a polynomial functions can help you understand the behavior of its graph.
- You can divide polynomials using steps that are similar to the long division steps that you use to divide whole numbers.
- The degree of a polynomial equation tells you how many roots the equation has.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 3 Exam (Chapter 4 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: For the Birds—Wildlife Management Performance Task

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Identify polynomial functions. (E/OL/H) Graph polynomial functions using tables and end behavior. (E/OL/H)	Content: polynomial polynomial function end behavior Skills: Determine if a polynomial is a function and write it in standard form, stating its degree, type, and leading coefficient. Evaluate a function for a given value. Describe the end behavior of the graph of a function. Graph polynomial functions.	Vocabulary and Core Concept Check (Big Ideas) p. 162 Activity: match cubic and quartic polynomial functions to their graphs and use calculator to confirm your answers.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	1.5 hours
Add, subtract, and multiply polynomials. (E/OL/H) Identify the operations under which polynomials are closed. (OL/H) Use Pascal's Triangle to expand binomials. (H)	Content: Pascal's Triangle Skills: Compute sums, differences and products of polynomials. Apply Pascal's Triangle to multiply binomials raised to higher degrees. (H)	Vocabulary and Core Concept Check (Big Ideas) p. 170 Writing Activity: Your friend claims the sum of two binomials is always a binomial and product of two binomials is always a trinomial. Is your friend correct? Explain your reasoning.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (H)	1 hour
Use long division to divide polynomials by other polynomials. (E/OL/H) Use synthetic division to divide polynomials by binomials of the form x - k. (E/OL/H) Use the Remainder Theorem. (E/OL/H)	Content: Polynomial long division Synthetic division Skills: Divide polynomials using long division. Divide using synthetic division.	Vocabulary and Core Concept Check (Big Ideas) p. 177 Match given division statements with graphs of the related cubic polynomials; explain your reasoning; verify your results with a calculator.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.2	1.5 hours

	Use synthetic division to evaluate a function for an indicated value.	Partner Activity: Steps of the proof of the Remainder Theorem are written on cards and students work together to put the steps of the proof in the correct order.	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). A.APR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Factor polynomials. (OL/H) Use the Factor Theorem. (OL/H)	Content: Factored completely Factor by grouping Quadratic form Skills: Factor polynomials completely. Show that a binomial is a factor of a polynomial. Use method of choice to factor a polynomial completely.	Activity: identify the x-intercepts of graphs of polynomial function; use the intercepts to write each polynomial in factored form. Vocabulary and Core Concept Check (Big Ideas) p. 184 Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	1.5 hours
Find solutions of polynomial equations and zeros of polynomial functions. (OL/H) Use the Rational Root Theorem. (OL/H) Use the Irrational Conjugates Theorem. (OL/H)	Content: Repeated solution Roots of an equation Real numbers Conjugates Skills: Solve polynomial equations. Use the Rational Root Theorem to determine which given solutions are not possible. Find all real solutions of an equation. Use the graph of a function to shorten the list of possible rational zeros of the function. Write a polynomial function of least degree given zeros of the function.	Vocabulary and Core Concept Check (Big Ideas) p. 194 Activity: Solve a cubic equation using two different methods and explain which method you prefer. Rational Root Theorem Activity: http://www.teacherspayteachers.com/Product/Rational-Root-Theorem-Activity-489144	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	1.5 hours

Use the Fundamental Theorem of Algebra. (OL/H) Find conjugate pairs of complex zeros of polynomial functions. (OL/H) Use Descartes's Rule of Signs. (OL/H) Extend polynomial identities to the complex numbers. (H) Develop the Fundamental Theorem of Algebra and show that it is true for quadratic polynomials. (H)	Content: Complex conjugates Repeated solution Degree of a polynomial Solution of an equation Zero of a function Conjugates Fundamental Theorem of Algebra (H) Skills: Identify the number of solutions or zeros. Write a polynomial function that has rational coefficients using given zeros. Determine the number of possible positive real zeros, negative real zeros and imaginary zeros for a function. Use Descartes's Rule of Signs to determine the number of positive and negative zeros of a function.	Vocabulary and Core Concept Check (Big Ideas) p. 202 Work with a partner: Use a graphing calculator to determine whether given quartic functions have imaginary solutions. Explain your reasoning and then find all the solutions. Quiz Question: Factor $x^2 + 9$. (H)	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. (H) N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (H) A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	1.5 hours
Describe transformations of polynomial functions. Write transformations of polynomial functions.	Content: Polynomial functions. Transformations Skills: Describe transformations of a given function and graph the function. Match a function with the graph of the correct transformation. Write a rule for a new function indicating the transformation of a given function.	Vocabulary and Core Concept Check (Big Ideas) p. 209 Have students go to Desmos online graphing tool to graph a parent function. Edit the parent function making one transformation at a time and observe and make note of the way the function changes. https://www.desmos.com/calculator	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	1.5 hours
Use x-intercepts to graph polynomial functions. (OL/H) Use the Location Principal to identify zeros of polynomial functions. (E/OL/H) Find turning points and identify local maximums and local minimums of graphs of polynomial functions. (E/OL/H)	Content: Local maximum Local minimum Even function Odd function End behavior Increasing Decreasing Symmetric about the y-axis Skills: Match a function to its graph. Graph a function.	Vocabulary and Core Concept Check (Big Ideas) p. 216 Writing activity: your friend claims that the product of two odd functions is an odd function. Is your friend correct? Explain your answer. Work with a partner: Use a graphing calculator to	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	1.5 hours

