## UNIT/STANDARD 9

Students are encouraged to copy down the completed notes onto their blank note sheet. Try the practice problems on the note sheet prior to checking your answers on the completed notes. This will help students process the information better. Links to instructional videos related to each lesson can be found on the teacher's website. Students are encouraged to refer to these videos if they have the means to do so. If students have access to their textbook, they may also refer to Lessons 10-6 and 10-7 of their text for additional support. After completing their note sheet, students should then complete the corresponding practice worksheet before moving on to the next page of notes. Below is the set of instructions and order in which to complete Unit/Standard 9.

1) Notes Handout: Geometry Review - Circumference and Area of Circles
2) Worksheet: Geometry - Circumference and Area of Circles

Complete problems 1-14 (charts) on the front side and problems 1-6 on the back side.
3) Notes Handout: Lesson 10-6: Arcs and Arc Measure (9-A)
4) Worksheet: Practice 9-A

Complete problems 1-4 (all problems on both sides)
5) Notes Handout: REVIEW: Multiplying fractions and a whole number
6) Notes Handout: Lesson 10-6: Arc Length (9-B)
7) Worksheet: Practice 9-B

Complete problems 1-10 all on front and problems 11-12 on the back
8) Notes Handout: Introduction to Sectors
9) Notes Handout: Lesson 10-7: Circles - Area of a Sector

* Try the practice problems prior to checking your answers on the completed notes.

10) Worksheet: Practice 9-C

Complete problems 1-8 all
I1) Worksheet: Standard 9 Review \#2
Complete problems 1-16 all
itherivifenc:
$\qquad$ 0 -20 + ab, +1









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2-10.1
$$

## STANDARD 9: Circles - Arc Length and Sector Area

(Find arc lengths and areas of sectors of circles)

|  | LEARNING TARGETS | TextSection and/or Notes | Common Core State Standards |
| :---: | :---: | :---: | :---: |
| REVIEW | Review circumference and area of circles. |  |  |
| A | Students will be able to identify minor arcs, major arcs and semicircles and find their measures given the measure of a central angle and/or other arc measures. | Lesson 10-6 |  |
| B | Students will be able to find the length of an arc (arc length). | Lesson 10-6 | G.C. 5 |
| C | Students will be able to find the area of a sector of a circle. | Lesson 10-7 | G.C. 5 |

KEY VOCABULARY TERMS:

* Circumference: perimeter of a circle: $C=2 \pi r=\pi d$
* Area of a Circle: $A=\pi r^{2}$
* Radius/Diameter of a circle
* Central angle: angle with vertex at center of circle and sides are radii
* Arc of a circle ("sections" of a circle, include minor arcs, major arcs and semicircles)
* Arc measure (in degrees)
* Arc length: length of an arc (fraction of the circumference)
* Sector of a Circle: region bounded by an arc of the circle and two radii


## NOTES: GEOMETRY REVIEW - CIRCUMFERENCE AND AREA OF CIRCLES

Circumference: $\mathrm{C}=2 \pi \mathrm{r}=\pi \mathrm{D} \quad$ Area: $\mathrm{A}=\pi \mathrm{r}^{2}$


## Complete the chart below. Refer to circle $\mathbf{P}$ below.



|  | Radius | Diameter | Circumference <br> (in terms of $\pi$ ) | Circumference <br> (nearest <br> hundredth) | Area <br> (in terms of $\pi$ ) | Area <br> (nearest <br> hundredth) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5 |  |  |  |  |  |
| 2 |  | 16 |  |  |  |  |
| 3 |  |  |  |  | $49 \pi$ |  |
| 4 |  |  | $12 \pi$ |  |  |  |

5) A bicycle's tires have a diameter of 24 inches. How far does the bicycle travel:
a) for each full rotation (one revolution) of a tire? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.

b) if the tires make 500 full rotations (revolutions)? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.
6) A rotating sprinkler sprays a circular pattern. The water covers a circular region that reaches 8 feet from the sprinkler. What is the area of the lawn that receives water from this sprinkler? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.

7) Find the (a) area and (b) perimeter of the figure below.


Sections of the circumference are called arcs. There are three types of arcs - minor, major and semicircles. Arcs have measure (in degrees) and length (in centimeters, inches, feet, etc.). Refer to the diagram of circle $P$ below when completing the chart.


D

| Type of Arc | Named by: | Example(s) in <br> Circle $\mathbf{P}$ | Measure |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Practice: Name $\mathbf{3}$ of each of the following in $\odot P$. $\overline{\mathrm{BE}}$ and $\overline{\mathrm{AC}}$ are diameters.

| Minor Ares |  |
| :---: | :---: |
| Semicircles |  |
| Major Ares |  |
| Central Angles |  |



Practice: Find the measure of each central angle in $\odot P$. $\overline{\mathrm{BE}}$ and $\overline{\mathrm{AC}}$ are diameters.

| Angle | Measure |
| :---: | :---: |
| $\angle A P B$ |  |
| $\angle \mathrm{CPD}$ |  |
| $\angle \mathrm{APE}$ |  |
| $\angle \mathrm{DPE}$ |  |
| $\angle \mathrm{CPE}$ |  |
| $\angle \mathrm{APD}$ |  |



Practice: Identify each arc type and then find the measure of each arc in $\odot \mathbf{P}$ above. $\overline{\mathrm{BE}}$ and $\overline{\mathrm{AC}}$ are diameters.

| Arc | Arc Type | Measure |
| :--- | :--- | :---: |
| $\overparen{\mathbf{A B}}$ |  |  |
| $\overparen{\mathbf{A B C}}$ |  |  |
| $\overparen{\mathbf{A E}}$ |  |  |
| $\overparen{\mathbf{A B D}}$ |  |  |
| $\overparen{\mathbf{B C}}$ |  |  |
| $\overparen{\mathbf{B C E}}$ |  |  |

## REVIEW: Multiplying fractions and a whole number.

In the lessons that follow in Unit/Standard 9, you will need to be able to multiply a fraction and a whole number. Here are some examples with different approaches and some problems to practice.

