Sequence of Grade 3 Modules Aligned with the Standards

Module 1: Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
Module 2: Place Value and Problem Solving with Units of Measure
Module 3: Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Geometry and Measurement Word Problems

Summary of Year

Grade 3 mathematics is about (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with a numerator of 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Key Areas of Focus for 3–5: Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving

Required Fluency:

3.OA.7	Multiply and divide within 100.
3.NBT.2	Add and subtract within 1000.

Major Emphasis Clusters

Operations and Algebraic Thinking
- Represent and solve problems involving multiplication and division.
- Understand the properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations and identify and explain patterns in arithmetic.

Number and Operations—Fractions
- Develop understanding of fractions as numbers.

Measurement and Data
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

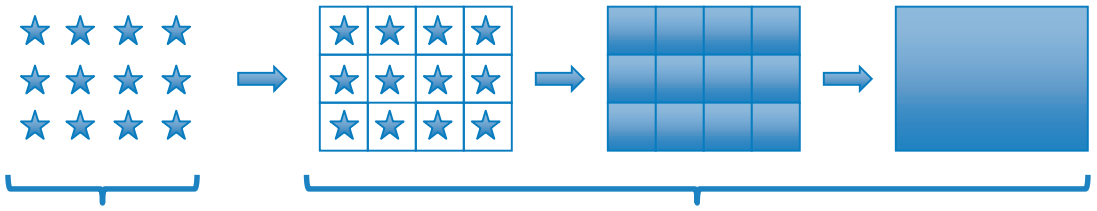
Rationale for Module Sequence in Grade 3

The first module builds upon the foundation of multiplicative thinking with units started in Grade 2. First, students concentrate on the meaning of multiplication and division and begin developing fluency for learning products involving factors of 2, 3, 4, 5, and 10 (see Key Areas of Focus and

Required Fluency above). The restricted set of facts keeps learning manageable, and also provides enough examples to do one- and two-step word problems and to start measurement problems involving weight, capacity, and time in the second module.

Module 2 focuses on measurement of time and metric weight and capacity. In exploratory lessons, students decompose a kilogram into 100 gram, 10 gram, and 1 gram weights and decompose a liter into analogous amounts of milliliters. Metric measurement thereby develops the concept of mixed units (e.g., 3 kilograms 400 grams is clearly related to 3 thousands, 4 hundreds). Students then apply their new understanding of number to place value, comparison and rounding, composing larger units when adding, decomposing into smaller units when subtracting. Students also draw proportional tape diagrams to solve word problems (e.g., "If this tape represents 62 kg, then a tape representing 35 kg needs to be slightly longer than half the 62 kg bar …"). Drawing the relative sizes of the lengths involved in the model prepares students to locate fractions on a number line in Module 5 (where they learn to locate points on the number line relative to each other and relative to the whole unit). Module 2 also provides students with internalization time for learning the 2, 3, 4, 5, and 10 facts as part of their fluency activities.

Students learn the remaining multiplication and division facts in Module 3 as they continue to develop their understanding of multiplication and division strategies within 100 and use those strategies to solve two-step word problems. The "2, 3, 4, 5, and 10 facts" module (Module 1) and the "0, 1, 6, 7, 8, 9, and multiples of 10 facts" module (Module 3) both provide important, sustained time for work in understanding the structure of rectangular arrays to prepare students for area in Module 4. This work is necessary because students initially find it difficult to distinguish the different units in a grid (the third array in the picture below), count them, and recognize that the count is related to multiplication. Tiling also supports a correct interpretation of the grid. Modules 1 and 3 slowly build up to the area model (the fourth model in the picture below), using rectangular arrays in the context of learning multiplication and division:



Progression from Rectangular Array to Area Model

By Module 4, students are ready to investigate area. They measure the area of a shape by finding the total number of same-size units of area (e.g., tiles) required to cover the shape without gaps or overlaps. When that shape is a rectangle with whole number side lengths, it is easy to partition the rectangle into squares with equal areas (as in the third stage of the illustration above).