Identify even and odd functions. (OL/H)	Find all the real zeros of a function. Graph a function and identify the x-intercepts and points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing. Determine whether the function is even, odd or neither.	approximate the coordinates of turning points of given functions.	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	
Write polynomial functions for a set of points and using finite differences. (OL/H) Use technology to find models for data sets. (OL/H)	Content: Finite differences, scatter plots Skills: Write a cubic function given the zeros of the graph. Determine the degree of a polynomial function using finite differences given a data set.	Vocabulary and Core Concept Check (Big Ideas) p. 216 Partner Activity: Write three different cubic functions that pass through the points (3, 0), (4, 0) and (2, 6). Justify your answers. Common Benchmark Assessment Unit 3 Performance Task: For the Birds - Wildlife Management (Big Ideas p. 225) Ch. 4 Standards Assessment (Big Ideas) p. 232- 233	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. NJSLSA.SL4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs. CRP11 Use technology to enhance productivity.	1.5 hours

Unit 4 Overview

Unit Title: Radical Functions

Unit Summary:
This unit provides students the ability to work with rational exponents, their properties, and radicals. Students will understand how to use a rational exponent to represent a power involving a radical. They will graph radical functions and solve radical equations and inequalities. Students will also perform function operations and find the inverse of a function.

Suggested Pacing: 11 hours

Learning Targets

Unit Essential Questions:

- How are expressions involving radicals and exponents related?
- How do power and radical functions model real-world problems and their solutions?
- When you square each side of an equation, is the resulting equation equivalent to the
- original?
- How are a function and its inverse function related?

Unit Enduring Understandings:

- Corresponding to every power there is a root.
- The characteristics of power and radical functions and their representations are useful in solving real-world problems.
- You can write a radical expression in an equivalent form using a rational exponent instead of a radical sign.
- Solving a square root equation may require that you square each side of the equation. This process can introduce extraneous solutions.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 4 Exam (Chapter 5 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Turning the Tables Performance Task

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Find the nth root of numbers. (E/OL/H) Evaluate expressions with rational exponents. (E/OL/H) Solve equations using nth roots. (E/OL/H)	Content: nth roots of *a* Index of a radical Square root Cube root Exponent Skills: Find the indicated real *n*th root of *a*. Match equivalent expressions. Find the reals solutions of an equation, both exact and decimal approximations.	Vocabulary and Core Concept Check (Big Ideas) p. 241 Writing Activity: Between what two consecutive integers does $\sqrt[4]{125}$ lie? Explain your reasoning.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	1.5 hours
Use properties of rational exponents to simplify expressions with rational exponents. (E/OL/H) Use properties of radicals to simplify and write radical expressions in simplest form. (E/OL/H)	Content: Simplest form Conjugate Like radicals Properties of integer exponents Rationalizing the denominator Absolute Value Skills: Use properties of rational exponents to simplify expressions. Perform indicated operations to simplify expressions.	Vocabulary and Core Concept Check (Big Ideas) p. 248 "I have…, Who has..?" cards for properties of radicals and rational exponents. http://www.doe.virginia.gov/testing/solsearch/sol/math/AII/m_ess_a2-1bc.pdf	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	1 hour

Graph radical functions. (E/OL/H) Write transformations of radical functions. (E/OL/H) Explain the effect on the graph given various radical functions. (OL/H) Graph parabolas and circles. (H)	Content: Radical function Transformations Parabolas Circle Skills: Match a function to its graph. Graph a function and identify its domain and range. Describe the transformation of the given function and graph the new function. Write a rule for a new function described by the transformation of the graph of given functions.	Vocabulary and Core Concept Check (Big Ideas) p. 256 Have students go to Desmos online graphing tool to graph a parent function. Edit the parent function making one transformation at a time and observe and make note of the way the function changes. https://www.desmos.com/calculator Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7b Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k,k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	2.5 hours
Solve equations containing radicals and rational exponents and explain each step in solving. (E/OL/H) Solve radical inequalities and explain each step in solving. (E/OL/H)	Content: Radical equation Extraneous solutions Rational exponents Radical expressions Solving quadratic equations Skills: Solve an equation. Solve an inequality. Solve a nonlinear system.	Vocabulary and Core Concept Check (Big Ideas) p. 266 Writing activity: Explain how you know the radical equation $\sqrt{x+4} = -5$ has no real solution without solving it. Equation Calculator Activity: Enter all different types of radical equations into the equation calculator and observe the steps taken to solve each problem. http://symbolab.com/solver/radical-equation-calculator/%5Csqrt%7Bx%2B1%7D%3D4/?origin=button	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	1.5 hours
Add, subtract, multiply and divide functions. (E/OL/H)	Content: Domain scientific notation Skills: Given functions *f* and *g*, find (f + g)(x), (f - g)(x), and evaluate the new function for a given value of x.	Vocabulary and Core Concept Check (Big Ideas) p. 273 Writing activity: Is the addition of functions and the multiplication of functions commutative? Justify your answer.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.BF.1b Combine standard function types using arithmetic operations.	1 hour

