



Priority Standards

Math 5-12

Judy Pennington, Director of Secondary Instruction

Chris Barnes, Director of Elementary Instruction

Freda Martin, 5th grade instructional facilitator

Michael Hines, math instructional coach

Pam Grimes, 5th grade

Melinda Swift, 6th grade

Amy Covey, 7th grade

Luke Humphreys, 8th grade

Terra Hale, Algebra I

Laura Kobs, Geometry

Kara Braeutigam, Algebra II



5th Grade Math

5th Grade Math: Measurement and Data

- ❖ **AR.Math.Content.5.MD.A.1**
 - Convert among different-sized standard measurement units within the metric system
For example: Convert 5 cm to 0.05 m.
 - Convert among different-sized standard measurement units within the customary system
For example: Convert 1 ½ ft to 18 in.
 - Use these conversions in solving multi-step, real world problems
- ❖ **AR.Math.Content.5.MD.B.2**
 - Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$)
 - Use operations on fractions for this grade to solve problems involving information presented in line plots

For example: Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. Given different measurements of length between the longest and shortest pieces of rope in a collection, find the length each piece of rope would measure if each rope's length were redistributed equally or other examples that demonstrate measures of center (mean, median, mode).
- ❖ **AR.Math.Content.5.MD.C.3**

Recognize volume as an attribute of solid figures and understand concepts of volume measurement:

 - A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume
 - A solid figure, which can be packed without gaps or overlaps using n unit cubes, is said to have a volume of n cubic units
- ❖ **AR.Math.Content.5.MD.C.4**

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units

❖ **AR.Math.Content.5.MD.C.5**

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume:

- **Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base (B)**
- **Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication)**
- **Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems**
- **Recognize volume as additive**
- **Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems**

5th Grade Math: Number and Operations in Base Ten

❖ AR.Math.Content.5.NBT.A.1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left

❖ AR.Math.Content.5.NBT.A.2

Understand why multiplying or dividing by a power of 10 shifts the value of the digits of a whole number or decimal:

- Explain patterns in the number of zeros of the product when multiplying a whole number by powers of 10
- Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10
- Use whole-number exponents to denote powers of 10

❖ AR.Math.Content.5.NBT.A.3

Read, write, and compare decimals to thousandths:

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form(s)

Examples could include:

- *Base-ten numerals “standard form” (347.392)*
- *Number name form (three-hundred forty seven and three hundred ninety-two thousandths)*
- *Expanded form(s):*

$$300 + 40 + 7 + .3 + .09 + .002 = 300 + 40 + 7 + \frac{3}{10} + \frac{9}{100} + \frac{2}{1000} =$$

$$3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \left(\frac{1}{10}\right) + 9 \times \left(\frac{1}{100}\right) + 2 \times \left(\frac{1}{1000}\right) =$$

$$3 \times 10^2 + 4 \times 10^1 + 7 \times 10^0 + 3 \times \left(\frac{1}{10^1}\right) + 9 \times \left(\frac{1}{10^2}\right) + 2 \times \left(\frac{1}{10^3}\right)$$

- *Compare two decimals to thousandths based on the value of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons*

❖ AR.Math.Content.5.NBT.A.4

Apply place value understanding to round decimals to any place

❖ AR.Math.Content.5.NBT.B.5

Fluently (efficiently, accurately and with some degree of flexibility) multiply multi-digit whole numbers using a standard algorithm

Note: A “standard algorithm” can be viewed as, but should not be limited to, the traditional recording system. A “standard algorithm” denotes any valid base-ten strategy.

❖ **AR.Math.Content.5.NBT.B.6**

• Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on:

- Place value
- The properties of operations
- Divisibility rules; and
- The relationship between multiplication and division

• Illustrate and explain calculations by using equations, rectangular arrays, and area models

❖ **AR.Math.Content.5.NBT.B.7**

Perform operations with multi-digit whole #’s and with decimals to hundredths.

- Add, subtract, multiply and divide decimals to hundredths

5th Grade Math: Number and Operations-Fractions

❖ AR.Math.Content.5.NF.A.1

Efficiently, accurately, and with some degree of flexibility, add and subtract fractions with unlike denominators (including mixed numbers) using equivalent fractions and common denominators

For example: Understand that $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$ (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$)

Note: The focus of this standard is applying equivalent fractions, not necessarily finding least common denominators or putting results in simplest form.

❖ AR.Math.Content.5.NF.A.2

•Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators

For example: Use visual fraction models or equations to represent the problem.

•Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers

For example: Recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.

❖ AR.Math.Content.5.NF.B.3

•Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$), where a and b are natural numbers

For example: Interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$.

•Solve word problems involving division of natural numbers leading to answers in the form of fractions or mixed numbers

For example: Use visual fraction models or equations to represent the problem. If 9 people want to share a 50- pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

❖ AR.Math.Content.5.NF.B.4

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction:

•Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$

For example: Use a visual fraction model to show $(\frac{2}{3}) \times 12$ means to take 12 and divide it into thirds ($\frac{1}{3}$ of 12 is 4) and take two of the parts (2×4 is 8), so $(\frac{2}{3}) \times 12 = 8$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)

•Find the area of a rectangle with fractional (less than and/or greater than 1) side lengths, by tiling it with unit squares of the appropriate unit fraction side lengths, by multiplying the fractional side lengths, and then show that both procedures yield the same area

❖ **AR.Math.Content.5.NF.B.5**

Interpret multiplication as scaling (resizing), by:

- Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication
- Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number
- Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number
- Relate the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1

❖ **AR.Math.Content.5.NF.B.6**

Solve real world problems involving multiplication of fractions and mixed numbers

❖ **AR.Math.Content.5.NF.B.7**

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions:

- Interpret division of a unit fraction by a natural number, and compute such quotients
- Interpret division of a whole number by a unit fraction, and compute such quotients
- Solve real world problems involving division of unit fractions by natural numbers and division of whole numbers by unit fractions

5th Grade Math: Operations and Algebraic Thinking

❖ AR.Math.Content.5.OA.A.1

Use grouping symbols including parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols

Note: Expressions should not contain nested grouping symbols such as $[4+2(10+3)]$ and they should be no more complex than the expressions one finds in an application of the associative or distributive property (e.g., $(8+7) \times 2$ or $\{6 \times 30\} + \{6 \times 7\}$).

❖ AR.Math.Content.5.OA.A.2

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For Example: Express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

❖ AR.Math.Content.5.OA.B.3

- Generate two numerical patterns, each using a given rule
- Identify apparent relationships between corresponding terms by completing a function table or input/output table
- Using the terms created, form and graph ordered pairs in the first quadrant of the coordinate plane

Note: Terms of the numerical patterns will be limited to whole number coordinates.

5th Grade Math: Geometry

❖ AR.Math.Content.5.G.A.1

- Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates
- Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate)

Note: Graphing will be limited to the first quadrant and the non-negative x- and y-axes only

❖ AR.Math.Content.5.G.A.2

- Represent real world and mathematical problems by graphing points in the first quadrant and on the non-negative x- and y-axes of the coordinate plane
- Interpret coordinate values of points in the context of the situation

❖ AR.Math.Content.5.G.B.3

Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category

For example: All rectangles have four right angles and squares are rectangles, so all squares have four right angles. All isosceles triangles have at least two sides congruent and equilateral triangles are isosceles. Therefore, equilateral triangles have at least two congruent sides.

❖ AR.Math.Content.5.G.B.4

Classify two-dimensional figures in a hierarchy based on properties

Note: Trapezoids will be defined to be a quadrilateral with at least one pair of opposite sides parallel, therefore all parallelograms are trapezoids.

