Mendham Borough Public Schools
Mendham, New Jersey

Curriculum and Instruction

Course of Study

Mathematics: Grade 4 **August 23, 2016**

I. RATIONALE, DESCRIPTION AND PURPOSE

We live in a mathematical world. Whenever we decide on a purchase, choose an insurance or health plan, or use a spreadsheet, we rely on mathematical understanding. Print and electronic media inundate us with quantitative information. The complexity of mathematical reasoning and problem solving needed in the workplace has increased dramatically.

In such a world, the ability to understand and use mathematics will yield opportunities in the workplace, in secondary and post-secondary study, and in the personal realm. Users of mathematics need to compute fluently and solve problems resourcefully.

Everyday Mathematics is designed to lead students to these two overarching mathematical competencies. Mathematics instruction in *Everyday Mathematics* proceeds in six strands. Overarching program goals are organized by strand and extend across grades pre-K-5. In *number and numeration*, students will understand the meanings, uses and representations of numbers; equivalent names for numbers; and common numerical relations. The *operations and computations* strand requires students to compute correctly; make reasonable estimates; and understand meanings of operations. *Data and chance* leads students to select and create appropriate graphical representations of collected or given data; analyze and interpret data; and understand and apply basic concepts of probability.

In *measurement and reference frames*, students will understand the systems and processes of measurement; use appropriate techniques, tools, units and formulas in making measurements; and use and understand reference frames. The *geometry* strand leads students to investigate characteristics and properties of two- and three-dimensional geometric shapes, and apply transformations and syMath Masteretry in geometric situations. In *patterns, functions and algebra*, students will understand patterns and functions, and use algebraic notation to represent and analyze situations and structures.

The goal of mathematics education in the intermediate grades is to develop students' view of the discipline as interesting and understandable. Three mathematical concepts emerge in the intermediate grades: multiplicative thinking, equivalence and computational fluency. The focus on multiplicative reasoning develops foundational understanding for the development of proportional reasoning in the middle grades. The concept of equivalence undergirds different mathematical representations for a given quantity and introduces students to algebraic ideas. Computational fluency empowers students with efficient and accurate methods for computing that are grounded in well understood properties and number relationships. Development of these three mathematical concepts supports two overarching goals of mathematics learning: making sense of mathematical ideas and acquiring the skills and understandings needed to solve problems.

II. DISTRICT OBJECTIVES

The district adopts the "Standards for Mathematical Practice" as outlined in the Common Core State Standards.

"*The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).*" (Common Core State Standards, 2010)

The standards are presented in the Common Core State Standards as follows:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

In addition, the mathematics curriculum will reflect the National Council of Teachers of Mathematics' "Curriculum Focal Points for Mathematics in Prekindergarten through Grade 8 Mathematics" through the incorporation of the following from the Focal Points document:

- *the use of mathematics to solve problems*
- *an application of logical reasoning to justify procedures and solutions; and*
- *an involvement in the design and analysis of multiple representations to learn, make connections among, and communicate about the ideas within and outside of mathematics*

III. ALIGNMENT TO STANDARDS

Mathematics development in fourth grade aligns with the following **New Jersey Student Learning Standards:**

Operations and Algebraic Thinking (OA)

• Use the four operations with whole numbers to solve problems.

• Gain familiarity with factors and multiples.
• Generate and analyze patterns.

Number and Operations in Base Ten (NBT)

• Generalize place value understanding for multidigit whole numbers.
• Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions (NF)

• Extend understanding of fraction equivalence and ordering.
• Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
• Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data (MD)

• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
• Represent and interpret data.
• Geometric measurement: understand concepts of angle and measure angles.

Geometry (G)

• Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

IV. CONTENT, SCOPE AND SEQUENCE, LEARNING OUTCOMES

The Common Core Standards for Mathematics in complete form are as follows:

CCSS.Math.Content.4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

CCSS.Math.Content.4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.1

CCSS.Math.Content.4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

CCSS.Math.Content.4.OA.B.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

CCSS.Math.Content.4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

CCSS.Math.Content.4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

CCSS.Math.Content.4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

CCSS.Math.Content.4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place.