Explore inverse functions. (E/OL/H) Find and verify inverses of nonlinear functions. (E/OL/H) Solve real-life problems using inverse functions. (OL/H) Verify that one function is the inverse of another by illustrating that f-1(f(x)) = f(f-1(x)) = x. (H)	Content: Inverse functions Input Output Inverse operations Reflection Line of reflection Skills: Find the inverse of a function and graph the function and its inverse. Determine whether the inverse of a function is a function. Determine whether two functions are inverses of each other.	Vocabulary and Core Concept Check (Big Ideas) p. 281 Partner activity: use a graphing calculator to graph two functions that are inverses of each other in the same window and list what you notice about the graphs Common Benchmark Assessment Unit 4 Performance Task: Turning the Tables (Big Ideas p. 285) Ch. 5 Standards Assessment (Big Ideas) p. 290 - 291	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. F-BF 4b (+) Verify by composition that one function is the inverse of another. (H)	1.5 hours

Unit 5 Overview

Unit Title: Exponential and Logarithmic Functions

Unit Summary:
The students will learn about the characteristics of exponential growth and decay functions and the natural base *e*. They we will work closely with logarithmic and exponential functions. Students will solve exponential and logarithmic equations by using their properties.

Suggested Pacing: 13 hours

Learning Targets

Unit Essential Questions:

- What characterizes exponential growth and decay and what are real world models of exponential growth and decay?
- How are exponential functions and logarithmic functions related?

Unit Enduring Understandings:

- Logarithms and exponents have corresponding properties.
- You can use logarithms to solve exponential equations as well as use exponents to solve logarithmic equations.
- A logarithmic function is the inverse of an exponential function and vice versa.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 5 Exam (Chapter 6 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Measuring Natural Disasters Performance Task

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Graph exponential growth and decay functions. (E/OL/H) Use exponential models to solve real-life problems. (OL/H) Interpret the parameters in an exponential function in terms of a context. (OL/H)	Content: Exponential function Exponential growth function Growth factor Asymptote Exponential decay function Decay factor Properties of exponents Skills: Evaluate exponential expressions. Justify steps in rewriting exponential functions. State the growth or decay rate of a given function. Calculate compounded interest.	Vocabulary and Core Concept Check (Big Ideas) p. 300 Illuminations Math Activity for working with compound interest: Predicting Your Financial Future http://illuminations.nctm.org/Lesson.aspx?id=2765 Partner activity: Match exponential functions with their graphs. Make a table of values to sketch the functions to verify your answers. Quiz question - Provide an exponential equation and have students write a word problem that accompanies the equation.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	2 hours
Define and use the natural base *e*. (OL/H) Graph natural base functions. (OL/H) Solve real-life problems. (OL/H)	Content: Natural base *e* Irrational number Properties of exponents Percent increase Percent decrease Compound interest Skills: Determine whether a function represents exponential growth or exponential decay. Match a function to its graph. Graph a function and identify its domain and range.	Vocabulary and Core Concept Check (Big Ideas) p. 307 Partner activity: Use an approximation of *e* and complete a table of values of x for $y = e^x$. Sketch the graphs and check your graphs with a graphing calculator.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. 9.1.12.B.7 Explain the meaning of income tax, describe how it is	1 hour

			calculated, and analyze its impact on one's personal budget.	
Define and evaluate logarithms. (E/OL/H) Use inverse properties of logarithmic and exponential functions. (E/OL/H) Graph logarithmic functions. (E/OL/H)	Content: Logarithm of y with base b function Common logarithm Natural logarithm Inverse function Skills: Write a logarithmic equation in exponential form. Write an exponential equation in logarithmic form. Evaluate a logarithm. Simplify an expression using the properties of logarithms. Find the inverse of a given exponential or logarithmic function. Graph a logarithmic function.	Vocabulary and Core Concept Check (Big Ideas) p. 314 Writing Activity: Does every logarithmic equations have to pass through the point (1, 0)? Explain your reasoning. Make a logarithms foldable emphasize both converting between exponential and logarithmic form and evaluating basic logarithms. http://mathequalslove.blogspot.com/2014/01/introducing-logarithms-with-foldables.html	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	1.5 hours
Transform graphs of exponential functions. (OL/H) Transform graphs of logarithmic functions. (OL/H) Write transformations of graphs of exponential and logarithmic functions. (OL/H)	Content: Exponential function Logarithmic function Transformations Skills: Match a function to its graph. Describe the transformation of a new function from the given function then graph the function. Write a rule for a new function from the transformation of the original function.	Vocabulary and Core Concept Check (Big Ideas) p. 322 Partner activity: Match each transformed function with its graph; describe the transformation; determine the domain, range and asymptote of each function and justify your answers. Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or	2 hours

			mathematically (e.g., in an equation) into words.	
Use the properties of logarithms to evaluate logarithms. (E/OL/H) Use the properties of logarithms to expand or condense logarithmic expressions. (E/OL/H) Use the change-of-base formula to evaluate logarithms. (OL/H)	Content: Base Properties of exponents Skills: Match an expression to a logarithm having the same value. Expand logarithmic expressions. Condense logarithmic expressions. Determine the correct equation from list of equations. Use the change-of-base formula to evaluate the logarithm.	Vocabulary and Core Concept Check (Big Ideas) p. 331 Develop properties of logarithms: Illuminations Math - Logarithms Demystified http://illuminations.nctm.org/Lesson.aspx?id=3024 http://illuminations.nctm.org/Lesson.aspx?id=3024	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.2 Use the structure of an expression to identify ways to rewrite it. F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	1.5 hours
Solve exponential equations and explain each step in solving. (E/OL/H) Solve logarithmic equations and explain each step in solving. (E/OL/H) Solve exponential and logarithmic inequalities. (OL/H) Understand the inverse relationship between exponents and logarithms. (OL/H) Use the inverse relationship between exponents and logarithms to solve problems. (H)	Content: Exponential equations Logarithmic equations Extraneous solutions Inequality Skills: Solve exponential equation. Solve logarithmic equation. Describe and correct the error in solving an equation. Approximate the solution of an equation using its graph. Solve an inequality.	Vocabulary and Core Concept Check (Big Ideas) p. 338 Writing activity: Solve $\log_5 x < 2$ algebraically and graphically. Explain why you prefer one method over the other. Explain how exponential and logarithmic functions are related.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. F-BF5 (+)Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (H)	1.5 hours
Classify data sets. (OL/H) Write exponential functions. (OL/H) Use technology to find exponential and logarithmic models. (OL/H)	Content: Finite differences Common ratio Point-slope form Skills: Determine type of function represented by table of data and explain reasoning.	Vocabulary and Core Concept Check (Big Ideas) p. 346 Performance Task: Measuring Natural Disasters (Big Ideas p. 349) Ch. 6 Standards Assessment (Big Ideas) p. 354 - 355	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this	1.5 hours