## Examples: Multiply.

a) $\frac{3}{4} \cdot 24$ Method $1: \frac{3}{4} \cdot 24=\frac{3}{4} \cdot \frac{24}{1}=\frac{72}{4}=18$

6
Method 2: $\frac{3}{4} \cdot 24=\frac{3}{4} \cdot \frac{24}{1}=3 \cdot 6=18 \quad$ * Cross cancel before multiplying 1
b) $\frac{1}{3} \cdot 27$ Method 1: $\frac{1}{3} \cdot 27=\frac{1}{3} \cdot \frac{27}{1}=\frac{27}{3}=9$

9
Method 2: $\frac{1}{3} \cdot 27=\frac{1}{\nexists} \cdot \frac{27}{1}=1 \cdot 9=9 \quad *$ Cross cancel before multiplying 1

NOTE: When you multiply a whole number by a fraction with a numerator of 1 (examples $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, etc.) like in problem (b) above, just divide the whole number by the denominator of the fraction.
c) $\frac{5}{8} \cdot 36$ Method $1: \frac{5}{8} \cdot 36=\frac{5}{8} \cdot \frac{36}{1}=\frac{180}{8}=\frac{45}{2} \quad$ *Reduce fraction, keeping it improper.

$$
9
$$

Method 2: $\frac{5}{8} \cdot 36=\frac{5}{8} \cdot \frac{36}{1}=\frac{45}{2} *$ Cross cancel common factor of 4 before multiplying

Practice: Multiply.

1) $\frac{5}{12} \cdot 36$
2) $\frac{3}{8} \cdot 36$
3) $\frac{1}{5} \cdot 35$

ARC LENGTH: The measure of an arc is in degrees. The arc's length depends on the size of the circle because it represents a fraction of the circumference. Arc Length $=$ $\qquad$ .

$$
\text { Length of } \operatorname{arc} \overparen{A B}=\frac{m \overparen{A B}}{360^{\circ}} \cdot 2 \pi r
$$



EXAMPLE: The diagram below is of two concentric circles (circles that share the same center $P$ ). Find the circumference (C) of each circle, the measure of the central angle ( $m \angle \mathrm{APB}$ ), measures of the intercepted arcs ( $m \overparen{\mathrm{AB}}$ and $m \overparen{C D}$ ), and lengths of $\overparen{\mathrm{AB}}$ and $\overparen{\mathrm{CD}}$. Write in terms of $\pi$.


C of $\mathrm{sm} \odot=$ $\qquad$ $C$ of $\lg \odot=$ $\qquad$ $m \angle \mathrm{APB}=$ $m \overparen{\mathrm{AB}}=$ $\qquad$ $m \overparen{\mathrm{CD}}=$ $\qquad$
length of $\overparen{\mathrm{AB}}=$ $\qquad$ length of $\overparen{C D}=$ $\qquad$

Examples: Find the length of $\overparen{A B B}$. Write answers in terms of $\pi$. Include units in answers.

2)



Fraction: - = $\qquad$ Fraction: $-=\square$
Fraction: - $=-$
Circumference $=$ $\qquad$
Circumference $=$ $\qquad$
Circumference $=$ $\qquad$

Arc Length $=$
Arc Length $=$

## Practice: Find the length of $\overparen{A B}$. Include units in your answers.

4) 


5)

6)

7) The minute hand of a large clock is 18 inches long. How far does the tip of the minute hand move for each given amount of time? Write answers in terms of $\pi$ and as decimals rounded to the nearest hundredth of an inch.
a) 1 hour
b) 30 minutes
c) 15 minutes
d) 40 minutes


## INTRODUCTION TO SECTORS

A pop-up sprinkler sitting in the middle of a lawn sprays water 15 feet. If the sprinkler is set to cover a $120^{\circ}$ sector, how much of the lawn receives water?
a) What is the radius of the circle in this problem?

b) What is the area of the whole circle?
c) What fraction of the circle is receiving water?
d) What area of the lawn is receiving water?


PRACTICE: A pop-up sprinkler is located in the center of a lawn. Find the (a) radius of the circle, (b) area of whole circle, (c) fraction of the circle receiving water (in lowest terms) and the (d) area of lawn receiving water (sector area) for each sprinkler.

1) Sprinkler sprays water 12 feet and is set to cover a $150^{\circ}$ sector.
2) Sprinkler sprays water 25 feet and is set to cover a $72^{\circ}$ sector.

(Standard 9 - LT C: TSWBAT find the area of a sector of a circle.)
SECTOR OF A CIRCLE: A region bounded by an arc of the circle and the two radii to the arc's endpoints. A sector is named using one arc endpoint, the center of the circle, and the other arc endpoint. The sector below is named sector AOB. The area of a sector is a fractional part of the area of the circle. That fraction can be determined by simplifying the corresponding arc's measure over $360^{\circ}$.


## FINDING THE AREA OF A SECTOR

To find the area of a sector (sector AOB above), find the area of the circle, then multiply by the fraction of the area covered by the arc of the sector.

$$
\text { Area of Sector } \mathrm{AOB}=\frac{m \widehat{\mathrm{AB}}}{360} \cdot \pi r^{2}
$$

EXAMPLES: Find the area of each shaded sector below. Write your answer in terms of $\pi$ (exact value) and rounded to the nearest tenth.
1.