CCSS.Math.Content.4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

CCSS.Math.Content.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

CCSS.Math.Content.4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

CCSS.Math.Content.4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

CCSS.Math.Content.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

CCSS.Math.Content.4.NF.B.3 Understand a fraction a/b with $a > 1$ as a sum of fractions 1/b.

CCSS.Math.Content.4.NF.B.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

CCSS.Math.Content.4.NF.B.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify

decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

CCSS.Math.Content.4.NF.B.3c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

CCSS.Math.Content.4.NF.B.3d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

CCSS.Math.Content.4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

CCSS.Math.Content.4.NF.B.4a Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).

CCSS.Math.Content.4.NF.B.4b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)

CCSS.Math.Content.4.NF.B.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

CCSS.Math.Content.4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

CCSS.Math.Content.4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

CCSS.Math.Content.4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

CCSS.Math.Content.4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

CCSS.Math.Content.4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple

fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

CCSS.Math.Content.4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

CCSS.Math.Content.4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

CCSS.Math.Content.4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

CCSS.Math.Content.4.MD.C.5a An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

CCSS.Math.Content.4.MD.C.5b An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

CCSS.Math.Content.4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

CCSS.Math.Content.4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

CCSS.Math.Content.4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

CCSS.Math.Content.4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

CCSS.Math.Content.4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

V. INSTRUCTIONAL TECHNIQUES

Mathematics instruction incorporates a variety of techniques to meet the continuum of learners' interests, learning profiles and readiness levels. Differentiation is the commitment and mechanism

through which the developmental needs of a range of readiness levels are met. Differentiated instruction is accomplished through pre-assessment and ongoing formative assessment that inform independent work, the small group strategy lesson and the individual conference. Differentiation in content, product and/or process addresses the needs of exceptionally able students, and scaffolding of varying degree is provided to support less ready students in meeting worthy and appropriately rigorous learning outcomes. Instructional objectives, strategies and materials emphasize relevance, authenticity, and student-centered learning.

Instructional techniques include the following:

- Mini-lessons (connection, teaching point, modeling, active student engagement, link to independent work)
- Daily routines for ongoing practice and application: math message, mental math and reflexes, study links, name-collection boxes, math boxes, function machines
- Daily routines to develop math fact automaticity; e.g., fact triangles, drills
- Regular cooperative routines; e.g., math games
- Use of manipulatives and simulations to model mathematical concepts
- Inquiry and inductive approaches to the discovery of mathematical concepts
- Teacher modeling/thinking aloud of mathematical reasoning and problem-solving processes
- Accountable talk ("turn and talk", "stop and jot")
- Small group strategy lesson
- Individual conference
- Writing about mathematics (problem-solving process, metacognitive awareness, affective response)
- Open-ended prompt response ("quick write")

VI. ASSESSMENT

Assessment in mathematics education includes **interim/formative assessments, including performance assessment:**

- Daily routines: math message, mental math and reflexes, study links, name-collection boxes, math boxes, function machines, math games

- Lesson and unit pre-assessment to differentiate instruction (i.e., curriculum compacting)

- End-of-unit assessments ("Progress Checks")

- Periodic teacher-student conferences to assess development of unit skills (performance assessment of application of concepts and problem-solving strategies) and concepts

- Periodic review of the Math Journal to assess mathematical reasoning strategies and processes

- Teachers' observation of students' independent mathematics work (i.e., stamina for focused work, algorithms and problem-solving skills)

- Teachers' observation of students' accountable talk in math partnerships (performance assessment of thinking in mathematics problem solving)

- Students' self-reflections regarding their metacognitive and affective responses to mathematics

- Periodic completion and review of timed NJASK-type prompts and tasks

Assessment in mathematics education includes **summative assessments, including performance assessment:**

- Baseline grade-level assessment to measure mastery of previously taught concepts and skills (pre-assessment)

- End-of-year assessment to measure mastery of fourth grade concepts and skills

- End-of-year "Skills to Review" checklist for each student