	Write an exponential function whose graph passes through 2 given points. Determine whether the data show an exponential relationship and write the function that models the data. Create a scatter plot of given point to determine whether an exponential model fits the data.	Common Benchmark Assessment for Unit 5	classroom and during structured learning experiences. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	

Unit 6 Overview

Unit Title: Rational Functions

Unit Summary:
This unit provides the students with the opportunity to work with inverse variation and the graphs of rational functions. Students will perform operations on rational expressions as well as solve rational equations.

Suggested Pacing: 10 hours

Learning Targets

Unit Essential Questions:

- Are two quantities inversely proportional if an increase in one corresponds to a decrease in the other?
- Are a rational expression and its simplified form equivalent?
- When are asymptotes used to graph rational functions?
- How do we decide which method is most appropriate when solving rational equations?

Unit Enduring Understandings:

- You can use much of what you know about operations with fractions to perform operations with rational expressions.
- When solving an equation involving rational expressions multiplying by the common denominator can result in extraneous solutions.
- The graph of the rational function will display asymptotic behavior if the rational function is in simplified form and the polynomial in the denominator is not a constant.
- The domain affects graphing and solving of rational functions.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 6 Exam (Chapter 7 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Circuit Design Performance Task

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Classify direct and inverse variation. (E/OL/H) Write inverse variation equations. (E/OL/H)	Content: Inverse variation Constant of variation Direct variation Ratios Skills: Determine whether an equation shows direct or inverse variation or neither. Using given values that relate x and y when they vary inversely, find y when given the value of x.	Vocabulary and Core Concept Check (Big Ideas) p. 363 Writing activity: Describe a real-life situation that can be modeled by an inverse variation equation.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	2 hours
Graph simple rational functions. (E/OL/H) Translate simple rational functions. (E/OL/H) Graph other rational functions. (OL/H) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. (H)	Content: Rational function Domain Range Asymptote Long Division Skills: Graph rational function. Match a function with its graph. Graph function, state domain and range. Name the vertical asymptote of the graph of the function. Name x-intercept of the graph of the function. Describe a rewritten equation as a transformation of the parent graph.	Vocabulary and Core Concept Check (Big Ideas) p. 370 Match a transformed function to the its graph and describe the transformation. Writing activity: Is it possible for a rational function to have two vertical asymptotes? Justify your answer. Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.6 Rewrite simple rational expressions in 1different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. F.IF.7d Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. (H) F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include	2 hours

			recognizing even and odd functions from their graphs and algebraic expressions for them. A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	
Simplify rational expressions. (E/OL/H) Multiply rational expressions. (E/OL/H) Divide rational expressions. (E/OL/H)	Content: Rational expression Simplified form of a rational expression Fractions Polynomials Domain Equivalent expressions Reciprocal Skills: Simplify rational expression. Find the product of rational expressions. Find the quotient of rational expressions. Perform indicated operations on rational expressions.	Vocabulary and Core Concept Check (Big Ideas) p. 380 Writing Activity: For the expression $(x^2 + 6x - 27)/(x^2 +4x-45)$ you think that the domain is all real numbers except 5 but your classmate thinks the domain is all real numbers except 5 and -9. Who is correct? Explain.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	1 hour
Add or subtract rational expressions. (E/OL/H) Rewrite rational expressions and graph the related function. (OL/H) Simplify complex fractions. (OL/H)	Content: Complex fraction Rational numbers Reciprocal Skills: Find the sum or difference of rational expressions. Find the least common multiple of given expressions. Rewrite a given function as a transformation of the parent function and graph the function. Simplify complex fractions.	Vocabulary and Core Concept Check (Big Ideas) p. 388 Work with a partner: Write a sum or difference of rational expressions that have a given domain and justify your answer.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	1.5 hours

Solve rational equations by cross multiplying. (E/OL/H) Solve rational equation by using the least common denominator. (OL/H) Use inverses of functions. (OL/H)	Content: Cross multiplying Proportion Extraneous solution Inverse of a function Skills: Solve the equation by cross multiplying. Identify the least common denominator. Solve the equation by using the LCD. Determine if the inverse of a given function is a function. Find the inverse of a function.	Vocabulary and Core Concept Check (Big Ideas) p. 396 Writing activity: Give an example of a rational equation that you would solve using cross multiplication and one that you would solve using the LCD. Explain your reasoning. Ch. 7 Standards Assessment (Big Ideas) p. 404 - 405 Common Benchmark Assessment for Unit 6	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	1.5 hours

Unit 7 Overview

Unit Title: Trigonometric Ratios and Functions

Unit Summary:
In this unit students gain a firm grasp of right triangle trigonometry which is essential to the understanding of new trigonometric functions. Students will become familiar with evaluating trigonometric functions in both degree and radian measure. This unit provides opportunities to graph and model trigonometric functions. The unit concludes with students learning and using the trigonometric identities and the sum and difference formulas.