3.


Fraction:


Fraction: $-=-$
Fraction: $-=-$
$\qquad$ Area of $\odot=$
Area of $\odot=$ $\qquad$

Sector Area $=$
Sector Area $=$
Sector Area $=$

EXAMPLES/PRACTICE: Complete the chart below. Refer to circle $\mathbf{O}$ below.


|  | Central <br> Angle <br> Measure <br> $(m \angle A O B)$ | Intercepted <br> Arc <br> Measure <br> $(m \overline{A B})$ | Fraction <br> of Circle | Radius <br> of <br> Circle | Area of <br> Circle <br> (in terms <br> of $\pi)$ | Area of Sector <br> (in terms of $\pi)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $60^{\circ}$ |  |  | 12 cm |  |  |
| 2 |  | $135^{\circ}$ |  |  | $64 \pi \mathrm{in}^{2}$ |  |
| 3 | $120^{\circ}$ |  |  | 9 m |  |  |
| 4 |  |  |  |  |  |  |

Circumference: $\mathrm{C}=2 \pi \mathrm{r}=\pi \mathrm{D} \quad$ Area: $\mathrm{A}=\pi \mathrm{r}^{2}$


Rearrange equal sized slices to form a parallelogram


$$
\text { Base }=\pi r
$$

Half of the circumference

$$
\text { Height }=r
$$

$$
\text { Area }=B \cdot H
$$

$$
\frac{1}{2} c=\frac{1}{z}(z \pi r)=\pi r
$$

$$
=\pi r \cdot r
$$

$$
=\pi r^{2}
$$

Complete the chart below. Refer to circle $P$ below.

$$
A=\pi r^{2}
$$



Diameter $=2 \cdot$ radius

$$
\text { Radius }=\frac{\text { Diameter }}{2}
$$


\#3) $r=\sqrt{49}=7$
5) A bicycle's tires have a diameter of 24 inches. How far does the bicycle travel:
a) for each full rotation (one revolution) of a tire? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.


As the tire rotates, the point $P$ makes a full circle equal to the tire's circumference.

I full rotation $=$ Circumference $=\pi D=24 \pi \mathrm{in}$
b) if the tires make 500 full rotations (revolutions)? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.

$$
\begin{aligned}
\text { Distance } & =500 \cdot 1 \text { full rotation } \\
& =500 \cdot 24 \pi \\
& =12,000 \pi \text { in } \approx 37,699.1 \mathrm{in}
\end{aligned}
$$

6) A rotating sprinkler sprays a circular pattern. The water covers a circular region that reaches 8 feet from the sprinkler. What is the area of the lawn that receives water from this sprinkler? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.


$$
\begin{aligned}
A & =\pi r^{2} \\
& =\pi \cdot 8^{2} \\
& =64 \pi \mathrm{ft}^{2} \\
& \approx 201.1 \mathrm{ft}^{2}
\end{aligned}
$$

7) Find the (a) area and (b) perimeter of the figure below.

(a) Area is $\frac{3}{4}$ of the whole circle

$$
\begin{aligned}
\text { Area } & =\frac{3}{4} \cdot \text { Area of Circle } \\
& =\frac{3}{4} \cdot \pi \cdot 10^{2} \\
& =\frac{3}{4} \cdot 100 \pi \\
& =75 \pi \mathrm{in}^{2} \approx 235.6 \mathrm{in}^{2}
\end{aligned}
$$

$\frac{1}{4}$ of circle is
missing, $50 \frac{3}{4}$
(b) Distance $=\frac{3}{4}$ of circumference +2 Radii of circle remains

$$
\left[\frac{3}{14} \cdot \frac{5}{T}=15\right]=\underset{\sim}{\frac{20}{4}} \cdot \frac{20 \pi+2(10)}{(15 \pi+20 \mathrm{in} \approx 67.1 \mathrm{in}}
$$

Sections of the circumference are called arcs. There are three types of arcs - minor, major and semicircles. Arcs have measure (in degrees) and length (in centimeters, inches, feet, etc.). Refer to the diagram of circle $P$ below when completing the chart.

* O symbol for circle
* $\widehat{A B}$ for arc
* $360^{\circ}$ in full circle
* Name a circle by its center point (©P)
* Central Angle - angle w/vertex at center and sides are radii ( $\angle B P C$ )
* $\overline{A C}$ is a diameter
$\rightarrow$ Minor Arcs - shortest "route" on circle between 2 points


Practice: Name 3 of each of the following in $\odot P$. $\overline{\mathrm{BE}}$ and $\overline{\mathrm{AC}}$ are diameters.



* The name of an arc can be reversed.
Ex: $\overparen{A B}$ or $\overparen{B A}, \overparen{A B C}$ or $\overparen{C B A}$ and $A D C$ name the
same semicircle

Practice: Find the measure of each central angle in $\odot P$. $\overline{\mathbf{B E}}$ and $\overline{\mathbf{A C}}$ are diameters.

| Angle | Measare |
| :---: | :---: |
| $\angle A P B$ | $115^{\circ}$ |
| $\angle \mathrm{CPD}$ | $90^{\circ}$ |
| $\angle \mathrm{APE}$ | $6.5^{\circ}$ |
| $\angle \mathrm{DPE}$ | $25^{\circ}$ |
| $\angle \mathrm{CPE}$ | $90+25=115^{\circ}$ |
| $\angle A P D$ | $90^{\circ}$ |



Practice: Identify each arc type and then find the measure of each arc in $\odot \mathbf{P}$ above. $\overline{\mathrm{BE}}$ and $\overline{\mathrm{AC}}$ are diameters.

