

# The trouble with long division

**The fourth graders** were ready to learn long division; however, their teachers were hesitant to begin the unit—just as they are every year. In a grade-level meeting with the school's math consultant, the teachers voiced their typical concerns. The math consultant was a university mathematics education professor spending a semester of sabbatical working with teachers to find ways to help elementary-aged students get excited about doing math and about learning to make sense of math through problem-solving activities.

## A meaningless procedure

In collaborative sessions with teachers and in interviews with some fourth graders, the math consultant discovered that teachers and students alike viewed the division algorithm as a set of procedures, not as a way to make any sense of performing the division operation. Teachers felt that students never grasped the meaning of the

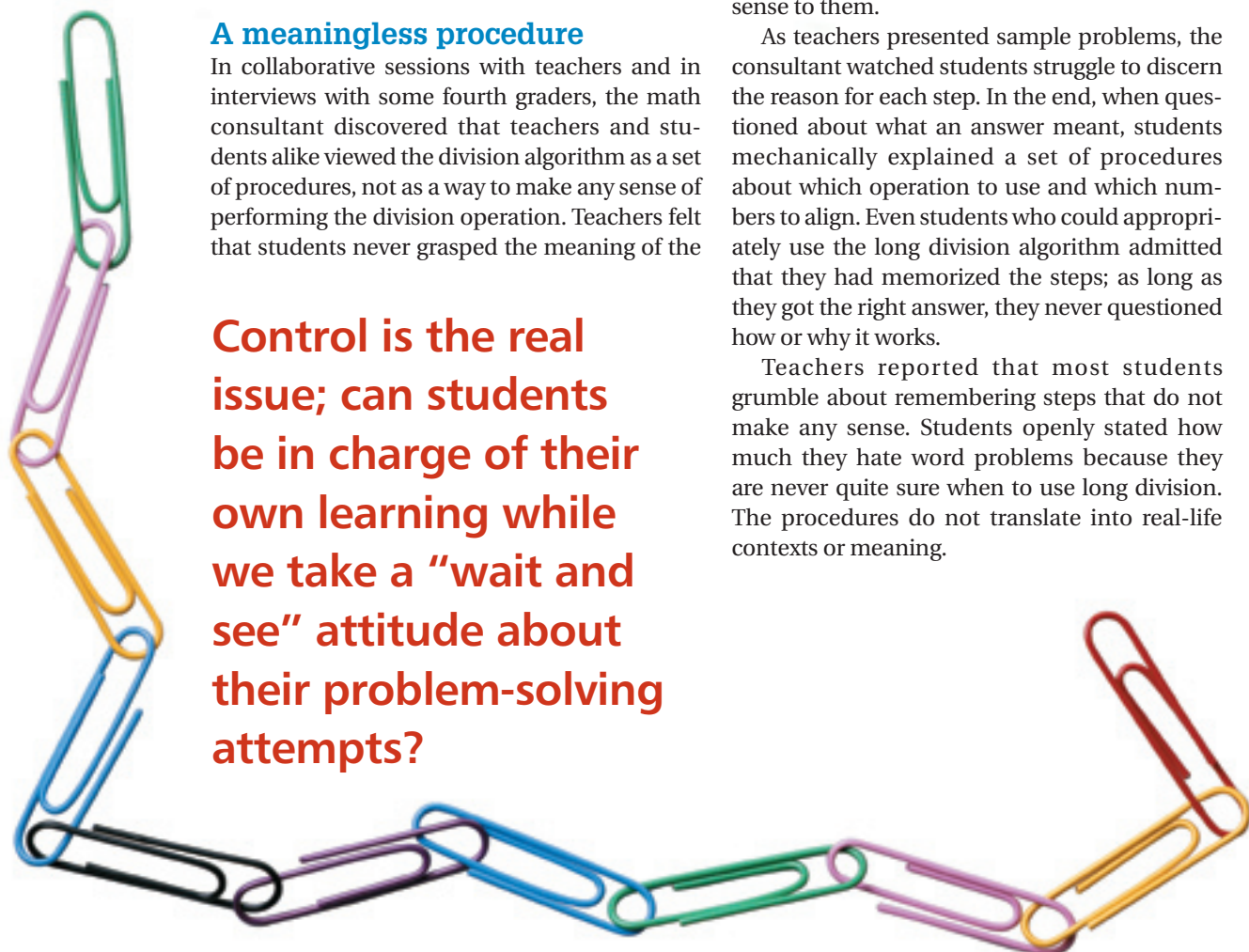
outcome, no matter how many times they were shown sample problems. Students' statements about division indicated that the procedures were meaningless to them.