Suggested Pacing: 14 hours

Learning Targets

Unit Essential Questions:

- What information does a trigonometric function provide about its graph and what can be determined about a trigonometric function from its graph?
- If you know the value of one trigonometric function, how can you find the value of the the five remaining trigonometric functions?
- How would you use the trigonometric functions to model real-life situations?

Unit Enduring Understandings:

- Periodic behavior is behavior that repeats over intervals of equal length.
- An angle with a full circle of rotation measures 2π radians or 360 degrees.
- You can translate periodic functions in the same way that you translate other functions.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 7 Exam (Chapter 9 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Lightening the Load Performance Task

Objectives (Students will be able to...)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Evaluate trigonometric functions of acute angles. (E/OL/H) Find unknown side lengths and angle measures of right triangles. (E/OL/H) Use trigonometric functions to solve real-life problems. (OL/H)	Content: Sine Cosine Tangent Cosecant Secant Cotangent Right triangle Hypotenuse Acute angle Pythagorean Theorem Reciprocal Complementary angles Skills: Evaluate the six trigonometric functions of the angle. Given an acute angle of the right triangle, find the other five trigonometric functions of the angle. Solve the right triangle given some measurements.	Vocabulary and Core Concept Check (Big Ideas) p. 466 Partner activity: Find the exact values of the sine, cosine and tangent functions for the angles 30, 45, and 60 degrees in special right triangles.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.	1.5 hours
Draw angles in standard position. (E/OL/H) Find coterminal angles. (E/OL/H) Understand, explain, and use radian measure. (E/OL/H) Construct the unit circle using special right triangles. (E/OL/H) Use special right triangles to determine the values of sine, cosine, tangent for $\pi/3,\ \pi/4,\ \pi/6$. (H)	Content: Initial side Terminal side Standard position Coterminal Radian Sector Central angle Radius of a circle Circumference of a circle Skills: Draw an angle with the given measure in standard position. Find one positive angle and one negative angle that are coterminal with the given angle. Convert the degree measure to radians or the radian measure to degrees. Match the angle measure with the angle. Use a calculator to evaluate the trigonometric function	Vocabulary and Core Concept Check (Big Ideas) p. 474 Group activity: Create the unit circle using compass and straightedge and label both degree and radian measure.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle F.TF 3 (+) Use special right triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3,\ \pi/4,\ \pi/6$ and use the unit circle to express the values of sine, cosine and tangent for $\pi - x.\pi + x,\ and\ 2\pi - x$ in terms of their values for x, where x is any real number. (H)	1 hour

Evaluate trigonometric functions of any angle. (OL/H) Find and use reference angles to evaluate trigonometric functions. (OL/H)	Content: Unit circle Quadrantal angle Reference angle Circle Radius Pythagorean Theorem Skills: Evaluate the six trigonometric functions of an angle. Use the unit circle to evaluate the six trigonometric functions of an angle. Sketch an angle and find its reference angle. Evaluate the function without using a calculator.	Vocabulary and Core Concept Check (Big Ideas) p. 482 Writing Activity: Write tan θ as ratio of two other trigonometric functions. Use the ratio to explain why tan 90^{o} is undefined but cot $90^{o} = 0$.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	1.5 hours
Explore characteristics of sine and cosine functions.(OL/H) Stretch and shrink graphs of sine and cosine functions. (OL/H) Translate graphs of sine and cosine functions. (OL/H) Reflect graphs of sine and cosine functions. (OL/H) Restrict a trigonometric function to a domain on which it is always increasing or always decreasing to allow the function to have an inverse. (H)	Content: Amplitude Periodic function Cycle Period Phase shift Midline Transformations x-intercept Skills: Determine whether a graph represents a periodic function. If so, identify the period. Identify the amplitude and period of the graph of the function. Graph the function. Describe the transformation of a function from the parent function. Match the function with its graph. Write a rule that represents the indicated transformation of a graph.	Vocabulary and Core Concept Check (Big Ideas) p. 491 Group activity: Complete a table for y = sin x, plot the points and draw a smooth curve through the points. Repeat for y = cos x Writing activity: Describe a transformation of the graph of f(x) = sin x that results in the graph of g(x) = cos x. Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. (H)	2 hours
Explore characteristics of tangent and cotangent functions. (OL/H) Graph tangent and cotangent functions. (OL/H) Graph secant and cosecant functions. (OL/H)	Content: Asymptote Period Amplitude x-intercept Transformations Skills: Graph one period of the function and describe the transformation of the	Vocabulary and Core Concept Check (Big Ideas) p. 502 Group activity: Complete a table for y = tan x, plot the points and draw a smooth curve through the points. Repeat for y = cot x , y = sec x, y = csc x	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.7e	1.5 hours