6th Grade Math

6th Grade Math: Expressions and Equations

- ❖ **AR.Math.Content.6.EE.A.1**
Write and evaluate numerical expressions involving whole-number exponents
- ❖ **AR.Math.Content.6.EE.A.2**
Write, read, and evaluate expressions in which letters (variables) stand for numbers:
 - Write expressions that record operations with numbers and with letters standing for numbers For example, express the calculation 'subtract y from 5' or ' y less than 5' as $5 - y$.
 - Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.
 - Evaluate expressions at specific values of their variables
 - Include expressions that arise from formulas used in real-world problems
 - Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations)
For example, use the formulas involved in measurement such as $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.
- ❖ **AR.Math.Content.6.EE.A.3**
Apply the properties of operations to generate equivalent expressions
For example: Apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$. Note: Includes but not limited to the distributive property
- ❖ **AR.Math.Content.6.EE.A.4**
Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them)
For example: The expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.
- ❖ **AR.Math.Content.6.EE.B.5**
Understand solving an equation or inequality as a process of answering a question:
 - Using substitution, which values from a specified set, if any, make the equation or inequality true?

- ❖ **AR.Math.Content.6.EE.B.6**
 - Use variables to represent numbers and write expressions when solving a real-world or mathematical problem
 - Understand that a variable can represent an unknown number or any number in a specified set
- ❖ **AR.Math.Content.6.EE.B.7**

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers
- ❖ **AR.Math.Content.6.EE.B.8**

For real world or mathematical problems:

 - Write an inequality of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ to represent a constraint or condition
 - Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions
 - Represent solutions of such inequalities on number line diagrams
- ❖ **AR.Math.Content.6.EE.C.9**

Use variables to represent two quantities in a real-world problem that change in relationship to one another:

 - Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable
 - Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation

For example: In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Note: The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable.

6th Grade Math: Geometry

❖ AR.Math.Content.6.G.A.1

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes
- Apply these techniques in the context of solving real-world and mathematical problems

Note: Trapezoids will be defined to be a quadrilateral with at least one pair of opposite sides parallel, therefore all parallelograms are trapezoids.

❖ AR.Math.Content.6.G.A.2

- Find the volume of a right rectangular prism including whole number and fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism
- Apply the formulas $V = l w h$ and $V = B h$ to find volumes of right rectangular prisms including fractional edge lengths in the context of solving real-world and mathematical problems

❖ AR.Math.Content.6.G.A.3

Apply the following techniques in the context of solving real-world and mathematical problems:

- Draw polygons in the coordinate plane given coordinates for the vertices
- Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate

❖ AR.Math.Content.6.G.A.4

Apply the following techniques in the context of solving real-world and mathematical problems:

- Represent three-dimensional figures using nets made up of rectangles and triangles
- Use the nets to find the surface area of these figures

6th Grade Math: Statistics and Probability

❖ AR.Math.Content.6.SP.A.1

Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers

For example, 'How old am I?' is not a statistical question, but 'How old are the students in my school?' is a statistical question because one anticipates variability in students' ages.

Note: Statistics is also the name for the science of collecting, analyzing and interpreting data.

Data are the numbers produced in response to a statistical question and are frequently collected from surveys or other sources (i.e. documents).

❖ AR.Math.Content.6.SP.A.2

Determine center, spread, and overall shape from a set of data

❖ AR.Math.Content.6.SP.A.3

Recognize that a measure of center for a numerical data set summarizes all of its values with a single number (mean, median, mode), while a measure of variation (interquartile range, mean absolute deviation) describes how its values vary with a single number

Example: If the mean height of the students in the class is 48" are there any students in the class taller than 48"?

❖ AR.Math.Content.6.SP.B.4

Display numerical data in plots on a number line, including dot plots, histograms, and box plots

❖ AR.Math.Content.6.SP.B.5

Summarize numerical data sets in relation to their context, such as by:

- Reporting the number of observations

- Describing the nature of the attribute under investigation, including how it was measured and its units of measurement

- Calculate quantitative measures of center (including but not limited to median and mean) and variability (including but not limited to interquartile range and mean absolute deviation)

- Use the calculations to describe any overall pattern and any striking deviations (outliers) from the overall pattern with reference to the context in which the data were gathered

Note: Instructional focus should be on summarizing and describing data distributions.

- Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. For example, demonstrate in the case where there are outliers in the data median would be a better measure of center than the mean.

6th Grade Math: The Number System

- ❖ **AR.Math.Content.6.NS.A.1**
 - Interpret and compute quotients of fractions
 - Solve word problems involving division of fractions by fractions (e.g., by using various strategies, including but not limited to, visual fraction models and equations to represent the problem) *For example: Create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. How many $3/4$ -cup servings are in $2/3$ of a cup of yogurt? Note: In general, $(a/b) \div (c/d) = ad/bc$.*
- ❖ **AR.Math.Content.6.NS.B.2**

Use computational fluency to divide multi-digit numbers using a standard algorithm

Note: A standard algorithm can be viewed as, but should not be limited to, the traditional recording system. A standard algorithm denotes any valid base-ten strategy.
- ❖ **AR.Math.Content.6.NS.B.3**

Use computational fluency to add, subtract, multiply, and divide multi-digit decimals and fractions using a standard algorithm for each operation. *Note: A standard algorithm can be viewed as, but should not be limited to, the traditional recording system. A standard algorithm denotes any valid base-ten strategy.*
- ❖ **AR.Math.Content.6.NS.B.4**
 - Find the greatest common factor of two whole numbers less than or equal to 100 using prime factorization as well as other methods
 - Find the least common multiple of two whole numbers less than or equal to 12 using prime factorization as well as other methods
 - Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express $36 + 8$ as $4(9 + 2)$.*
- ❖ **AR.Math.Content.6.NS.C.5**

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, explaining the meaning of 0 (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge)
- ❖ **AR.Math.Content.6.NS.C.6**

Understand a rational number as a point on the number line

Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates:

 - Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line
 - Recognize that the opposite of the opposite of a number is the number itself (e.g., $-(-3) = 3$, and that 0 is its own opposite)
 - Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane

- Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram
- Find and position pairs of integers and other rational numbers on a coordinate plane

❖ AR.Math.Content.6.NS.C.7

Understand ordering and absolute value of rational numbers:

- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*
- Write, interpret, and explain statements of order for rational numbers in real-world contexts *For example, write $-30\text{ }^{\circ}\text{C} > -70\text{ }^{\circ}\text{C}$ to express the fact that $-30\text{ }^{\circ}\text{C}$ is warmer than $-70\text{ }^{\circ}\text{C}$.*
- Understand the absolute value of a rational number as its distance from 0 on the number line
- Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation
For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.
- Distinguish comparisons of absolute value from statements about order
For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.

❖ AR.Math.Content.6.NS.C.8

- ❖ •Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane
- ❖ •Use coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate

6th Grade Math: Ratios and Proportional

❖ AR.Math.Content.6.RP.A.1

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities

For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

❖ AR.Math.Content.6.RP.A.2

Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship

For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."

Note: Expectations for unit rates in this grade are limited to non-complex fractions.

❖ AR.Math.Content.6.RP.A.3

Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations):

- Use and create tables to compare equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane

- Solve unit rate problems including those involving unit pricing and constant speed

For example: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity)

- Solve problems involving finding the whole, given a part and the percent

- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities

Example: How many centimeters are in 7 feet, given that $1 \text{ inch} \approx 2.54 \text{ cm}$?

Note: Conversion factors will be given. Conversions can occur both between and across the metric and English system. Estimates are not expected.

7th Grade Math

7th Grade Math: Ratios and Proportional

❖ AR.Math.Content.7.RP.A.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units

For example: If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.