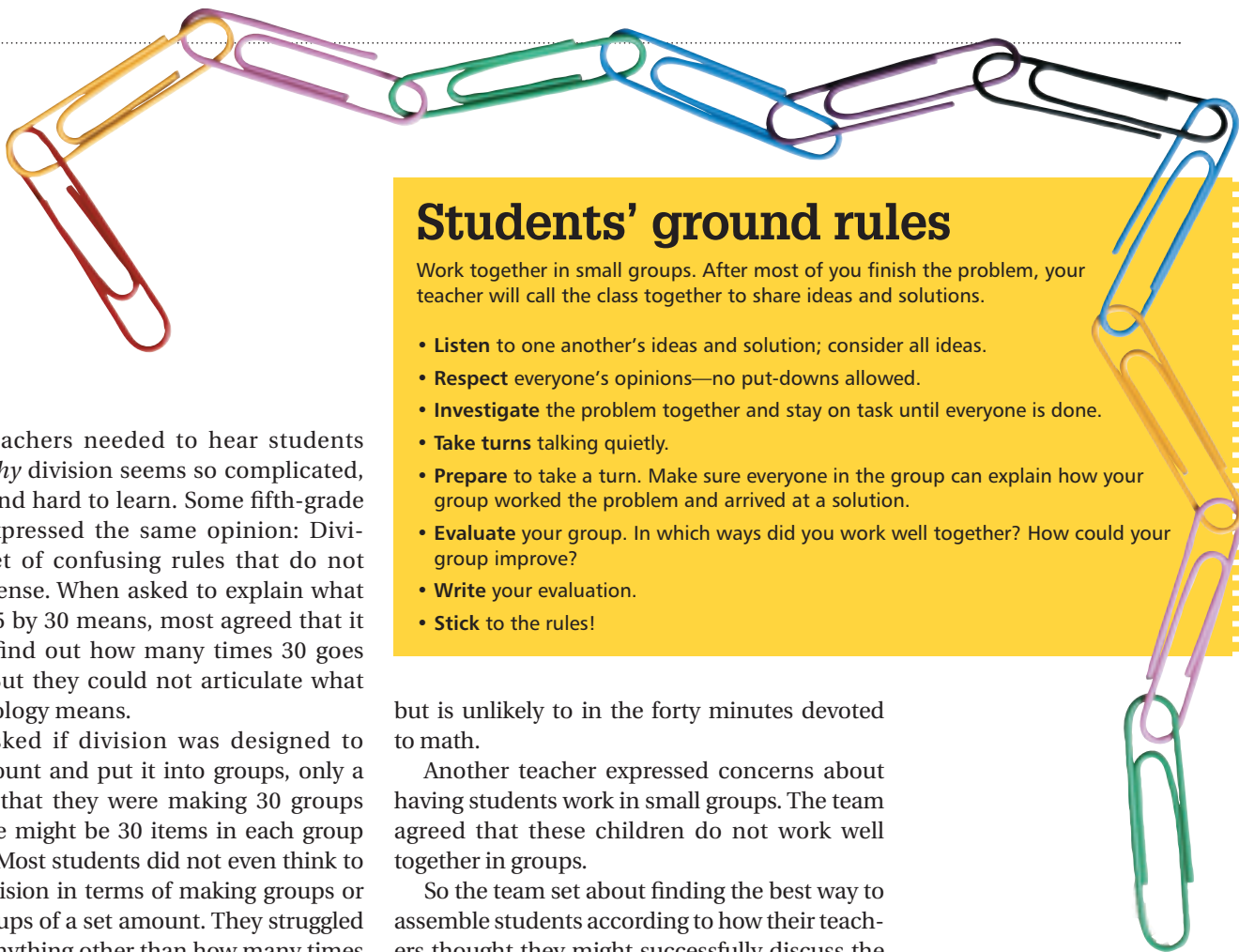
Until this point in their math careers, students learn to work from right to left for every operation. The division algorithm involves steps that have them work from left to right and keep making new little division problems that involve only parts and pieces of the original numbers. It is no wonder that students are confused and find it hard to remember steps that make little sense to them.

