Mendham Borough Public Schools
Mendham, New Jersey

Curriculum and Instruction

Course of Study

Mathematics: Grade 3 **August 23, 2016**

I. RATIONALE, DESCRIPTION AND PURPOSE

We live in a mathematical world. Whenever we decide on a purchase, choose an insurance or health plan, or use a spreadsheet, we rely on mathematical understanding. Print and electronic media inundate us with quantitative information. The complexity of mathematical reasoning and problem solving needed in the workplace has increased dramatically.

In such a world, the ability to understand and use mathematics will yield opportunities in the workplace, in secondary and post-secondary study, and in the personal realm. Users of mathematics need to compute fluently and solve problems resourcefully.

Everyday Mathematics is designed to lead students to these two overarching mathematical competencies. Mathematics instruction in *Everyday Mathematics* proceeds in six strands. Overarching program goals are organized by strand and extend across grades pre-K-5. In *number and numeration*, students will understand the meanings, uses and representations of numbers; equivalent names for numbers; and common numerical relations. The *operations and computations* strand requires students to compute correctly; make reasonable estimates; and understand meanings of operations. *Data and chance* leads students to select and create appropriate graphical representations of collected or given data; analyze and interpret data; and understand and apply basic concepts of probability.

In *measurement and reference frames*, students will understand the systems and processes of measurement; use appropriate techniques, tools, units and formulas in making measurements; and use and understand reference frames. The *geometry* strand leads students to investigate characteristics and properties of two- and three-dimensional geometric shapes, and apply transformations and symmetry in geometric situations. In *patterns, functions and algebra*, students will understand patterns and functions, and use algebraic notation to represent and analyze situations and structures.

The goal of mathematics education in the intermediate grades is to develop students' view of the discipline as interesting and understandable. Three mathematical concepts emerge in the intermediate grades: multiplicative thinking, equivalence and computational fluency. The focus on multiplicative reasoning develops foundational understanding for the development of proportional reasoning in the middle grades. The concept of equivalence undergirds different mathematical representations for a given quantity and introduces students to algebraic ideas. Computational fluency empowers students with efficient and accurate methods for computing that are grounded in well understood properties and number relationships. Development of these three mathematical concepts supports two overarching goals of mathematics learning: making sense of mathematical ideas and acquiring the skills and understandings needed to solve problems.

II. DISTRICT OBJECTIVES

The district adopts the "Standards for Mathematical Practice" as outlined in the Common Core State Standards.

"*The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).*" (Common Core State Standards, 2010)

The standards are presented in the Common Core State Standards as follows:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

In addition, the mathematics curriculum will reflect the National Council of Teachers of Mathematics' "Curriculum Focal Points for Mathematics in Prekindergarten through Grade 8 Mathematics" through the incorporation of the following from the Focal Points document:

- *the use of mathematics to solve problems*
- *an application of logical reasoning to justify procedures and solutions; and*
- *an involvement in the design and analysis of multiple representations to learn, make connections among, and communicate about the ideas within and outside of mathematics*

III. ALIGNMENT TO STANDARDS

Mathematics development in third grade aligns with the following **New Jersey Student Learning Standards:**

Operations and Algebraic Thinking (OA)

• Represent and solve problems involving multiplication and division.

• Understand properties of multiplication and the relationship between multiplication and division.
• Multiply and divide within 100.
• Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten (NBT)

• Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions (NF)

• Develop understanding of fractions as numbers.

Measurement and Data (MD)

• Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
• Represent and interpret data.
• Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
• Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry (G)

• Reason with shapes and their attributes.

IV. CONTENT, SCOPE AND SEQUENCE, LEARNING OUTCOMES

The Common Core Standards for Mathematics in complete form are as follows:

CCSS.Math.Content.3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

CCSS.Math.Content.3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

CCSS.Math.Content.3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1

CCSS.Math.Content.3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?