| Arc | Arc Type | Measure | $\left\{\begin{array}{l} \begin{array}{c} 360-90 \\ \text { or } \end{array}=270^{\circ} \\ \underbrace{115+65}_{180}+90=270^{\circ} \end{array}\right.$ |
| :---: | :---: | :---: | :---: |
| $\overparen{A B}$ | Minor | $115^{\circ}$ |  |
| $\widehat{\text { ABC }}$ | Semicircle | $180^{\circ}$ |  |
| $\overparen{\text { AE }}$ | Minor | $65^{\circ}$ |  |
| $\overparen{\text { ABD }}$ | Major | 270 |  |
| $\overparen{B C}$ | Minor | $65^{\circ}$ |  |
| $\widehat{\text { BCE }}$ | Semicircle | $180^{\circ}$ |  |

## REVIEW: Multiplying fractions and a whole number.

In the lessons that follow in Unit/Standard 9, you will need to be able to multiply a fraction and a whole number. Here are some examples with different approaches and some problems to practice.

## Examples: Multiply.

a) $\frac{3}{4} \cdot 24$ Method 1: $\frac{3}{4} \cdot 24=\frac{3}{4} \cdot \frac{24}{1}=\frac{72}{4}=18$

6
Method 2: $\frac{3}{4} \cdot 24=\frac{3}{\not 4} \cdot \frac{24}{1}=3 \cdot 6=18 \quad *$ Cross cancel before multiplying 1
b) $\frac{1}{3} \cdot 27$ Method 1: $\frac{1}{3} \cdot 27=\frac{1}{3} \cdot \frac{27}{1}=\frac{27}{3}=9$

9
Method 2: $\frac{1}{3} \cdot 27=\frac{1}{\not x} \cdot \frac{27}{1}=1 \cdot 9=9 \quad *$ Cross cancel before multiplying

NOTE: When you multiply a whole number by a fraction with a numerator of 1 (examples $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, etc.) like in problem (b) above, just divide the whole number by the denominator of the fraction.
c) $\frac{5}{8} \cdot 36$ Method $1: \frac{5}{8} \cdot 36=\frac{5}{8} \cdot \frac{36}{1}=\frac{180}{8}=\frac{45}{2} \quad *$ Reduce fraction, keeping it improper.

9
Method 2: $\frac{5}{8} \cdot 36=\frac{5}{8} \cdot \frac{36}{1}=\frac{45}{2} *$ Cross cancel common factor of 4 before multiplying 2

## Practice: Multiply.

1) $\frac{5}{12} \cdot 36$
2) $\frac{3}{8} \cdot 36$
3) $\frac{1}{5} \cdot 35$
$\frac{5}{12} \cdot \frac{36}{1}=\frac{180}{12}=15$
$\frac{3}{8} \cdot \frac{36}{1}=\frac{108}{8}=\frac{27}{2}$
$\frac{35}{5}=7$
$\frac{5}{12} \cdot \frac{36}{1}=5 \cdot 3=15$

$$
\frac{3}{8} \cdot \frac{36}{1}=\frac{3 \cdot 9}{2}=\frac{27}{2}
$$



ARC LENGTH: The measure of an arc is in degrees. The arc's length depends on the size of the circle because it represents a fraction of the circumference.

Arc Length = $\qquad$


EXAMPLE: The diagram below is of two concentric circles (circles that share the same center $P$ ). Find the circumference ( $C$ ) of each circle, the measure of the central angle ( $m \angle \mathrm{APB}$ ), measures of the intercepted arcs ( $m \overparen{\mathrm{AB}}$ and $m \overparen{\mathrm{CD}}$ ), and lengths of $\overparen{\mathrm{AB}}$ and $\overparen{\mathrm{CD}}$. Write in terms of $\pi$.

$C$ of $\lg \odot=\ldots 20 \pi^{\prime \prime}$ $m \angle \mathrm{APB}=72^{\circ}$ $\left.\overparen{m \mathrm{AB}}=\frac{72^{\circ}}{}\right\} \begin{aligned} & \text { same } \\ & m \text { measure }\end{aligned}$
$\begin{aligned} & \overparen{\mathrm{CD}}=72^{\circ}\end{aligned}$ different
length of $\left.\overparen{A B}=2 \pi^{\prime \prime}\right\} \begin{aligned} & \text { Different } \\ & \text { lengths }\end{aligned}$ length of $\overparen{C D}=4 \pi^{\prime \prime}$ lengths

Examples: Find the length of $\overparen{A B}$. Write answers in terms of $\boldsymbol{\pi}$. Include units in answers.

2)



Fraction: $\frac{60}{360}=\frac{1}{6}$
Fraction: $\frac{120}{360}=\frac{1}{3}$
Circumference $=36 \pi \mathrm{~m}$
Circumference $=24 \pi \mathrm{~cm}$
Fraction: $\frac{90}{360}=\frac{1}{4}$

Arc Length $=\frac{1}{3} \cdot 24 \pi$


Arc Length $=\frac{1}{6} \cdot \frac{36 \pi}{6}$
$=\frac{1}{3} \cdot 24 \pi$
$=8 \pi \mathrm{~cm}$

Circumference $=16 \pi \mathrm{ft}$
Arc Length $=\frac{1}{4} \cdot 16 \pi$

$$
=\frac{1}{14} \cdot 6 \pi
$$

Practice: Find the length of $\overparen{\mathbf{A B}}$. Include units in your answers.
4)


$$
\begin{aligned}
& F=\frac{72}{360}=\frac{1}{5} \\
& C=30 \pi \mathrm{~m} \\
& \text { Length }=\frac{1}{5} \cdot 30 \pi
\end{aligned}
$$

$$
L=6 \pi \mathrm{~m}
$$

5) 