❖ AR.Math.Content.7.RP.A.2

Recognize and represent proportional relationships between quantities:

- Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin)
- Identify unit rate (also known as the constant of proportionality) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships
- Represent proportional relationships by equations (e.g., if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$)
- Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate

Note: Unit rate connects to slope concept in 8th grade.

❖ AR.Math.Content.7.RP.A.3

Use proportional relationships to solve multi-step ratio and percent problems

Note: Examples include but are not limited to simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease..

7th Grade Math: The Number System

❖ AR.Math.Content.7.NS.A.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers

Represent addition and subtraction on a horizontal or vertical number line diagram:

- Describe situations in which opposite quantities combine to make 0 and show that a number and its opposite have a sum of 0 (additive inverses) (e.g., A hydrogen atom has 0 charge because its two constituents are oppositely charged.)
- Understand $p + q$ as a number where p is the starting point and q represents a distance from p in the positive or negative direction depending on whether q is positive or negative
- Interpret sums of rational numbers by describing real-world contexts (e.g., $3 + 2$ means beginning at 3, move 2 units to the right and end at the sum of 5; $3 + (-2)$ means beginning at 3, move 2 units to the left and end at the sum of 1; $70 + (-30) = 40$ could mean after earning \$70, \$30 was spent on a new video game, leaving a balance of \$40)
- Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$
- Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts (e.g., the distance between -5 and 6 is 11. -5 and 6 are 11 units apart on the number line)

❖ AR.Math.Content.7.NS.A.2

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers:

- Understand that multiplication is extended from fractions to all rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, and the rules for multiplying signed numbers
- Interpret products of rational numbers by describing real-world contexts
- Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number (e.g., if p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$)
- Interpret quotients of rational numbers by describing real-world contexts
- Fluently multiply and divide rational numbers by applying properties of operations as strategies
- Convert a fraction to a decimal using long division
- Know that the decimal form of a fraction terminates in 0s or eventually repeats

❖ AR.Math.Content.7.NS.A.3

Solve real-world and mathematical problems involving the four operations with rational numbers, including but not limited to complex fractions

7th Grade Math: Expressions and Equations

❖ AR.Math.Content.7.EE.A.1

Apply properties of operations as strategies to add, subtract, expand, and factor linear expressions with rational coefficients

❖ AR.Math.Content.7.EE.A.2

Understand how the quantities in a problem are related by rewriting an expression in different forms

For example: $a + 0.05a = 1.05a$ means that 'increase by 5%' is the same as 'multiply by 1.05' or the perimeter of a square with side length s can be written as $s+s+s+s$ or $4s$.

❖ AR.Math.Content.7.EE.B.3

Solve multi-step, real-life, and mathematical problems posed with positive and negative rational numbers in any form using tools strategically:

- Apply properties of operations to calculate with numbers in any form (e.g., $-(1/4)(n-4)$)
- Convert between forms as appropriate (e.g., if a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50, for a new salary of \$27.50)

- Assess the reasonableness of answers using mental computation and estimation strategies (e.g., if you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation)

❖ AR.Math.Content.7.EE.B.4

- Use variables to represent quantities in a real-world or mathematical problem

- Construct simple equations and inequalities to solve problems by reasoning about the quantities

- Solve word problems leading to equations of these forms $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently

- Write an algebraic solution identifying the sequence of the operations used to mirror the arithmetic solution (e.g., The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? Subtract $2*6$ from 54 and divide by 2; $(2*6) + 2w = 54$)

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers

- Graph the solution set of the inequality and interpret it in the context of the problem (e.g., As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.)

7th Grade Math: Geometry

❖ AR.Math.Content.7.G.A.1

- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale

Note: This concept ties into ratio and proportion.

❖ AR.Math.Content.7.G.A.2

- Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions:
 - Given three measures of angles or sides of a triangle, notice when the conditions determine a unique triangle, more than one triangle, or no triangle
 - Differentiate between regular and irregular polygons

❖ AR.Math.Content.7.G.A.3

Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids

❖ AR.Math.Content.7.G.B.4

- Know the formulas for the area and circumference of a circle and use them to solve problems.
- Give an informal derivation of the relationship between the circumference and area of a circle

❖ AR.Math.Content.7.G.B.5

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure

❖ AR.Math.Content.7.G.B.6

Solve real-world and mathematical problems involving area of two-dimensional objects and volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms

7th Grade Math: Statistics and Probability

❖ AR.Math.Content.7.SP.A.1

Understand that:

- Statistics can be used to gain information about a population by examining a sample of the population
- Generalizations about a population from a sample are valid only if the sample is representative of that population
- Random sampling tends to produce representative samples and support valid inferences

❖ AR.Math.Content.7.SP.A.2

- Use data from a random sample to draw inferences about a population with a specific characteristic
- Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions

For example: Estimate the mean word length in a book by randomly sampling words from the book, or predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

❖ AR.Math.Content.7.SP.B.3

Draw conclusions about the degree of visual overlap of two numerical data distributions with similar variability such as interquartile range or mean absolute deviation, expressing the difference between the centers as a multiple of a measure of variability such as mean, median, or mode

For example: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

❖ AR.Math.Content.7.SP.B.4

Draw informal comparative inferences about two populations using measures of center and measures of variability for numerical data from random samples

For example: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

❖ AR.Math.Content.7.SP.C.5

- Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
- A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event

❖ **AR.Math.Content.7.SP.C.6**

- Collect data to approximate the probability of a chance event
- Observe its long-run relative frequency
- Predict the approximate relative frequency given the probability

For example: When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

Note: Emphasis should be given to the relationship between experimental and theoretical probability

❖ **AR.Math.Content.7.SP.C.7**

Develop a probability model and use it to find probabilities of events

Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy:

- **Develop a uniform probability model, assigning equal probability to all outcomes, and use the model to determine probabilities of events (e.g., If a student is selected at random from a class of 6 girls and 4 boys, the probability that Jane will be selected is .10 and the probability that a girl will be selected is .60.)**
- **Develop a probability model, which may not be uniform, by observing frequencies in data generated from a chance process (e.g., Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?)**

❖ **AR.Math.Content.7.SP.C.8**

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation:

- **Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs**
- **Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams**
- **Identify the outcomes in the sample space which compose the event**

Generate frequencies for compound events using a simulation (e.g., What is the frequency of pulling a red card from a deck of cards and rolling a 5 on a die?)

8th Grade Math

The Number System

❖ AR.Math.Content.8.NS.A.1

Know that numbers that are not rational are called irrational:

- Understand that every number has a decimal expansion

For example: $2=2.00\dots$

- *Write a fraction a/b as a repeating decimal*

- *Write a repeating decimal as a fraction*

❖ AR.Math.Content.8.NS.A.2

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2)

For example: By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations..

Expressions and Equations

❖ AR.Math.Content.8.EE.A.1

Know and apply the properties of integer exponents to generate equivalent numerical expressions using product, quotient, power to a power, or expanded form

❖ AR.Math.Content.8.EE.A.2

Use square root and cube root symbols to represent solutions to equations:

- Use square root symbols to represent solutions to equations of the form $x^2 = p$, where p is a positive rational number

Evaluate square roots of small perfect squares.

- Use cube root symbols to represent solutions to equations of the form $x^3 = p$, where p is a rational number.

Evaluate square roots and cube roots of small perfect cubes

❖ AR.Math.Content.8.EE.A.3

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other

For example: Estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9 , and determine that the world population is more than 20 times larger.