	graph from its parent function. Match the equation with its graph. Write a rule for a function that represents the transformations of a given function from the parent function.		Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	
Interpret and use frequency. (OL/H) Write trigonometric functions. (OL/H) Use technology to find trigonometric models. (OL/H)	Content: Frequency Sinusoid Amplitude Period Midline Skills: Find the frequency of the function. Write a function for the sinusoid. Determine whether you would use a sine or cosine function to model each sinusoid with the y-intercept given and explain your reasoning.	Vocabulary and Core Concept Check (Big Ideas) p. 510 Calculator Activity: Plot data and use the regression capabilities of the a graphing calculator to model the data.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. 8.1.12.F.1 Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs. RST.11-12.9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.	1.5 hours
Use trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions. (OL/H) Verify trigonometric identities. (OL/H)	Contents: Trigonometric identity Unit circle Skills: Find the values of the other five trigonometric functions of an angle. Simplify the expression. Verify the identity.	Vocabulary and Core Concept Check (Big Ideas) p. 517 Partner activity: Derive remaining two Pythagorean identities from $\sin^2 x + \cos^2 x = 1$. Writing activity: Explain how transformations of the graph of the parent function f(x) = sin x	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos	1.5 hours

		support the cofunction identity $\sin(\pi/2 - \theta) = \cos\theta$.	(θ), or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle.	
Use sum and difference formulas to evaluate and simplify trigonometric expressions. (OL/H) Use sum and difference formulas to solve trigonometric equations and rewrite real-life formulas. (OL/H) Prove and apply trigonometric identities. (H) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them solve problems. (H)	Content: Sum and difference formulas Ratio Trigonometric identities (H) Skills: Find the exact value of the expression. Evaluate the expression given specific values for cos a and sin a. Simplify the expression. Solve the equation.	Vocabulary and Core Concept Check (Big Ideas) p. 523 Performance Task: Lightening the Load (Big Ideas p. 525) Ch. 9 Standards Assessment (Big Ideas) p. 532 - 533 Common Benchmark Assessment for Unit 7	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.TF.9 (+)Prove and apply trigonometric identities. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. (H) WHST.11-12.1. Write arguments focused on discipline-specific content. (H)	1.5 hours

Unit 8 Overview

Unit Title: Sequences and Series

Unit Summary:
In this unit students define sequences and series and use them to write rules and find sums. Students will have the opportunity to analyze both arithmetic and geometric sequences and series. The students will then move on to finding sums of infinite geometric series and using recursive rules with sequences.

Suggested Pacing: 10 hours

Learning Targets

Unit Essential Questions:

- What is the difference between series and sequences?
- How are sequences and series used to model real-life situations?
- How is a graphing calculator used to work with sequences and series?

Unit Enduring Understandings:

- Sequences and series provide the basis for subsequent mathematics courses, especially calculus.
- Sequences and series are a direct result of finding patterns.
- Arithmetic and geometric sequences and series are mathematical patterns that stem from practical situations.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 8 Exam (Chapter 8 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Integrated Circuits and Moore's Law Performance Task

Objectives (Students will be able to…)	**Essential Content/Skills**	**Suggested Assessments**	**Standards** (NJCCCS CPIs, CCSS, NGSS)	**Pacing**
Use sequence notation to write terms of sequences. (E/OL/H) Write a rule for the nth term of a sequence. (E/OL/H) Sum the terms of a sequence to obtain a series and use summation notations. (OL/H)	Content: Sequence Terms of a sequence Series Summation notation Sigma notation Domain Range Skills: Write the first six terms of a sequence. Describe the pattern, write the next term, and write a rule for the nth term of the sequence. Write the series using summation notation. Find the sum.	Vocabulary and Core Concept Check (Big Ideas) p. 414 Partner activity: Match a sequence with its graph. Write a rule for the nth term of the sequence and use the rule to find the tenth term.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	2 hours
Identify arithmetic sequences. (E/OL/H) Write rules for arithmetic sequences. (E/OL/H) Find sums of finite arithmetic series. (OL/H)	Content: Arithmetic sequence Common difference Arithmetic series Linear function Mean Skills: Tell whether a sequence is arithmetic and explain your reasoning. Write a rule for the nth term of the sequence. Write a rule for a sequence from the graph of the sequence. Find the sum of the arithmetic sequence.	Vocabulary and Core Concept Check (Big Ideas) p. 422 Writing activity: Describe how doubling each term in an arithmetic sequence changes the common difference of the sequence. Justify your answer. Partner activity: Determine whether given graphs show an arithmetic sequence. If it does, write a rule for the nth term and determine the sum of the first 20 terms. Describe what you notice about the graph of an arithmetic sequence.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	1.5 hours
Identify geometric sequences. (E/OL/H) Write rules for geometric sequences. (E/OL/H) Find sums of finite geometric series. (OL/H)	Content: Geometric sequence Common ratio Geometric series Exponential function Properties of exponents Skills: Tell whether the sequence is geometric.	Vocabulary and Core Concept Check (Big Ideas) p. 430 Illuminations Math Activity: Exploring Measurement, Sequences, and Curves with Stringed Instrument http://illuminations.nctm.org/Lesson.aspx?id=1938	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences.	2 hours