$F=\frac{30}{360}=\frac{1}{12}$
$C=18 \pi \mathrm{~cm}$
$L=\frac{1}{12} \cdot{ }^{3} 18 \pi$
$L=\frac{3 \pi}{2} \mathrm{~cm}$
6)


$$
F=\frac{150}{360}=\frac{5}{12}
$$

$$
C=36 \pi \mathrm{ft}
$$

$$
L=\frac{5}{1 x} \cdot 33 \pi=15 \pi \mathrm{ft}
$$

7) The minute hand of a large clock is 18 inches long. How far does the tip of the minute hand move for each given amount of time? Write answers in terms of $\pi$ and as decimals rounded to the nearest hundredth of an inch.
a) 1 hour
b) 30 minutes
c) 15 minutes
d) 40 minutes

I hour = Circumference


$$
\begin{aligned}
& F=\frac{15}{60}=\frac{1}{4} \\
& C=36 \pi \mathrm{~m} \\
& L=\frac{1}{4} \cdot 36 \pi \\
& =9 \pi \mathrm{~m} \\
& \approx 28.27 \pi \mathrm{in}
\end{aligned}
$$

$$
F=\frac{40}{60}=\frac{2}{3}
$$

$$
c=36 \pi \mathrm{in}
$$

$$
L=\frac{2}{3} \cdot 36 \pi
$$

$$
=\frac{2}{3} \cdot 36 \pi
$$

$$
\begin{aligned}
& =2 \cdot 12 \pi \\
& =24 \pi \mathrm{in}
\end{aligned}
$$

$$
\begin{aligned}
& =24 \pi \mathrm{in} \\
& \approx 75.40 \mathrm{in}
\end{aligned}
$$

* In a clock, 60 minutes $=360^{\circ}$ (full circle)

So on a clock, the length the tip of a minute hand travels is an arc length. To calculate this length, use the formula:

$$
\begin{array}{r}
\text { Length }=\left(\frac{\text { minutes }}{60}\right) \cdot 2 \pi r \text { where } r \text { is the } \\
T_{c} \text { length of minute hand }
\end{array}
$$

INTRODUCTION TO SECTORS
A pop-up sprinkler sitting in the middle of a lawn sprays water 15 feet. If the sprinkler is set to cover a $120^{\circ}$ sector, how much of the lawn receives water?
a) What is the radius of the circle in this problem?

$$
\text { Radius }=15 \mathrm{ft}
$$

b) What is the area of the whole circle?

$$
A=\pi r^{2}=\pi \cdot 15^{2}=225 \pi \mathrm{ft}^{2}
$$

c) What fraction of the circle is receiving water?

$$
\text { Fraction }=\frac{120}{360}=\frac{1}{3}
$$

d) What area of the lawn is receiving water?

$$
\begin{aligned}
\text { Sector Area } & =\text { Fraction } \cdot \text { Area of } \odot \\
& =\frac{1}{3} \cdot 225 \pi \\
& =\frac{225 \pi}{3}=75 \pi \mathrm{ft}^{2} \approx 235.6 \mathrm{ft}^{2}
\end{aligned}
$$

PRACTICE: A pop-up sprinkler is located in the center of a lawn. Find the (a) radius of the circle, (b) area of whole circle, (c) fraction of the circle receiving water ( in lowest terms) and the (d) area of lawn receiving water (sector area) for each sprinkler.

1) Sprinkler sprays water 12 feet and is set to cover a $150^{\circ}$ sector.
2) Sprinkler sprays water 25 feet and is set to cover a $72^{0}$ sector.

a) $r=12 \mathrm{ft}$
b)

$$
\begin{aligned}
A & =\pi \cdot 12^{2} \\
& =144 \pi \mathrm{ft}^{2}
\end{aligned}
$$

$$
\text { c) } F=\frac{150}{360}=\frac{5}{12}
$$

d)

$$
\begin{aligned}
\text { Sector Area } & =F \cdot \text { Area of Circle } \\
& =\frac{5}{12} \cdot \not 24 \pi=60 \pi \mathrm{ft}^{2} \\
& \approx 188.5 \mathrm{ff}^{2}
\end{aligned}
$$

a) $r=25 \mathrm{ft}$
b) $A=\pi \cdot 25^{2}$

$$
=625 \pi \mathrm{ft}^{2}
$$

c) $F=\frac{72}{360}=\frac{1}{5}$
(Standard 9-LT C: TSWBAT find the area of a sector of a circle.)
SECTOR OF A CIRCLE: A region bounded by an arc of the circle and the two radii to the arc's endpoints. A sector is named using one arc endpoint, the center of the circle, and the other arc endpoint. The sector below is named sector AOB. The area of a sector is a fractional part of the area of the circle. That fraction can be determined by simplifying the corresponding arc's measure over $360^{\circ}$.


## FINDING THE AREA OF A SECTOR

To find the area of a sector (sector AOB above), find the area of the circle, then multiply by the fraction of the area covered by the arc of the sector.

$$
\text { Area of Sector } \mathrm{AOB}=\frac{m \widehat{\mathrm{AB}}}{360} \cdot \pi r^{2}
$$

EXAMPLES: Find the area of each shaded sector below. Write your answer in terms of $\pi$ (exact value) and rounded to the nearest tenth.
1.