As teachers presented sample problems, the consultant watched students struggle to discern the reason for each step. In the end, when questioned about what an answer meant, students mechanically explained a set of procedures about which operation to use and which numbers to align. Even students who could appropriately use the long division algorithm admitted that they had memorized the steps; as long as they got the right answer, they never questioned how or why it works.

Teachers reported that most students grumble about remembering steps that do not make any sense. Students openly stated how much they hate word problems because they are never quite sure when to use long division. The procedures do not translate into real-life contexts or meaning.

**Control is the real issue; can students be in charge of their own learning while we take a "wait and see" attitude about their problem-solving attempts?**





## Students' ground rules

Work together in small groups. After most of you finish the problem, your teacher will call the class together to share ideas and solutions.

- **Listen** to one another's ideas and solution; consider all ideas.
- **Respect** everyone's opinions—no put-downs allowed.
- **Investigate** the problem together and stay on task until everyone is done.
- **Take turns** talking quietly.
- **Prepare** to take a turn. Make sure everyone in the group can explain how your group worked the problem and arrived at a solution.
- **Evaluate** your group. In which ways did you work well together? How could your group improve?
- **Write** your evaluation.
- **Stick to the rules!**

These teachers needed to hear students tell them *why* division seems so complicated, senseless, and hard to learn. Some fifth-grade students expressed the same opinion: Division is a set of confusing rules that do not make any sense. When asked to explain what dividing 345 by 30 means, most agreed that it means "to find out how many times 30 goes into 345." But they could not articulate what this terminology means.

When asked if division was designed to take an amount and put it into groups, only a few agreed that they were making 30 groups or that there might be 30 items in each group they made. Most students did not even think to describe division in terms of making groups or forming groups of a set amount. They struggled to express anything other than how many times one number goes into another number.

When asked if they could think of the 345 as marbles, then explain why one would want to divide 345 marbles by 30, only a handful of students could accurately describe either a partitive or a quotative situation to fit the problem. At this point, some of the teachers began to understand why word problems cause confusion: Division in context can take on more than one meaning.

### A new way to prepare

Kamii and Livingston (1994) describe the importance of a meaningful context in an equal-sharing exercise with large enough numbers that students must find their own way of doing the problem. When this equal-sharing situation was described to the fourth-grade teachers, they agreed to try some lessons with students before introducing their standard long-division algorithm methods. Teachers were nervous about whether students would be able to do the problem. In fact, one of the three teachers did not think the children would be able to complete the large problem. He thought that students would attempt to draw 345 items and then divide them into groups of 30—a process that works

but is unlikely to in the forty minutes devoted to math.

Another teacher expressed concerns about having students work in small groups. The team agreed that these children do not work well together in groups.

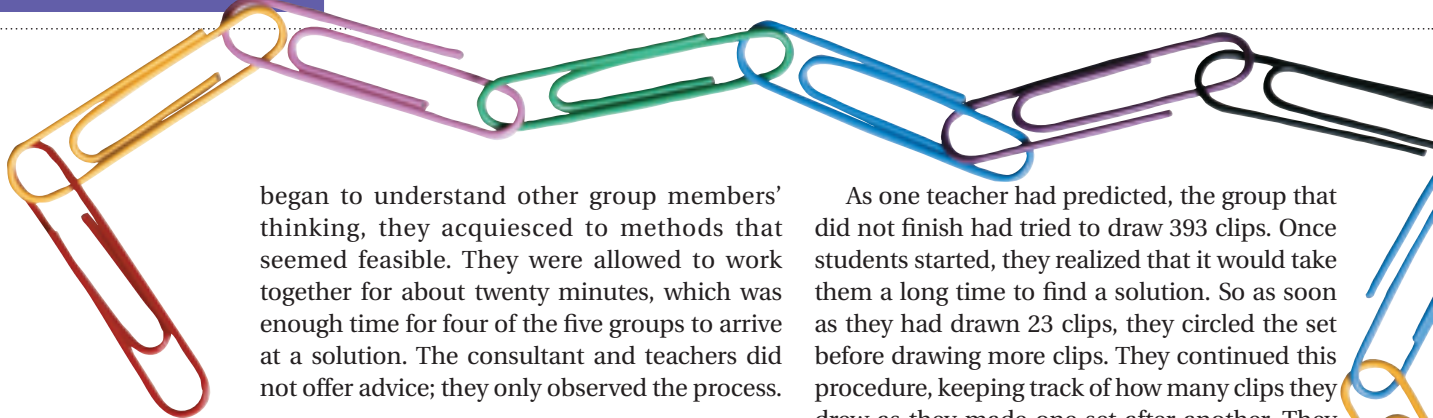
So the team set about finding the best way to assemble students according to how their teachers thought they might successfully discuss the problem, complement one another's different perspectives or methods, and work the problem together in the classroom.