	Write a rule for the nth term of the sequence. Then find a_7 Write a rule for a sequence from a graph of the sequence. Find the sum.	Quiz	A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
Find partial sums of infinite geometric series. (OL/H) Find sums of infinite geometric series. (OL/H)	Content: Partial sum Repeating decimal Fraction in simplest form Rational number Skills: Find and graph the partial sums of given geometric series. Find the sum of the infinite geometric series, if it exists. Write the repeating decimal as a fraction in simplest form.	Vocabulary and Core Concept Check (Big Ideas) p. 439 Partner activity: Enter given geometric series in a spreadsheet. Use the spreadsheet to determine whether the infinite geometric series has a fine sum. If it does, find the sum and explain your reasoning. Writing activity: Is 0.999... equal to 1? Justify your answer.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.	1 hour
Evaluate recursive rules for sequences. (OL/H) Write recursive rules for sequences. (OL/H) Translate between recursive and explicit rules for sequences. (OL/H) Use recursive rules to solve real-life problems. (OL/H)	Content: Explicit rule Recursive rule Skills: Write the first six terms of the sequence. Write a recursive rule for the sequence. Write an explicit rule for the sequence.	Vocabulary and Core Concept Check (Big Ideas) p. 447 Ch. 8 Standards Assessment (Big Ideas) p. 456- 457 Common Benchmark Assessment for Unit 8 Administer PBA practice test	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.2	1.5 hours

			Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	

Unit 9 Overview

Unit Title: Probability

Unit Summary:
This unit provides the skills and techniques for finding sample spaces and calculating probability. Students will discuss ways of determining whether events are dependent or independent and construct and interpret two-way tables of data. Students will have the opportunity to work with the probability of disjoint and overlapping events. The unit concludes with the students becoming familiar with the use and applications formulas for permutations and combinations and binomial distributions.

Suggested Pacing: 12 hours

Learning Targets

Unit Essential Questions:
- How is probability used in everyday life?
- Why might it be necessary to determine if there is a difference between two data sets?

Unit Enduring Understandings:
- Probability leads to reasonable predictions in everyday life situations.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 9 Exam (Chapter 10 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: A New Dartboard Performance Task

Objectives (Students will be able to...)	Essential Content/Skills	Suggested Assessments	Standards (NJCCCS CPIs, CCSS, NGSS)	Pacing
Find sample spaces. (E/OL/H) Find theoretical probabilities. (E/OL/H) Find experimental probabilities. (E/OL/H)	Content: Probability experiment Outcome Event Sample space Probability of an event Theoretical probability Geometrical probability Experimental probability Tree diagram Skills: Find the number of possible outcomes in the sample space and list the outcomes.	Vocabulary and Core Concept Check (Big Ideas) p. 542 Quiz Question: Describe a real-life event that has a probability of 0. Then describe a real-life event that has a probability of 1.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	2 hours
Determine whether events are independent events. (E/OL/H) Find probabilities of independent and dependent events. (OL/H) Find conditional probabilities. (OL/H)	Content: Independent events Dependent events Conditional events Probability Sample space Skills: Tell whether events are independent or dependent.	Vocabulary and Core Concept Check (Big Ideas) p. 550 Partner activity: Determine if the following events are independent or dependent: 1) Two six-sided dice are rolled. 2) Six pieces of paper, numbered 1 through 6, are in a ag. Two pieces of paper are selected one at a time without replacement. Experimentally estimate the probability that the sum of the two numbers rolled is 7 and the sum of the two numbers selected is 7.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	1.5 hours

			S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	
Make two-way tables. (OL/H) Find relative and conditional relative frequencies. (OL/H) Use conditional relative frequencies to find conditional probabilities. (OL/H)	Contents: Two-way table Joint frequency Marginal frequency Joint relative frequency Marginal relative frequency Conditional relative frequency Conditional probability Skills: Complete the two-way table.	Vocabulary and Core Concept Check (Big Ideas) p. 558 Writing Activity: Compare two-way tables and Venn diagrams. Then describe the advantages and disadvantages of each. Quiz	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	2 hours
Find probabilities of compound events. (OL/H) Use more than one probability rule to solve real-life problems. (OL/H)	Contents: Compound event Overlapping events Disjoint or mutually exclusive event Venn diagram Skills: Given events A and B are disjoint, find P(A or B). Problem solving Error analysis	Vocabulary and Core Concept Check (Big Ideas) p. 567 Probability of Compound Events Activity: http://www.erusd.k12.ca.us/ProjectAlphaWeb/index_files/SDAP/Probability%20of%20Compound%20Events%20(6).pdf	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.CP.1	1.5 hours

	Drawing conclusions		Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	
Use the formula for the number of permutations. (OL/H) Use the formula for the number of combinations. (OL/H) Use combinations and the Binomial Theorem to expand binomials. (OL/H)	Content: Permutation *n* factorial Combination Binomial Theorem Fundamental Counting principle Pascal’s Triangle Skills: Find all the ways you can arrange all the letters of a given word. Evaluate the expression. Count the possible combinations of r letters chosen from a given list. Use the Binomial Theorem to write the binomial expansion.	Vocabulary and Core Concept Check (Big Ideas) p. 575 Calculator Activity: Finding permutations and combinations using the graphing calculator.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal’s Triangle. S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	1.5 hours
Construct and interpret probability distributions. (OL/H) Construct and interpret binomial distributions. (OL/H) Use permutations and combinations to compute probabilities of compound events and solve problems. (H)	Content: Random variable Probability distribution Binomial distribution Binomial experiment Histogram Skills: Make a table and draw a histogram showing the probability distribution for the random variable. Use given probability distribution to determine outcomes of a spinner.	Vocabulary and Core Concept Check (Big Ideas) p. 583 Performance Task: A New Dart board (Big Ideas p. 585) Ch. 10 Standards Assessment (Big Ideas) p. 590 - 591 Common Benchmark Assessment for Unit 9 Administer EOY practice test	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems. (H)	1.5 hours

Unit 10 Overview

Unit Title: Data Analysis and Statistics

Unit Summary:
This unit studies normal distribution finding normal probability and interpreting normally distributed data. Students will study populations, samples and analyze hypotheses. Students will learn the importance of sampling techniques and survey questions when collecting data. This unit continues with the analysis of experimental designs. It concludes with students learning to make inferences from sample surveys and experiments.