Fraction: $\frac{90}{360}=\frac{1}{4}$
Area of $\odot=64 \pi \mathrm{~cm}^{2}$
Sector Area $=\frac{1}{4} \cdot \frac{64 \pi}{16}$
$\left.\begin{array}{rl}\frac{64 \pi}{4}=16 \pi \longleftrightarrow & =\frac{1}{4} \cdot 64 \pi \\ \longrightarrow & =16 \pi \mathrm{~cm}^{2} \\ & \approx 50.3 \mathrm{~cm}^{2}\end{array}\right]$
2.


Fraction: $\frac{150}{360}=\frac{5}{12}$
Area of $C=\frac{36 \pi m_{3}^{2}}{5}$
Sector Area $=\frac{5}{12} \cdot 36 \pi$



Fraction: $\frac{288}{360}=\frac{4}{5}$

$$
\text { Area of } \odot=100 \pi \mathrm{~m}^{2}
$$

$$
20
$$

$\begin{aligned} \text { Sector Area } & =\frac{4}{5} \cdot 100 \pi \\ & =4 \cdot 20 \pi \\ & =80 \pi \mathrm{~m}^{2} \\ & \approx 251.3 \mathrm{~m}^{2}\end{aligned}$

EXAMPLES/PRACTICE: Complete the chart below. Refer to circle $\mathbf{O}$ below.


|  | Central <br> Angle <br> Measure <br> $(m \angle A O B)$ | Intercepted <br> Arc <br> Measure <br> $(m \overline{A B})$ | Fraction <br> of Circle | Radius <br> of <br> Circle | Area of <br> Circle <br> (in terms <br> of $\pi)$ | Area of Sector <br> (in terms of $\pi)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $60^{\circ}$ | $60^{\circ}$ | $\frac{60}{360}=\frac{1}{6}$ | 12 cm | $144 \pi$ | $\frac{1}{6} \cdot 144 \pi=\frac{144 \pi}{6}=24 \pi$ |
| $\mathrm{~cm}^{2}$ |  |  |  |  |  |  |$|$

## PRACTICE: GEOMETRY - CIRCUMFERENCE AND AREA OF CIRCLES



|  | Radius | Diameter | Circumference <br> (in terms of $\pi$ ) | Circumference <br> (nearest hundredth) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 3 |  |  |  |
| 2 |  | 10 |  |  |
| 3 | 6 |  |  |  |
| 4 |  |  | $18 \pi$ |  |
| 5 | 1 |  |  |  |
| 6 |  | 8 |  |  |
| 7 |  |  | $24 \pi$ |  |


|  | Radius | Diameter | Area <br> (in terms of $\pi$ ) | Area <br> (nearest hundredth) |
| :---: | :---: | :---: | :---: | :---: |
| 8 | 3 |  |  |  |
| 9 |  | 10 |  |  |
| 10 | 9 |  |  |  |
| 11 |  |  | $36 \pi$ |  |
| 12 | 1 |  |  |  |
| 13 |  | 4 |  |  |
| 14 |  |  | $64 \pi$ |  |

$$
\text { Circumference: } \quad \mathrm{C}=2 \pi \mathrm{r}=\pi \mathrm{D} \quad \text { Area: } \mathrm{A}=\pi \mathrm{r}^{2}
$$

1-2: Find the circumference (distance around figure) and area of each. Write answers in terms of $\pi$ and as decimals to the nearest tenth.
1)

2)

3) A bicycle's tires have a diameter of 18 inches. How far does the bicycle travel:
a) for each full rotation (one revolution) of a tire? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.
b) if the tires make 250 full rotations (revolutions)? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.
4) A rotating sprinkler sprays a circular pattern. The water covers a circular region that reaches 8 feet from the sprinkler. What is the area of the lawn that receives water from this sprinkler? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.
5) A dog is on a leash that is attached to a pole in the ground. If the leash is 8 ft long, in how much area can the dog move around? Round to the nearest tenth.
6) A Ferris wheel has a $50-\mathrm{m}$ radius. How many kilometers will a passenger travel during a ride if the wheel makes 10 revolutions? Round your answer to the nearest tenth of a kilometer. (Remember to convert units from meters to kilometers; $1 \mathrm{~km}=1000 \mathrm{~m}$ )

1) Refer to the diagram of circle $P$ below. $\overline{\mathrm{AC}}$ is a diameter. Complete the chart.


| Arc | Arc Type | Measure |
| :--- | :--- | :--- |
| $\overparen{\mathbf{A B}}$ |  |  |
| $\overparen{\mathbf{A B C}}$ |  |  |
| $\overparen{\mathbf{A D}}$ |  |  |
| $\overparen{\mathbf{B D}}$ |  |  |
| $\overparen{\mathbf{B C}}$ |  |  |
| $\overparen{\mathbf{B A C}}$ |  |  |
| $\overparen{\mathbf{C D}}$ |  |  |
| $\overparen{\mathbf{A D C}}$ |  |  |
| $\overparen{\mathbf{A B D}}$ |  |  |

2) Refer to the diagram of circle $\mathbf{P}$ below. $\overline{\mathbf{A C}}$ is a diameter. Find the measure of each arc or angle.


| Arc or Angle | Measure |
| :---: | :---: |
| $\overparen{\text { AB }}$ |  |
| $\overparen{A D C}$ |  |
| $\overparen{\text { AD }}$ |  |
| $\overparen{\text { BD }}$ |  |
| 二CPD |  |
| LAPD |  |
| $\angle B P E$ |  |
| $\overparen{\text { BE }}$ |  |
| ABD |  |
| ADB |  |

3) If $m \overparen{\mathrm{AB}}=50^{\circ}$ in a circle, what does $m \overparen{\mathrm{ACB}}$ equal in the same circle?
4) If $m \overparen{\mathrm{AB}}=70^{\circ}$ and $m \overparen{\mathrm{BC}}=20^{\circ}$, what are the possible measures of $\overparen{\mathrm{AC}}$ ?