- ❖ **AR.Math.Content.8.EE.A.4**
 - Perform operations with numbers expressed in scientific notation, including problems where both standard form and scientific notation are used
 - Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading)
 - Interpret scientific notation that has been generated by technology
- ❖ **AR.Math.Content.8.EE.B.5**
 - Graph proportional relationships, interpreting the unit rate as the slope of the graph.
 - Compare two different proportional relationships represented in different ways (graphs, tables, equations)

For example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- ❖ **AR.Math.Content.8.EE.B.6**
 - Using a non-vertical or non-horizontal line, show why the slope m is the same between any two distinct points by creating similar triangles
 - Write the equation $y=mx + b$ for a line through the origin
 - Be able to write the equation $y = mx + b$ for a line intercepting the vertical axis at b
- ❖ **AR.Math.Content.8.EE.C.7**

Solve linear equations in one variable:

 - Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions

Note: Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers)

 - Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms

Note: Students should solve equations with variables on both sides.
- ❖ **AR.Math.Content.8.EE.C.8**

Analyze and solve pairs of simultaneous linear equations:

 - Find solutions to a system of two linear equations in two variables so they correspond to points of intersection of their graphs
 - Solve systems of equations in two variables algebraically using simple substitution and by inspection (e.g., $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6)
 - Solve real-world mathematical problems by utilizing and creating two linear equations in two variables.

For example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions

- ❖ **AR.Math.Content.8.F.A.1**
 - Understand that a function is a rule that assigns to each input exactly one output
 - The graph of a function is the set of ordered pairs consisting of an input and the corresponding output

Note: An informal discussion of function notation is needed; however, student assessment is not required.
- ❖ **AR.Math.Content.8.F.A.2**

Compare properties (e.g., y-intercept/initial value, slope/rate of change) of two functions each represented in a different way (e.g., algebraically, graphically, numerically in tables, or by verbal descriptions)

For example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
- ❖ **AR.Math.Content.8.F.A.3**

Identify the unique characteristics of functions (e.g., linear, quadratic, and exponential) by comparing their graphs, equations, and input/output tables
- ❖ **AR.Math.Content.8.F.B.4**

Construct a function to model a linear relationship between two quantities:

 - Determine the rate of change and initial value of the function from:
 - a verbal description of a relationship
 - two (x, y) values
 - a table
 - a graph
 - Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values
- ❖ **AR.Math.Content.8.F.B.5**
 - Describe the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear)
 - Sketch a graph that exhibits the features of a function that has been described verbally

Geometry

- ❖ **AR.Math.Content.8.G.A.1**
Verify experimentally the properties of rotations, reflections, and translations:
 - Lines are taken to lines, and line segments to line segments of the same length
 - Angles are taken to angles of the same measure
 - Parallel lines are taken to parallel lines
- ❖ **AR.Math.Content.8.G.A.2**
 - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations
 - Given two congruent figures, describe a sequence that exhibits the congruence between them
- ❖ **AR.Math.Content.8.G.A.3**
Given a two-dimensional figure on a coordinate plane, identify and describe the effect (rule or new coordinates) of a transformation (dilation, translation, rotation, and reflection):
 - Image to pre-image
 - Pre-image to image
- ❖ **AR.Math.Content.8.G.A.4**
 - Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations
 - Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them
- ❖ **AR.Math.Content.8.G.A.5**
Use informal arguments to establish facts about:
 - The angle sum and exterior angle of triangles
For example: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line.
 - The angles created when parallel lines are cut by a transversal
For example: Give an argument in terms of translations about the angle relationships.
 - The angle-angle criterion for similarity of triangles
- ❖ **AR.Math.Content.8.G.B.6**
Model or explain an informal proof of the Pythagorean Theorem and its converse
- ❖ **AR.Math.Content.8.G.B.7**
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions
- ❖ **AR.Math.Content.8.G.B.8**
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system
- ❖ **AR.Math.Content.8.G.C.9**
Develop and know the formulas for the volumes and surface areas of cones, cylinders, and spheres and use them to solve real-world and mathematical problems

Statistics and Probability

❖ AR.Math.Content.8.SP.A.1

- Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities
- Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association

❖ AR.Math.Content.8.SP.A.2

- Know that straight lines are widely used to model relationships between two quantitative variables
- For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line
For example: Identify weak, strong, or no correlation.

❖ AR.Math.Content.8.SP.A.3

- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercepts
- For example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

❖ AR.Math.Content.8.SP.A.4

- Understand that patterns of association can also be seen in *bivariate* categorical data by displaying frequencies and relative frequencies in a two-way table
- Construct and interpret a two-way table on two categorical variables collected from the same subjects
- Use relative frequencies calculated for rows or columns to describe possible association between the two variables

Example: Two-Way Frequency Table

<http://mathbitsnotebook.com/Algebra1/StatisticsReg/ST2TwoWayTable.html>

	 Sport Utility Vehicle (SUV)	 Sports Car	Totals
male	21	39	60
female	135	45	180
Totals	156	84	240

Row Totals: 60, 180, 240
Column Totals: 156, 84, 240

MathBits.com

Example: Two-Way Relative Frequency Table

	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	$\frac{21}{240} = 0.09$	$\frac{39}{240} = 0.16$	$\frac{60}{240} = 0.25$
female	$\frac{135}{240} = 0.56$	$\frac{45}{240} = 0.19$	$\frac{180}{240} = 0.75$
Totals	$\frac{156}{240} = 0.65$	$\frac{84}{240} = 0.35$	$\frac{240}{240} = 1.00$

Whole Table
Relative Frequencies -
Divide all cells by 240.

MathBits.com

For example: Students might be asked to interpret from the tables above, if they saw an SUV in the parking lot, would it be more likely to belong to a male or female?

Note: Suggested connections for instruction: Standard 8.NS.1. On the Two-Way Relative Frequency Table, it is not required to include the fractional representation for each value, this is simply provided as an example.

Algebra I

The Real Number System

Goal Statement: Use properties of rational and irrational numbers.

- ❖ **HSN.RN.B.3**
Explain why:
 - The sum/difference or product/quotient (where defined) of two rational numbers is rational
 - The sum/difference of a rational number and an irrational number is irrational
 - The product/quotient of a nonzero rational number and an irrational number is irrational
 - The product/quotient of two nonzero rational numbers is a nonzero rational
- ❖ **HSN.RN.B.4**
 - Simplify radical expressions
 - Perform operations (add, subtract, multiply, and divide) with radical expressions
 - Rationalize denominators and/or numerators

Quantities

Goal Statement: Reason quantitatively and use units to solve problems.

- ❖ **HSN.Q.A.1**
 - Use units as a way to understand problems and to guide the solution of multi-step problems
 - Choose and interpret units consistently in formulas
 - Choose and interpret the scale and the origin in graphs and data displays
- ❖ **HSN.Q.A.2**
Define appropriate quantities for the purpose of descriptive modeling (i.e., use units appropriate to the problem being solved)
Limitation: This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.
- ❖ **HSN.Q.A.3-** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities

Seeing Structure in Expressions

Goal Statement: Interpret the structure of expressions, and write expressions in equivalent forms to solve problems

❖ HSA.SSE.A.1

Interpret expressions that represent a quantity in terms of its context*

- Interpret parts of an expression using appropriate vocabulary, such as terms, factors, and coefficients
- Interpret complicated expressions by viewing one or more of their parts as a single entity
For example: Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P .

❖ HSA.SSE.A.2

Use the structure of an expression to identify ways to rewrite it

For example: See that $(x + 3)(x + 3)$ is the same as $(x + 3)^2$ or $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Limitation:

- Tasks are limited to numerical expressions and polynomial expressions in one variable.*
- Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 + 47)(53 - 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$.*

❖ HSA.SSE.B.3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression*

- Factor a quadratic expression to reveal the zeros of the function it defines
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines *Note: Students should be able to identify and use various forms of a quadratic expression to solve problems.*

Standard Form: $ax^2 + bx + c$

Factored Form: $a(x - r_1)(x - r_2)$

Vertex Form: $a(x - h)^2 + k$

Limitation: i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.

ii) Tasks are limited to exponential expressions with integer exponents.