Suggested Pacing: 12 hours

Learning Targets

Unit Essential Questions:

- Why is an understanding of the normal curve essential to statistics?
- How do I know that the data I am looking is unbiased?
- What are some considerations when managing statistical study?

Unit Enduring Understandings:

- The statistics of many real life events can be approximated by the normal curve.
- You can get good statistical information about population by studying a sample of the population.

Evidence of Learning

Formative Assessments: A variety of formative assessments will be used throughout the lesson, such as warm-up and closure questions completed on paper and handed in and on Google Classroom, Four Corners, Hand It In, Pass It Out, Self-Evaluation, Think-Pair-Share, Jigsaw, Socrative, Two Roses and a Thorn, etc...

Summative Assessments: Unit 10 Exam (Chapter 11 in Big Ideas), Section and Multi-section quizzes

Alternative Assessments: Curving the Test Performance Task

Objectives **(Students will be able to…)**	**Essential Content/Skills**	**Suggested Assessments**	**Standards** **(NJCCCS CPIs, CCSS, NGSS)**	**Pacing**
Calculate probabilities using normal distributions. (E/OL/H) Use z-scores and the standard normal table to find probabilities. (OL/H) Recognize data sets that are normal. (OL/H)	Content: Normal distribution Normal curve Standard normal distribution z-score Probability distribution Symmetric Mean Standard deviation Skewed Median Skills: Give the percent of the area under the normal curve represented by the shaded region(s) Find the indicated probability for a randomly selected x-value from the distribution.	Vocabulary and Core Concept Check (Big Ideas) p. 600 Writing activity: Explain how you can convert ACT scores into corresponding SAT scores when you know the mean and standard deviation of each distribution	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to	2 hours
Distinguish between populations and samples. (E/OL/H) Analyze hypotheses. (OL/H)	Content: Population Sample Parameter Statistic Hypotheses Venn Diagram Proportion Skills: Determine whether the data are collected from a population or a sample. Explain your reasoning. Identify the population and sample. Describe the sample Determine whether the data are collected from a population or a sample. Explain your reasoning. Identify the population and sample. Describe the sample. Determine whether the numerical value is a parameter or a statistic. Explain your reasoning.	Vocabulary and Core Concept Check (Big Ideas) p. 607 Create a Venn diagram that compares and contrasts the contributions of three individuals. Find a newspaper or magazine article that describes a survey. Identify the population and sample describe the sample.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	1.5 hours
Identify types of sampling methods in statistical studies. (OL/H) Recognize bias in sampling. (OL/H)	Content: Random sample Self-selected sample Systematic sample Stratified sample Cluster sample Convenience sample Bias	Vocabulary and Core Concept Check (Big Ideas) p. 614 Writing activity: What is the difference between a “blind experiment” and a	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom	2 hours

Analyze methods of collecting data. (OL/H) Recognize bias in survey questions. (OL/H)	Unbiased sample Biased sample Experiment Observational study Survey Simulation Biased question Population Sample Skills: Identify the type of sample described. Determine whether the sample is biased. Identify the method of data collection the situation describes. Make an argument.	"double-blind" experiment? Describe a possible advantage of the second type of experiment over the first.	and during structured learning experiences. S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	
Describe experiments. (E/OL/H) Recognize how randomization applies to experiments and observational studies. (OL/H) Analyze experimental designs. (OL/H)	Content: Controlled experiment Control group Treatment group Randomization Randomized comparative experiment Placebo Replication Sample size Skills: Determine whether the study is a randomized comparative experiment.	Vocabulary and Core Concept Check (Big Ideas) p. 623 Designing Experiments Activity http://www.thirteen.org/edonline/lessons/experiments/b.html#close	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.6 Evaluate reports based on data.	1.5 hours
Estimate population parameters. (OL/H) Analyze estimated population parameters. (OL/H) Find margins of error or surveys. (OL/H)	Content: Descriptive statistics Inferential statistics Margin of error Statistic Parameter Skills: Estimate the population mean	Vocabulary and Core Concept Check (Big Ideas) p. 630 Writing activity: Is it possible to have a margin of error between 0 and 100 percent, not including 0 to 100 percent? Explain your reasoning.	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop	1.5 hours

			a margin of error through the use of simulation models for random sampling.	
Organize data from an experiment with two samples. (E/OL/H) Resample data using a simulation to analyze a hypothesis. (OL/H) Make inferences about a treatment. (OL/H)	Content: Randomized comparative experiment Control group Treatment group Mean Dot plot Outlier Simulation Hypothesis Skills: Use error analysis to describe and correct interpreting the experimental difference of the means.	Vocabulary and Core Concept Check (Big Ideas) p. 637 Writing activity: Describe an example of an observation that can be made from an experiment. Then give 4 possible inferences that could be made from the observation. Performance Task: Curving the Test (Big Ideas p. 639) Ch. 11 Standards Assessment (Big Ideas) p. 644 - 645 Common Benchmark Assessment for Unit 10	9.1.12.A.1 Apply critical thinking and problem-solving strategies during structured learning experiences. 9.1.12.F.2 Demonstrate a positive work ethic in various settings, including this classroom and during structured learning experiences. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	1.5 hours