### The equal-sharing task

Teachers chose one fourth-grade class, and the consultant introduced the lesson. Teachers agreed to refrain from interfering or commenting on students' answers. In the Kamii and Livingston (1994) example, students were shown a large bag of small candies. They were told that the teacher had opened and counted 393 candies in a similar bag and that they could share the candies if they could determine how to share them equally.

Instead of candies, this class used colored paper clips that the children had been collecting to make chains. Students worked in five groups of four students each and one group of three. Several students indicated that this was a "dumb" activity, but they cooperated when classmates reminded them that everyone would get a share of the paper clips if they could divide 393 clips among 23 students.

Power struggles arose when the fourth graders disagreed about how to proceed. As they



began to understand other group members' thinking, they acquiesced to methods that seemed feasible. They were allowed to work together for about twenty minutes, which was enough time for four of the five groups to arrive at a solution. The consultant and teachers did not offer advice; they only observed the process.

### Group presentations and methods

The children were told to decide on one student to present the respective group's solution to the whole class. Amazingly, each of the five groups had a slightly different approach to the problem.

As one teacher had predicted, the group that did not finish had tried to draw 393 clips. Once students started, they realized that it would take them a long time to find a solution. So as soon as they had drawn 23 clips, they circled the set before drawing more clips. They continued this procedure, keeping track of how many clips they drew as they made one set after another. They did not finish, due to disagreement about how to draw the clips and how to keep track of how many they had drawn. But they did conclude that each time they circled a set, it meant that each person in the class would get one more paper clip. In the end, they would know how many clips each child would get by counting the number of sets they had drawn.

A member of the first group to present explained that they had drawn 23 circles and had used tick marks to "deal out" clips to each of the 23 circles. Another group had also used this method.

The next group presented this solution: Every time students wrote the numeral 23 on the paper, each person got a paper clip. Their process was to add 23 repeatedly until they got as close as possible to 393 without going over. Their answer was 17 clips for each student, with 2 clips remaining.


The next group was anxious to present a solution because students had approached the problem quite differently. Their presenter started by writing the problem in the standard algorithmic form, showing 393 divided by 23. She indicated that 23 multiplied by 10 is 230. Then she subtracted:  $393 - 230 = 163$ . Students had gotten "stuck" because they did not know what to do after putting 10 where the quotient goes in the standard algorithm. They knew they still had to use the 163 clips that were left. They had decided to subtract 23, then another 23, and so on until they could not subtract 23 any more. They subtracted 23 seven more times. When they added that 7 to the 10, their answer was also that each student receives 17 clips with a remainder of 2.

The last group did not want to present; students thought their trial-and-error method would be considered unacceptable. Interestingly, though, they had used much more than trial and error and had shown excellent number sense. They multiplied 23 by 10 because that answer was easy to get. They indicated that

## The teacher's role in problem solving

Remember that struggle has value. Do *not* interrupt students' work unless you think a question is in order to keep them on task. Do *not* give hints, and do *not* tell students how to solve a problem. Students can do their own thinking and reasoning if we let them.