Arithmetic with Polynomials and Rational Expressions

Goal Statement: Perform arithmetic operations on polynomials, and rewrite rational expressions

- ❖ **HSA.APR.A.1**
 - Add, subtract, and multiply polynomials
 - Understand that polynomials, like the integers, are closed under addition, subtraction, and multiplication

Note: If p and q are polynomials $p + q$, $p - q$, and pq are also polynomials.
- ❖ **HSA.APR.B.3**
 - Identify zeros of polynomials (linear, quadratic only) when suitable factorizations are available
 - Use the zeros to construct a rough graph of the function defined by the polynomial
- ❖ **HSA.APR.C.4**

Prove polynomial identities and use them to describe numerical relationships

Note: Examples of Polynomial Identities may include but are not limited to the following:

 - $(a + b)^2 = a^2 + 2ab + b^2$ (Algebra 1)
 - $a^2 - b^2 = (a - b)(a + b)$ (Algebra 1)
- ❖ **HSA.APR.D.7**
 - Add, subtract, multiply, and divide by nonzero rational expressions
 - Understand that rational expressions, like the integers, are closed under addition, subtraction, and multiplication

Creating Equations

Goal Statement: Create equations that describe numbers or relationships

- ❖ **HSA.CED.A.1- Create equations and inequalities in one variable and use them to solve problems**

Note: Including but not limited to equations arising from:

 - Linear functions
 - Quadratic functions
 - Exponential functions
 - Absolute value functions
- ❖ **HSA.CED.A.2**
 - Create equations in two or more variables to represent relationships between quantities
 - Graph equations, in two variables, on a coordinate plane
- ❖ **HSA.CED.A.3**
 - Represent and interpret constraints by equations or inequalities, and by systems of equations and/or inequalities
 - Interpret solutions as viable or nonviable options in a modeling and/or real-world context
- ❖ **HSA.CED.A.4**

Rearrange literal equations using the properties of equality

Reasoning with Equations and Inequalities

Goal Statement: Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable. Solve systems of equations and inequalities graphically. Solve systems of equations.

❖ HSA.REI.A.1

Assuming that equations have a solution, construct a solution and justify the reasoning used

Note: Students are not required to use only one procedure to solve problems nor are they required to show each step of the process. Students should be able to justify their solution in their own words. (limited to quadratics)

❖ HSA.REI.A.2

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

For example: The area of a square equals 49 square inches. The length of the side is 7 inches. Although -7 is a solution to the equation, $x^2 = 49$, -7 is an extraneous solution.

❖ HSA.REI.B.3

Solve linear equations, inequalities and absolute value equations in one variable, including equations with coefficients represented by letters

❖ HSA.REI.B.4

Solve quadratic equations in one variable

•Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions

Note: This would be a good opportunity to demonstrate/explore how the quadratic formula is derived. This standard also connects to the transformations of functions and identifying key features of a graph (F-BF3). Introduce this with a leading coefficient of 1 in Algebra I. Finish mastery in Algebra II.

•Solve quadratic equations (as appropriate to the initial form of the equation) by:

- Inspection of a graph
- Taking square roots
- Completing the square
- Using the quadratic formula
- Factoring

Recognize complex solutions and write them as $a + bi$ for real numbers a and b (Algebra 2 only)

Limitation:

Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions.

Note: Solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials (cluster A-APR.B). Cluster A-APR.B is formally assessed in Algebra II.

- ❖ **HSA.REI.C.5**
 - Solve systems of equations in two variables using substitution and elimination
 - Understand that the solution to a system of equations will be the same when using substitution and elimination
- ❖ **HSA.REI.C.6**

Solve systems of equations algebraically and graphically

Limitation:

Tasks have a real-world context.

Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle).
- ❖ **HSA.REI.C.7**

Solve systems of equations consisting of linear equations and nonlinear equations in two variables algebraically and graphically

For example: Find the points of intersection between $y = -3x$ and $y = x^2 + 2$.
- ❖ **HSA.REI.D.10**

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane
- ❖ **HSA.REI.D.11**

Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;

Find the solutions approximately by:

 - Using technology to graph the functions
 - Making tables of values
 - Finding successive approximations

Include cases (but not limited to) where $f(x)$ and/or $g(x)$ are:

 - Linear
 - Polynomial
 - Absolute value
 - Exponential (Introduction in Algebra 1, Mastery in Algebra 2)

Teacher notes: Modeling should be applied throughout this standard.
- ❖ **HSA.REI.D.12**

Solve linear inequalities and systems of linear inequalities in two variables by graphing

Interpreting Functions

Goal Statement: Understand the concept of a function and use function notation.

Interpret functions that arise in applications in terms of the concept.

❖ **HSF.IF.A.1**

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range
- Understand that if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x
- Understand that the graph of f is the graph of the equation $y = f(x)$

❖ **HSF.IF.A.2**

In terms of a real-world context:

- Use function notation
- Evaluate functions for inputs in their domains
- Interpret statements that use function notation

❖ **HSF.IF.A.3**

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers

For example: The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + (n - 1)$ for $n \geq 1$.

❖ **HSF.IF.B.4**

For a function that models a relationship between two quantities:

- Interpret key features of graphs and tables in terms of the quantities, and
- Sketch graphs showing key features given a verbal description of the relationship

*Note: Key features may include but not limited to: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

Limitation:

Tasks have a real-world context.

Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

Compare note (ii) with standard F-IF.7.

The function types listed here are the same as those listed in the Algebra I column for standards F-IF.6 and F-IF.9.

❖ HSF.IF.B.5

- Relate the domain of a function to its graph
- Relate the domain of a function to the quantitative relationship it describes

*For example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

❖ HSF.IF.B.6

- Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval*
- Estimate the rate of change from a graph*

Limitation:

Tasks have a real-world context.

Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.9.

❖ HSF.IF.C.7

Graph functions expressed algebraically and show key features of the graph, with and without technology

- Graph linear and quadratic functions and, when applicable, show intercepts, maxima, and minima
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions
- Graph exponential functions, showing intercepts and end behavior

❖ HSF.IF.C.8

Write expressions for functions in different but equivalent forms to reveal key features of the function

- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values (vertex), and symmetry of the graph, and interpret these in terms of a context.

Note: Connection to A.SSE.B.3

❖ HSF.IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)

Limitation:

Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.6.

Building Functions

Goal Statement: Build new functions from existing functions

❖ HSF.BF.A.1

Write a function that describes a relationship between two quantities*

- From a context, determine an explicit expression, a recursive process, or steps for calculation

Limitation:

Tasks have a real-world context.

Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

Goal Statement: Build new functions from existing functions

❖ HSF.BF.B.3

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (k , a constant both positive and negative)

- Find the value of k given the graphs of the transformed functions

- Experiment with multiple transformations and illustrate an explanation of the effects on the graph with or without technology

Note: Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Limitation:

Identifying the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions.

Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

Tasks do not involve recognizing even and odd functions.

The function types listed in note (ii) are the same as those listed in the Algebra I column for standards F-IF.4, F-IF.6, and F-IF.9.

Geometry

Geometry: Congruence

Goal Statement: Investigate transformations in plane.

❖ HSG.CO.A.1

Based on the undefined notions of point, line, plane, distance along a line, and distance around a circular arc, define:

- Angle
- Line segment
- Circle
- Perpendicular lines
- Parallel lines

❖ HSG.CO.A.2

- Represent transformations in the plane (e.g., using transparencies, tracing paper, geometry software)
- Describe transformations as functions that take points in the plane as inputs and give other points as outputs
- Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation)

❖ HSG.CO.A.3

Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself

❖ HSG.CO.A.4

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments

❖ HSG.CO.A.5

- Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure (e.g., using graph paper, tracing paper, miras, geometry software)
- Specify a sequence of transformations that will carry a given figure onto another

Goal Statement: Understand congruence in terms of rigid motions.