## PRACTICE 9-B

## Find the length of each arc.

1) 


2)

3)

4)

5)

6)

7)

8)

9)

10)



ax ins inger of puit

## PRACTICE 9-B

11) Find the length of the outfield fence ( $\overparen{A B}$ ) on the field below. Write answer in terms of $\pi$ and as a decimal to the nearest tenth.

12) The minute hand of a large clock is 18 inches long. How far does the tip of the minute hand move for each given amount of time. Write answers in terms of $\pi$ and as decimals rounded to the nearest hundredth of an inch.
a) 45 minutes
b) 20 minutes
c) 55 minutes
d) 2 hours





1-3: Find the area of each shaded sector below. Write your answer in terms of $\pi$ (exact value).
1.



4: Write the answer to problem \#3 above as a decimal rounded to the nearest tenth.

5-8: Find each answer. Write answers in terms of $\pi$ and as a decimal rounded to the nearest tenth.
5) A rotating sprinkler sitting in the middle of a lawn sprays water 20 feet. If the sprinkler is set to cover a $90^{\circ}$ sector, how much of the lawn receives water?
6) A pop-up sprinkler sprays water a maximum of 10 feet from the sprinkler head. If the sprinkler is set to cover a $135^{\circ}$ sector, what is the maximum area of lawn that receives water?
7) A rotating sprinkler is sitting in the middle of a square lawn that is 20 feet on a side. The sprinkler sprays a radius of 10 feet. See the diagram below. The shaded area receives water. What is the area of the lawn that does not receive any water from the sprinkler? Show work.

8) A rotating sprinkler is sitting in the middle of a circular lawn that has a radius of 25 feet. The sprinkler sprays a radius of 20 feet. See the diagram below. The shaded area receives water. What is the area of the lawn that does not receive any water from the sprinkler? Show work.


Standard 9 Review \#2
Circles - Ares, Arc Length and Sector Area

Name
Period
$\qquad$

1-4: Refer to the diagram of circle $P$ below. (a) Identify the arc as a minor arc, major arc or semicircle and then (h) find its measure. [LT A]

1) $\overparen{\mathrm{AD}}$
2) $\overparen{\mathrm{BCD}}$
3) $\overparen{\mathrm{ACB}}$
4) $\overparen{A C}$

5) $a$ $\qquad$ b. $\qquad$
6) $a$ $\qquad$ b $\qquad$
7) $a$ $\qquad$ b $\qquad$
8) a $\qquad$ b $\qquad$

5-7: Refer to the diagram of circle $P$ above.
5) Name a minor arc not listed in problems 1-4 above. State its measure.
6) Name a major arc not listed im problems 1-4 above. State its measure.
7) Name the major are that corresponds to minor arc AB . State its measure.

8-11: Find the lenpth of each specified arc of the circle P. Write answers in terms of $\pi$. Include units in each answer. Show work. [LT B]
8) length of $\overparen{A B}$

9) length of CD
8)

9)
11) length of $\overparen{A B C}$

10) $\qquad$
11)
$\qquad$ of the minute hand travel in 45 minutes? Write answer in terms of $\pi$ and as a decimal rounded to the nearest tenth. Show work and include units. [LT B]

13-14: Find the area of each shaded sector. Write answers (a) in terms of $\pi$ and as (b) decimals to the nearest hundredth. Include units in each answer. [LT C]
13) Diameter $=16 \mathrm{~cm}$

14)

13) a $\qquad$
b $\qquad$
14) $\qquad$
b $\qquad$
15) A rotating sprinkler is setup in a large lawn. The sprinkler sprays water
15) $\qquad$ with a $15^{\prime}$ radius. The sprinkler is setup to spray a $72^{\circ}$ sector. How much of the lawn is the sprinkler watering? Write answer as a decimal to the nearest tenth. Show work.
16) A rotating sprinkler is setup in the center of a square lawn that is $50^{\prime}$ on
16) side. The water from the sprinkler reaches to the edge of each side of the square lawn. How much of the square lawn (shaded region) does not get water from the sprinkler? Write answer as a decimal to the nearest tenth. Show work.


1) Refer to the diagram of circle $P$ below. $\overline{\mathbf{A C}}$ is a diameter. Complete the chart.


| Are | Arc Type | Measure |
| :---: | :---: | :---: |
| $\stackrel{\text { AB }}{ }$ | Minor | $120^{\circ}$ |
| $\overparen{\text { ABC }}$ | Semicircle | $180^{\circ}$ |
| $\stackrel{\text { AD }}{ }$ | Minor | $108^{\circ}$ |
| BD | Minor | $132^{\circ}$ |
| BC | Minor | $60^{\circ}$ |
| $\widehat{\text { BAC }}$ | Major | $300^{\circ}$ |
| - ${ }_{\text {BD }}$ | Minor | $72^{\circ}$ |
| $\widehat{\text { ADC }}$ | Semicircle | $180^{\circ}$ |
| $\overparen{\text { ABD }}$ | Major | $252^{\circ}$ |

2) Refer to the diagram of circle $P$ below. $\overline{\mathrm{AC}}$ is a diameter. Find the measure of each arc of angle.