- **Review** student rules for cooperation (or partner rules for grades K, 1, and 2).
- **Show** a video of a class similar to theirs doing cooperative group problem solving.
- **Discuss** whether the group they saw in the video was working well together:
  - Were they listening to one another?
  - Were they polite to one another?
  - Were they all working on the problem?
- **Introduce** the problem of the day to your students. Ask them if they know what the problem requires them to do. Refrain from giving hints or having a group discussion of how to solve the problem, but be sure that they understand the task.
- **Remind** them to follow group rules and to take any assigned roles.
- **Encourage** investigation of the problem and be sure students are on task until most are finished.
- **Prompt** groups that finish early to work on a challenge portion of the task.
- **Ensure** that students follow the ground rules for cooperation.
- **Note** groups or individual students you want to call on during the whole-class sharing phase. Choose students who worked through a struggle and who need practice explaining how to work a problem. Try not to pick students who could work the problem instantly unless they need practice articulating and sharing their work with the class. When most students are finished, bring the class together to share ideas and solutions according to the student guidelines.
- **Collect** students' solutions after the discussion and grade them using a holistic grading scale:
  - Students used what the problem gave them.
  - Strategy would work to solve the problem. Why, or why not?
  - Solution is reasonable.
  - Solution is correct.
  - Students used multiple solution approaches and found multiple answers (if multiple answers exist).



$23 \times 10$  would give each student 10 clips but would use only 230 clips. Then they calculated that  $230 + 230$ , or 460, would yield 20 clips apiece; but they knew that was too many, as they had only 393 clips in total. They had tried  $23 \times 15$ , reasoning that the answer was somewhere between 230 and 460. When the answer of 345 turned out to be too low, they knew they had to multiply by more than 15 but less than 20. One group member noted that they “still had a ways to go with only  $23 \times 15$ ,” so the group had tried  $23 \times 17 = 391$ . Students were surprised when they got the same answer as the other groups but remained embarrassed by their method.

### Lessons learned

Several teachers had predicted that students would draw pictures; they were pleasantly surprised that students could make sense of the problem in so many other ways. Because addition and subtraction both mirror the process of “dealing out” the clips, these methods did not

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surprise them as much as other methods did. The successive approximations were a complete surprise. Although the teachers had been using a problem-solving curriculum, which often asks students to make predictions or estimates about the magnitude of their answers, the teachers were amazed when students used so much number sense after their initial trial-and-error guess.

In once-a-month grade-level meetings, these educators always discuss the importance of



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## Procedures do not translate into real-life contexts or meaning.

allowing—and encouraging—multiple methods. However, they had not recognized the value of the process until they observed students' solutions to the equal-sharing problem.

Most teachers agree that promoting a risk-free, problem-centered environment is a worthy goal, but they often do not recognize how little they actually do to provide such an environment. Teachers find it difficult to step out of their comfort zone because they feel that students need guidance. After the equal-sharing lesson, these teachers were convinced that students could do more of their own thinking and reasoning with the right kind of activities and environment. The hardest part was to trust students to work out a real problem with so little guidance.

The consultant recognized a mistake she had made when setting up the appropriate risk-free environment for real problem solving with real student-to-student dialogue: She had failed to advise teachers to establish teachers' and students' rules and a teacher-student agreement, both of which help such situations work smoothly.

### Conclusions and implications

During a summer in-service session, these same teachers had spent a week as students while the consultant modeled problem-solving activities and demonstrated how to provide a student-centered, risk-free environment. Teachers liked what they experienced, but they had to see the approach work with their own students before they agreed to try

teaching mathematics differently from how they had been taught as youngsters.

Changing our beliefs can be difficult. When teachers are stressed or pressed for preparation time, even those who see the value of problem-centered lessons will revert to comfortable methods. Control is the real issue; can we let students be in charge of their own learning while we take a "wait and see" attitude about their problem-solving attempts? Most teachers are afraid to confront unfamiliar situations, students' questions they cannot answer, and the inability to follow students' thinking. No one wants to appear incompetent. Sadly, until teachers insist that students be accountable for their own learning, many students will learn to distrust their mathematical thinking.

When the process was modeled, these teachers saw the value of changing their instructional methods, but actually witnessing their students solving problems with so little guidance was the impetus that moved many of them to open the door to better methods and allow students to do their own thinking.

### REFERENCE

Kamii, Constance, and Sally Jones Livingston. *Young Children Continue to Reinvent Arithmetic—3rd Grade: Implications of Piaget's Theory*. New York: Teachers College Press, 1994.

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