❖ HSG.CO.B.6

- Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure
- Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent

❖ HSG.CO.B.7

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent

❖ HSG.CO.B.8

Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions

Investigate congruence in terms of rigid motion to develop the criteria for triangle congruence (ASA, SAS, AAS, SSS, and HL)

Note: The emphasis in this standard should be placed on investigation.

Goal Statement: Apply and prove geometric theorems.

❖ HSG.CO.C.9

Apply and prove theorems about lines and angles

Note: Theorems include but are not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Note: Proofs are not an isolated topic and therefore should be integrated throughout the course.

❖ HSG.CO.C.10

Apply and prove theorems about triangles

Note: Theorems include but are not limited to: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Note: Proofs are not an isolated topic and therefore should be integrated throughout the course.

❖ **HSG.CO.C.11**

Apply and prove theorems about quadrilaterals

Note: Theorems include but are not limited to relationships among the sides, angles, and diagonals of quadrilaterals and the following theorems concerning parallelograms: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Note: Proofs are not an isolated topic and therefore should be integrated throughout the course.

Goal Statement: Make geometric constructions

❖ **HSG.CO.D.12**

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software)

Note: Constructions may include but are not limited to: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

❖ *Note: Constructions are not an isolated topic and therefore should be integrated throughout the course.*

❖ **HSG.CO.D.13**

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle

Note: Constructions are not an isolated topic and therefore should be integrated throughout the course.

Goal Statement: Logic and Reasoning

❖ **HSG.CO.E.14**

Apply inductive reasoning and deductive reasoning for making predictions based on real world situations using:

- Conditional Statements (inverse, converse, and contrapositive)
- Venn Diagrams

Note: This is not intended to be an isolated topic but instead to support concepts throughout the course.

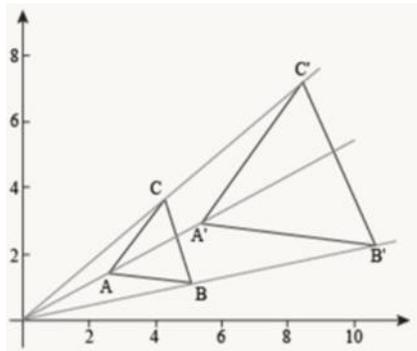
Geometry: Similarity, Right Triangles, and Trigonometry

Goal Statement: Understand similarity in terms of similarity transformations

❖ HSG.SRT.A.1

Verify experimentally the properties of dilations given by a center and a scale factor

- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor



❖ HSG.SRT.A.2

Given two figures:

- Use the definition of similarity in terms of similarity transformations to determine if they are similar
- Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides

❖ HSG.SRT.A.3

Use the properties of similarity transformations to establish the AA \sim , SAS \sim , SSS \sim criteria for two triangles to be similar

Goal Statement: Apply and prove theorems involving similarity

❖ HSG.SRT.B.4

Use triangle similarity to apply and prove theorems about triangles

❖ Note: Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

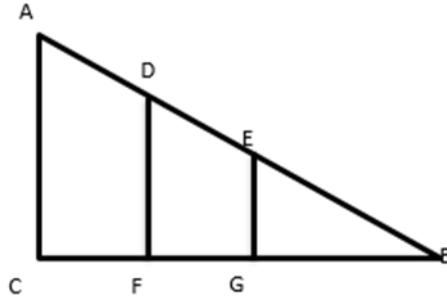
❖ HSG.SRT.B.5

- Use congruence (SSS, SAS, ASA, AAS, and HL) and similarity (AA \sim , SSS \sim , SAS \sim) criteria for triangles to solve problems
- Use congruence and similarity criteria to prove relationships in geometric figures

Goal Statement: Define trigonometric ratios and solve problems involving right triangles

❖ HSG.SRT.C.6

❖ Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles



❖ *For example: Trigonometric ratios are related to the acute angles of a triangle, not the right angle. The values of the trigonometric ratio depend only on the angle. Consider the following three similar right triangles, why are they similar?*

❖ HSG.SRT.C.7

Explain and use the relationship between the sine and cosine of complementary angles

❖ HSG.SRT.C.8

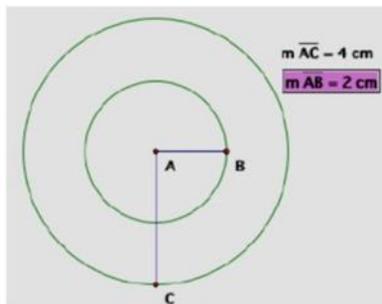
Use trigonometric ratios, special right triangles, and the Pythagorean Theorem to find unknown measurements of right triangles in applied problems

Note: Examples should include, but are not limited to angles of elevation, angles of depression, navigation, and surveying.

Geometry: Circles

Goal Statement: Understand and apply theorems about circles

- ❖ **HSG.C.A.1**
Prove that all circles are similar



- ❖ **HSG.C.A.2**
Identify, describe, and use relationships among angles, radii, segments, lines, arcs, and chords as related to circles
Note: Examples include but are not limited to the following: the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; angles inscribed in a semi-circle are right angles; the radius of a circle is perpendicular to a tangent line of the circle at the point of tangency.
- ❖ **HSG.C.A.3**
 - Construct the inscribed and circumscribed circles of a triangle
 - Prove properties of angles for a quadrilateral inscribed in a circle

Goal Statement: Find arc lengths and areas of sectors of circles

- ❖ **HSG.C.B.5**
 - Derive using similarity that the length of the arc intercepted by an angle is proportional to the radius
 - Derive and use the formula for the area of a sector
 - Understand the radian measure of the angle as a unit of measure*Note: Connected to F.TF.1 (+)*

Geometry: Expressing Geometric Properties with Equations

Goal Statement: Translate between the geometric description and the equation of a conic section

❖ HSG.GPE.A.1

- Derive the equation of a circle of given center and radius using the Pythagorean Theorem
- Complete the square to find the center and radius of a circle given by an equation

Note: Students should also be able to identify the center and radius when given the equation of a circle and write the equation given a center and radius.

Goal Statement: Use coordinates to prove simple geometric theorems algebraically

❖ HSG.GPE.B.4

Use coordinates to prove simple geometric theorems algebraically

For example: Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.

❖ HSG.GPE.B.5

- Prove the slope criteria for parallel and perpendicular lines
- Use the slope criteria for parallel and perpendicular lines to solve geometric problems

Note: Examples should include but are not limited to finding the equation of a line parallel or perpendicular to a given line that passes through a given point.

❖ HSG.GPE.B.6

Find the midpoint between two given points; and find the endpoint of a line segment given the midpoint and one endpoint

Note: An extension of this standard would be to find the point on a directed line segment between two given points that partitions the segment in a given ratio.

❖ HSG.GPE.B.7

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles

Note: Examples should include, but are not limited using the distance formula and area of composite figures.

Geometry: Geometric measurement and dimension

Goal Statement: Explain volume formulas and use them to solve problems

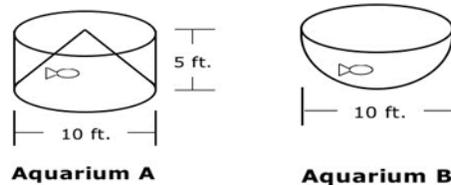
❖ **HSG.GMD.A.1**

Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume and surface area of a cylinder, pyramid, and cone

For example: Use dissection arguments, Cavalieri's principle, and informal limit arguments.