CCSS.Math.Content.3.OA.B.5 Apply properties of operations as strategies to multiply and divide.2 Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

CCSS.Math.Content.3.OA.B.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

CCSS.Math.Content.3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

CCSS.Math.Content.3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.3

CCSS.Math.Content.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

CCSS.Math.Content.3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

CCSS.Math.Content.3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

CCSS.Math.Content.3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

CCSS.Math.Content.3.NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

CCSS.Math.Content.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CCSS.Math.Content.3.NF.A.2a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

CCSS.Math.Content.3.NF.A.2b Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

CCSS.Math.Content.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

CCSS.Math.Content.3.NF.A.3a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

CCSS.Math.Content.3.NF.A.3b Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

CCSS.Math.Content.3.NF.A.3c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

CCSS.Math.Content.3.NF.A.3d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

CCSS.Math.Content.3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

CCSS.Math.Content.3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).1 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.2

CCSS.Math.Content.3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

CCSS.Math.Content.3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

CCSS.Math.Content.3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

CCSS.Math.Content.3.MD.C.5a A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.

CCSS.Math.Content.3.MD.C.5b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

CCSS.Math.Content.3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

CCSS.Math.Content.3.MD.C.7 Relate area to the operations of multiplication and addition.

CCSS.Math.Content.3.MD.C.7a Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

CCSS.Math.Content.3.MD.C.7b Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
CCSS.Math.Content.3.MD.C.7c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.

CCSS.Math.Content.3.MD.C.7d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

CCSS.Math.Content.3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

CCSS.Math.Content.3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

CCSS.Math.Content.3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

V. INSTRUCTIONAL TECHNIQUES

Mathematics instruction incorporates a variety of techniques to meet the continuum of learners' interests, learning profiles and readiness levels. Differentiation is the commitment and mechanism through which the developmental needs of a range of readiness levels are met. Differentiated instruction is accomplished through pre-assessment and ongoing formative assessment that inform independent work, the small group strategy lesson and the individual conference. Differentiation in content, product and/or process addresses the needs of exceptionally able students, and scaffolding of varying degree is provided to support less ready students in meeting worthy and appropriately rigorous learning outcomes. Instructional objectives, strategies and materials emphasize relevance, authenticity, and student-centered learning.

Instructional techniques include the following:

- Mini-lessons (connection, teaching point, modeling, active student engagement, link to independent work)
- Daily routines for ongoing practice and application: math message, mental math and reflexes, study links, name-collection boxes, math boxes, function machines

- Daily routines to develop math fact automaticity; e.g., fact triangles, drills
- Regular cooperative routines; e.g., math games
- Use of manipulatives and simulations to model mathematical concepts
- Inquiry and inductive approaches to the discovery of mathematical concepts
- Teacher modeling/thinking aloud of mathematical reasoning and problem-solving processes
- Accountable talk ("turn and talk", "stop and jot")
- Small group strategy lesson
- Individual conference
- Writing about mathematics (problem-solving process, metacognitive awareness, affective response)
- Open-ended problem-solving response
- Math notebook notes to consolidate learning in the spiral approach
- Exit slips

VI. ASSESSMENT

Assessment in mathematics education includes **interim/formative assessments, including performance assessment:**

- Daily routines: math message, mental math and reflexes, study links, name-collection boxes, math boxes, function machines, math games

- Lesson and unit pre-assessment to differentiate instruction (i.e., curriculum compacting)

- End-of-unit assessments ("Progress Checks"), including open-ended problem-solving response

- Periodic teacher-student conferences to assess development of unit skills (performance assessment of application of concepts and problem-solving strategies)

- Periodic review of the math journal to assess mathematical reasoning strategies and processes

- Teachers' observation of students' independent mathematics work (i.e., stamina for focused work, algorithms and problem-solving skills) and cooperative activities (e.g., math games)

- Teachers' observation of students' accountable talk in math partnerships (performance assessment of thinking in mathematics problem solving)

- Students' self-reflections regarding their metacognitive and affective responses to mathematics

- Periodic completion and review of timed NJASK-type prompts and tasks

- Exit slips

- Teacher-created mini-quizzes to assess skill mastery
- Mid-year assessment (formative for the second semester)

Assessment in mathematics education includes **summative assessments, including performance assessment:**

- Baseline grade-level assessment to measure mastery of previously taught concepts and skills (pre-assessment)
- Mid-year assessment (summative for the first semester)
- End-of-year assessment to measure mastery of third grade concepts and skills
- End-of-year "Skills to Review" checklist for each student