3) If $m \overparen{\mathrm{AB}}=50^{\circ}$ in a circle, what does $m \overparen{\mathrm{ACB}}$ equal in the same circle?

$$
\left.\begin{array}{rl}
{[m \overparen{A C B}=360-m \overparen{A B}}
\end{array}\right] \begin{aligned}
\mathrm{ACB} & =360-50 \\
& =310^{\circ}
\end{aligned}
$$

4) If $\overparen{m \mathrm{AB}}=70^{\circ}$ and $\overparen{m \mathrm{BC}}=20^{\circ}$, what are the possible measures of AC ?


$$
\begin{aligned}
& m \overparen{A C}=m \widehat{A B}+m \widehat{B C} \\
& =70^{\circ}+20^{\circ} \\
& =90^{\circ} \quad 50^{\circ} \text { or } 90^{\circ} \\
& m \overparen{A C}=m \overparen{A B}-m \widehat{B C} \\
& =70^{\circ}-20^{\circ} \\
& =50^{\circ}
\end{aligned}
$$

PRACTICE: GEOMETRY -CIRCUMFERENCE AND AREA OF CIRCLES


Circumference: $\mathrm{C}=2 \pi \mathrm{r}=\pi \mathrm{D} \quad$ Area: $\mathrm{A}=\pi \mathrm{r}^{2}$
1-2: Find the circumference (distance around figure) and area of each. Write answers in terms of $\pi$ and as decimals to the nearest tenth. $D=$ distance around

2) figure

3) A bicycle's tires have a diameter of 18 inches. How far does the bicycle travel:
a) for each full rotation (one revolution) of a tire? Write answer in terms of $\pi$ and as a decimal to the nearest tenth. I revolution $=C$

$$
C=18 \pi \mathrm{in} \approx 56.5 \mathrm{~m}
$$

b) if the tires make 250 full rotations (revolutions)? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.


$$
250 \cdot 18 \pi=14,137.2 \mathrm{in} / 4,500 \pi \mathrm{in}
$$

4) A rotating sprinkler sprays a circular pattern. The water covers a circular region that reaches 8 feet from the sprinkler. What is the area of the lawn that receives water from this sprinkler? Write answer in terms of $\pi$ and as a decimal to the nearest tenth.

$$
r=8 \text { feet } \quad \begin{aligned}
A & =\pi r^{2} \\
& =\pi \cdot 8^{2} \\
& =64 \pi \mathrm{ft}^{2}
\end{aligned} \quad A=64 \pi \mathrm{ff}^{2} \approx 201.1 \mathrm{ft}^{2}
$$

5) A dog is on a leash that is attached to a pole in the ground. If the leash is 8 ft long, in how much area can the dog move around? Round to the nearest tenth.

$$
\begin{aligned}
r=8 \text { feet } \quad A & =\pi r^{2} \\
& =\pi \cdot 8^{2} \\
& =64 \pi \mathrm{ft}^{2}
\end{aligned} \quad A=64 \pi \mathrm{ft}^{2} \approx 201.1 \mathrm{ft}^{2}
$$

6) A Ferris wheel has a 50 -m radius. How many kilometers will a passenger travel during a ride if the wheel makes 10 revolutions? Round your answer to the nearest tenth of a kilometer. (Remember to convert units from meters to kilometers; $1 \mathrm{~km}=1000 \mathrm{~m}$ )

$$
\begin{aligned}
& C=\pi D=2 \pi r \quad 10 \text { revolutions } \Rightarrow 10 \cdot 100 \pi \\
&=1,000 \pi \\
& c=2 \cdot \pi \cdot 50 \\
&=100 \pi \mathrm{~m} \\
& \quad[1 \mathrm{~km}=1000 \mathrm{~m}] \quad \\
&=3,141.6 \mathrm{~m} \\
&=3.1 \mathrm{~km}
\end{aligned}
$$

LESSON 10-6: ARC LENGTH
PRACTICE 9-B
Find the length of each arc.

$$
\begin{aligned}
& \text { Arc length }=C \cdot \\
& =2 \pi r \cdot \frac{\theta}{3602)}
\end{aligned}
$$ $\underset{v e d}{\text { Name }}$



$$
\begin{aligned}
& 2 \cdot \pi \cdot 12 \cdot \frac{135 \div 45}{360: 45} \\
& =\frac{24 \pi}{1} \cdot \frac{3}{8,}=9 \pi \mathrm{mi} .
\end{aligned}
$$



$$
\begin{aligned}
& 2 \cdot \pi \cdot 9 \cdot \frac{240}{360}^{\div 120} \\
& 18 \pi \cdot \frac{2}{3}=12 \pi \mathrm{mi}
\end{aligned}
$$

3) 


4)

5)

9)

10)
$\frac{600^{\circ}}{12 \mathrm{mi}}$


PRACTICE 9-B
5) Find the length of the outfield fence $(\underset{A B}{ })$ on the field below. Write answer in terms of $\pi$ and as a decimal to the nearest tenth.
$90^{\circ}$ angle; $r=280 \mathrm{ff}$.
$2 \cdot \pi \cdot 280 \cdot \frac{90}{360}$
$560 \pi \cdot \frac{1}{4}=140 \pi$

6) The minute hand of a large clock is 18 inches long. How far does the tip of the minute hand move for each given amount of time. Write answers in terms of $\pi$ and as decimals rounded to the nearest hundredth of an inch.
a) 45 minutes
b) 20 minutes
c) 55 minutes
d) 2 hours


$$
\text { (a) } 2 \cdot \pi \cdot 18 \cdot \frac{45}{60} \frac{3}{4}
$$

(c)
$2 \cdot \pi \cdot 18 \cdot \frac{80}{80} \frac{1}{3}$

$$
\begin{equation*}
2 \cdot \pi \cdot 18 \cdot \frac{55}{11} \frac{11}{12} \tag{b}
\end{equation*}
$$

$33 \pi \mathrm{in}$ 103.7 in
(d)
$2 \cdot \pi \cdot 18 \cdot 2$
$72 \pi$ in 226.2 in .