One goal of Module 5 is for students to transition from thinking of fractions as area or parts of a figure to points on a number line and finally, as numbers. To make that jump, students think of fractions as being constructed out of unit fractions: *1 fourth* is the length of a segment on the number line such that the length of four concatenated fourth segments on the line equals 1 (the whole). Once the unit *1 fourth* has been established, counting them is as easy as counting whole numbers: 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, etc. Students also compare fractions, find equivalent fractions in special cases, and solve problems that involve fractions. They realize that equivalent fractions share the same point on the number line.

In Module 6, by applying their knowledge of fractions from Module 5, students round lengths to the nearest halves and fourths of an inch and record that information on line plots. This module also prepares students for the multiplicative comparison problems of Grade 4 by asking students "how many more" and "how many less" questions about scaled bar graphs.

The year rounds out with plenty of time to solve two-step word problems involving the four operations and to improve fluency for concepts and skills initiated earlier in the year. In Module 7, students also describe, analyze, and compare properties of two-dimensional shapes. By now, students have done enough work with both linear and area measurement models to understand that there is no relationship in general between the area of a figure and its perimeter, which is one of the concepts taught in the last module.

Alignment Chart[48]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
Module 1: **Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10** (25 days)	**Represent and solve problems involving multiplication and division.[49]** **3.OA.1** Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

[48] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.
[49] In this module, work is limited to factors of 2–5 and 10 and the corresponding dividends.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 2.) **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = _ ÷ 3, 6 x 6 = ?* **Understand properties of multiplication and the relationship between multiplication and division.**[50] **3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*[51] **3.OA.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* **Multiply and divide within 100.**[52] **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[53] **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard

[50] In this module, work is limited to factors of 2–5 and 10 and the corresponding dividends.
[51] The associative property is addressed in Module 3.
[52] In this module, work is limited to factors of 2–5 and 10 and the corresponding dividends.
[53] In this module, problem solving is limited to multiplication and division and limited to factors of 2–5 and 10 and the corresponding dividends. 3.OA.9 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
Module 2: **Place Value and Problem Solving with Units of Measure** (25 days)	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**[54] **3.NBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100. **3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems, i.e., problems involving notions of "times as much"; see Standards Glossary, Table 2.)

[54] From this point forward, fluency practice with addition and subtraction is part of students' on-going experience. 3.NBT.3 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
Module 3: **Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10** (25 days)	**Represent and solve problems involving multiplication and division.**[55] **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 2.) **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 =___÷ 3, 6 x 6 = ?* **Understand properties of multiplication and the relationship between multiplication and division.**[56] **3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **Multiply and divide within 100.**[57] **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[58] **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students

[55] The balance of this cluster is addressed in Module 1.
[56] The balance of this cluster is addressed in Module 1.
[57] From this point forward, fluency practice with multiplication and division facts is part of students' on-going experience.
[58] After being fully taught in Module 3, this standard (as well as 3.OA.3) continues being practiced throughout the remainder of the school year.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.) **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)**[59] **3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
Module 4: **Multiplication and Area** (20 days)	**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **3.MD.7** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

[59] The balance of this cluster is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
Module 5: **Fractions as Numbers on the Number Line** (35 days)	**Develop understanding of fractions as numbers. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)** **3.NF.1** Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. **3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. **3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Reason with shapes and their attributes.**[60] **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area and describe the area of each part as ¼ of the area of the shape.
Module 6: **Collecting and Displaying Data** (10 days)	**Represent and interpret data.** **3.MD.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two- step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
Module 7: **Geometry and Measurement Word Problems**[61] (40 days)	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[62] **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard

[60] 3.G.1 is addressed in Module 7.
[61] The seemingly eclectic set of standards in Module 7 allows for a new level of word problems, including perimeter and measurement word problems.
[62] 3.OA.9 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.) **Represent and interpret data.**[63] **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **3.MD.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **Reason with shapes and their attributes.**[64] **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

[63] 3.MD.3 is addressed in Module 6.
[64] 3.G.2 is addressed in Module 5.