❖ **HSG.GMD.A.2**

(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures



❖ **HSG.GMD.A.3**

•Use volume formulas for cylinders, pyramids, cones, spheres, and to solve problems which may involve composite figures

•Compute the effect on volume of changing one or more dimension(s)

For example: How is the volume affected by doubling, tripling, or halving a dimension?

Goal Statement: Visualize relationships between two-dimensional and three-dimensional objects

❖ **HSG.GMD.B.4**

•Identify the shapes of two-dimensional cross-sections of three-dimensional objects

•Identify three-dimensional objects generated by rotations of two-dimensional objects

Goal Statement: Apply geometric concepts in modeling situations

❖ **HSG.MG.A.1**

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder)

❖ **HSG.MG.A.2**

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)

❖ **HSG.MG.A.3**

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios)

Algebra II

Algebra II: The Real Number System & Quantities

Goal Statement: Extend the properties of exponents to rational exponents

❖ **HSN.RN.A.1**

Explain how extending the properties of integer exponents to rational exponents provides an alternative notation for radicals

For example: We define $54^{1/3}$ to be the cube root of 54 because we want $(54^{1/3})^3 = 54$ to hold.

❖ **HSN.RN.A.2**

Rewrite expressions involving radicals and rational exponents using the properties of exponents

Goal Statement: Use properties of rational and irrational numbers

❖ **HSN.RN.B.4**

- Simplify radical expressions
- Perform operations (add, subtract, multiply, and divide) with radical expressions
- Rationalize denominators and/or numerators

Goal Statement: Reason quantitatively and use to solve problems

❖ **HSN.Q.A.2**

Define appropriate quantities for the purpose of descriptive modeling (i.e., use units appropriate to the problem being solved)

Algebra II: The Complex Number System

Goal Statement: Perform arithmetic operations with complex numbers

- ❖ **HSN.CN.A.1**
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real
- ❖ **HSN.CN.A.2**
Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers
- ❖ **HSN.CN.A.3**
 - Find the conjugate of a complex number
 - Use conjugates to find quotients of complex numbers

Goal Statement: Use complex numbers in polynomial identities and equations

- ❖ **HSN.CN.C.7**
Solve quadratic equations with real coefficients that have real or complex solutions
- ❖ **HSN.CN.C.8**
(+) Extend polynomial identities to the complex numbers
For example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
- ❖ **HSN.CN.C.9**
 - (+) Know the Fundamental Theorem of Algebra
 - (+) Show that it is true for quadratic polynomials

Algebra II: Vector and Matrix

Goal Statement: Perform operations on matrices and use matrices in applications

- ❖ **HSN.VM.C.6**
(+) Use matrices to represent and manipulate data (e.g., to represent payoffs or incidence relationships in a network)
- ❖ **HSN.VM.C.7**
(+) Multiply matrices by scalars to produce new matrices (e.g., as when all of the payoffs in a game are doubled)
- ❖ **HSN.VM.C.8**
(+) Add, subtract, and multiply matrices of appropriate dimensions
- ❖ **HSN.VM.C.9**
(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties
- ❖ **HSN.VM.C.10**
Understand that:
 - (+) The zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers
 - (+)The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse
- ❖ **HSN.VM.C.12**
(+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area

Algebra II: Seeing Structure in Expressions

Goal Statement: Interpret the structure of expressions

❖ HSA.SSE.A.1

Interpret expressions that represent a quantity in terms of its context

- Interpret parts of an expression using appropriate vocabulary, such as terms, factors, and coefficients
- Interpret complicated expressions by viewing one or more of their parts as a single entity

For example: Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P .

❖ HSA.SSE.A.2

Use the structure of an expression to identify ways to rewrite it

For example: See that $(x + 3)(x + 3)$ is the same as $(x + 3)^2$ or $x^2 - y^2$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Goal Statement: Write expressions in equivalent forms to solve problems

❖ HSA.SSE.B.3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

- Factor a quadratic expression to reveal the zeros of the function it defines
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines

Note: Students should be able to identify and use various forms of a quadratic expression to solve problems.

o *Standard Form: $ax^2 + bx + c$*

o *Factored Form: $a(x - r_1)(x - r_2)$*

o *Vertex Form: $a(x - h)^2 + k$*

- Use the properties of exponents to transform expressions for exponential functions

For example: The expression $1.15t$ can be rewritten as $(1.151/12)^{12t} \approx 1.01212t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Algebra II: Arithmetic with Polynomials and Rational Expressions

Goal Statement: Perform arithmetic operations on polynomials

❖ **HSA.APR.A.1**

- Add, subtract, and multiply polynomials
- Understand that polynomials, like the integers, are closed under addition, subtraction, and multiplication

Note: If p and q are polynomials $p + q$, $p - q$, and pq are also polynomials

Goal Statement: Understand the relationships between zeros and factors of polynomials

❖ **HSA.APR.B.2**

Know and apply the Factor and Remainder Theorems: for a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

❖ **HSA.APR.B.3**

- Identify zeros of polynomials when suitable factorizations are available
- Use the zeros to construct a rough graph of the function defined by the polynomial

Note: Algebra I is limited to the use of quadratics.

Goal Statement: Use polynomial identities to solve problems

❖ **HSA.APR.C.4**

Prove polynomial identities and use them to describe numerical relationships

Note: Examples of Polynomial Identities may include but are not limited to the following:

- $(a + b)^2 = a^2 + 2ab + b^2$ (Algebra 1)
- $a^2 - b^2 = (a - b)(a + b)$ (Algebra 1)
- $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples (Algebra 2).

Goal Statement: Rewrite rational expressions

❖ HSA.APR.D.6

Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, (where $a(x)$ is the dividend, $b(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder) are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system

Note: Students should understand that this method of dividing polynomials can be used for any polynomial expression, but that synthetic division should only be used when the divisor is a first-degree polynomial. Students should also recognize that when using synthetic division with a first-degree polynomial divisor that has a leading coefficient other than one, (such as $3x + 1$, where $x = -1/3$ is the “synthetic divisor” as in the example above), that the denominator of the “synthetic divisor” must be factored out of the quotient and multiplied by the divisor after the synthetic division has taken place.

❖ HSA.APR.D.7

- Add, subtract, multiply, and divide by nonzero rational expressions
- Understand that rational expressions, like the integers, are closed under addition, subtraction, and multiplication

Algebra II: Creating Equations

Goal Statement: Create equations that describe numbers or relationships

❖ HSA.CED.A.1

Create equations and inequalities in one variable and use them to solve problems

Note: Including but not limited to equations arising from:

- *Linear functions*
- *Quadratic functions*
- *Simple rational functions*
- *Exponential functions*
- *Absolute value functions*

❖ HSA.CED.A.2

- Create equations in two or more variables to represent relationships between quantities
- Graph equations, in two variables, on a coordinate plane

❖ HSA.CED.A.3

- Represent and interpret constraints by equations or inequalities, and by systems of equations and/or inequalities
- Interpret solutions as viable or nonviable options in a modeling and/or real-world context

❖ HSA.CED.A.4

Rearrange literal equations using the properties of equality

Algebra II: Reasoning with Equations and Inequalities

Goal Statement: Understand solving equations as a process of reasoning and explain the reasoning

❖ **HSA.REI.A.1**

Assuming that equations have a solution, construct a solution and justify the reasoning used

Note: Students are not required to use only one procedure to solve problems nor are they required to show each step of the process. Students should be able to justify their solution in their own words.

❖ **HSA.REI.A.2**

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

For example: The area of a square equals 49 square inches. The length of the side is 7 inches. Although -7 is a solution to the equation, $x^2 = 49$, -7 is an extraneous solution.

Goal Statement: Solve equations and inequalities in one variable

❖ **HSA.REI.B.4**

Solve quadratic equations in one variable

• Use the method of completing the square to transform any quadratic equation in x into an equation of the form

$(x - p)^2 = q$ that has the same solutions

Note: This would be a good opportunity to demonstrate/explore how the quadratic formula is derived. This standard also connects to the transformations of functions and identifying key features of a graph (F-BF3).

Introduce this with a leading coefficient of 1 in Algebra I. Finish mastery in Algebra II.

• Solve quadratic equations (as appropriate to the initial form of the equation) by:

- o Inspection of a graph
- o Taking square roots
- o Completing the square
- o Using the quadratic formula
- o Factoring

Recognize complex solutions and write them as $a + bi$ for real numbers a and b

Goal Statement: Solve systems of equations and inequalities graphically

- ❖ **HSA.REI.C.5**
 - Solve systems of equations in two variables using substitution and elimination
 - Understand that the solution to a system of equations will be the same when using substitution and elimination
- ❖ **HSA.REI.C.6**

Solve systems of equations algebraically and graphically
- ❖ **HSA.REI.C.7**

Solve systems of equations consisting of linear equations and nonlinear equations in two variables algebraically and graphically

For example: Find the points of intersection between $y = -3x$ and $y = x^2 + 2$.
- ❖ **HSA.REI.C.8**

(+) Represent a system of linear equations as a single matrix equation in a vector variable
- ❖ **HSA.REI.C.9**

(+) Find the inverse of a matrix (matrix inverse) if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater)

Goal Statement: Solve systems of equations

- ❖ **HSA.REI.D.11**

Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;
Find the solutions approximately by:

 - Using technology to graph the functions
 - Making tables of values
 - Finding successive approximations

Include cases (but not limited to) where $f(x)$ and/or $g(x)$ are:

 - Linear
 - Polynomial
 - Rational
 - Exponential (Introduction in Algebra 1, Mastery in Algebra 2)
 - Logarithmic functions

Teacher notes: Modeling should be applied throughout this standard.
- ❖ **HSA.REI.D.12**

Solve linear inequalities and systems of linear inequalities in two variables by graphing

Algebra II: Interpreting Functions

Goal Statement: Understand the concept of a function and use function notation

❖ **HSF.IF.A.3**

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

For example: The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + (n - 1)$ for $n \geq 1$.

Goal Statement: Interpret functions that arise in applications in terms of the context

❖ **HSF.IF.B.4**

For a function that models a relationship between two quantities:

- Interpret key features of graphs and tables in terms of the quantities, and
- Sketch graphs showing key features given a verbal description of the relationship

Note: Key features may include but not limited to: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

❖ **HSF.IF.B.6**

• Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval

• Estimate the rate of change from a graph

Goal Statement: Analyze functions using different representations

❖ **HSF.IF.C.7**

Graph functions expressed algebraically and show key features of the graph, with and without technology:

- Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior
- (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior
- Graph exponential and logarithmic functions, showing intercepts and end behavior
- (+) Graph trigonometric functions, showing period, midline, and amplitude

❖ **HSF.IF.C.8**

Write expressions for functions in different but equivalent forms to reveal key features of the function

Use the properties of exponents to interpret expressions for exponential functions

Note: Connection to A.SSE.B.3

Note: Various forms of exponentials might include representing the base as $1 \pm r$, where r is the rate of growth or decay.

Algebra II: Interpreting Categorical and Quantitative Data

Goal Statement: Summarize, represent, and interpret data on a single count or measurement variable

❖ HSS.ID.A.4

- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages

- Recognize that there are data sets for which such a procedure is not appropriate.

- Use calculators and/or spreadsheets to estimate areas under the normal curve

Note: Limit area under the curve to the empirical rule (68-95-99.7) to estimate the percent of a normal population that falls within 1, 2, or 3 standard deviations of the mean. Also, recognize that normal distributions are only appropriate for unimodal and symmetric shapes.

Goal Statement: Summarize, represent, and interpret data on two categorical and quantitative variables

❖ HSS.ID.B.6

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related

- Fit a function to the data; use functions fitted to data to solve problems in the context of the data

Note: Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. The focus of Algebra I should be on linear and exponential models while the focus of Algebra II is more on quadratic and exponential models.

Algebra II: Building Functions

Goal Statement: Build a function that models a relationship between two equalities

❖ **HSF.BF.A.1**

Write a function that describes a relationship between two quantities

- From a context, determine an explicit expression, a recursive process, or steps for calculation
- Combine standard function types using arithmetic operations. (e.g., given that $f(x)$ and $g(x)$ are functions developed from a context, find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, $(f/g)(x)$, and any combination thereof, given $g(x) \neq 0$.)
- Compose functions

❖ **HSF.BF.A.2**

- Write arithmetic and geometric sequences both recursively and with an explicit formula, and translate between the two forms
- Use arithmetic and geometric sequences to model situations

Goal Statement: Build new functions from existing functions

❖ **HSF.BF.B.3**

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (k a constant both positive and negative);

Find the value of k given the graphs of the transformed functions

Experiment with multiple transformations and illustrate an explanation of the effects on the graph with or without technology

Note: Include recognizing even and odd functions from their graphs and algebraic expressions for them.

❖ **HSF.BF.B.4**

Find inverse functions.

- Solve an equation of the form $y = f(x)$ for a simple function f that has an inverse and write an expression for the inverse

For example, $f(x) = 2x^2$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

- Verify by composition that one function is the inverse of another (Algebra II)
- Read values of an inverse function from a graph or a table, given that the function has an inverse (Algebra II)
- (+) Produce an invertible function from a non-invertible function by restricting the domain

❖ **HSF.IF.B.5**

- Relate the domain of a function to its graph

- Relate the domain of a function to the quantitative relationship it describes

For example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Algebra II: Linear, Quadratic, and Exponential Models

Goal Statement: Construct and compare linear, quadratic, and exponential models and solve problems

❖ **HSF.LE.A.2**

Construct linear and exponential equations, including arithmetic and geometric sequences,

- given a graph,
- a description of a relationship, or
- two input-output pairs (include reading these from a table)

❖ **HSF.LE.A.4**

- Express exponential models as logarithms
- Express logarithmic models as exponentials
- Use properties of logarithms to simplify and evaluate logarithmic expressions (expanding and/or condensing logarithms as appropriate)
- Evaluate logarithms with or without technology

Note: For exponential models, express the solution to $ab^{ct} = d$ where a , c , and d are constants and b is the base (Including, but not limited to: 2, 10, or e) as a logarithm; then evaluate the logarithm with or without technology. Connection to F.BF.B.5

Algebra II: Making Inferences and Justifying Conclusions

Goal Statement: Understand and evaluate random processes underlying statistical experiments

- ❖ **HSS.IC.A.1**
Recognize statistics as a process for making inferences about population parameters based on a random sample from that population
- ❖ **HSS.IC.A.2**
Compare theoretical and empirical probabilities using simulations (e.g. such as flipping a coin, rolling a number cube, spinning a spinner, and technology)

Goal Statement: Make inferences and justify conclusions from sample surveys, experiments and observational studies

- ❖ **HSS.IC.B.3**
 - Recognize the purposes of and differences among sample surveys, experiments, and observational studies
 - Explain how randomization relates to sample surveys, experiments, and observational studies
- ❖ **HSS.IC.B.6**
Read and explain, in context, the validity of data from outside reports by
 - Identifying the variables as quantitative or categorical.
 - Describing how the data was collected.
 - Indicating any potential biases or flaws.
 - Identifying inferences the author of the report made from sample data

Note: As a strategy, students could collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed.
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Algebra II: Expressing Geometric Properties with Equations

Goal Statement: Translate between the geometric description and the equation of a conic section

❖ **HSG.GPE.A.2**

(+) Derive the equation of a parabola given a focus and directrix