Think It Through

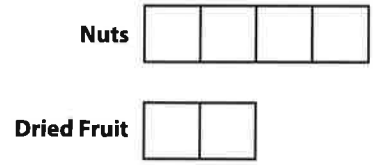
How are ratios, rates, and unit rates related?

Ratios, rates, and **unit rates** are all comparisons. They compare one quantity to another quantity.

A **ratio** compares any two quantities.

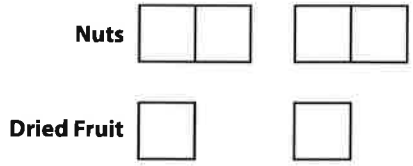
Yolanda uses 4 cups of nuts and 2 cups of dried fruit to make trail mix.

You can use a tape diagram to show this comparison.



The ratio is 4 cups to 2 cups or 4:2. Notice that the quantity of nuts is double the quantity of dried fruit.

Think Every ratio has a related rate.



A related **rate** is an equivalent ratio that compares the first quantity in a ratio to only one of the second quantity. In this example, you know that the amount of nuts is double the amount of dried fruit. So, what if you want the same kind of mix but you only have 1 cup of dried fruit? How many cups of nuts would you use?

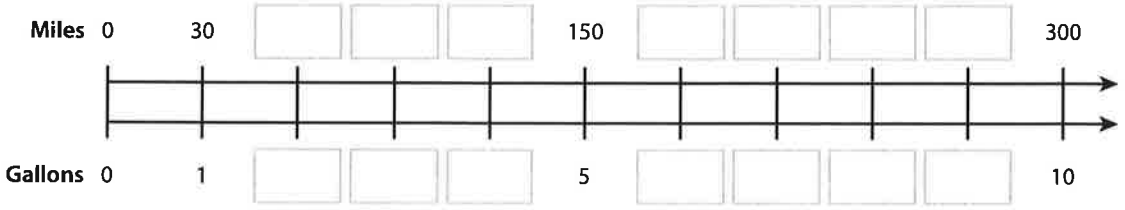
Think: 4:2 is the same as _____:1.

The rate is 2 cups of nuts to 1 cup of dried fruit. You can also say the rate is 2 cups of nuts per cup of fruit.

Think About Finding Unit Rates

Let's Explore the Idea A double number line can be used to find rate and unit rate.

A car can travel 300 miles on 10 gallons of gas. The ratio is 300 miles to 10 gallons.



2 What do the 300 and 10 in the double number line represent?

3 Fill in the remaining numbers on both number lines.

4 Look at the corresponding pairs of numbers on the bottom and top number lines. Write a multiplication sentence to show how 10 gallons and 300 miles are related. How are 5 gallons and 150 miles related? How are 1 gallon and 30 miles related?

5 Use words to describe the relationship between the number of miles and each corresponding number of gallons.

6 What is the rate of miles per gallon for this car?

7 What is the unit rate of miles to gallons?

Connect

Talk through these problems as a class. Then write your answers below.

14 Identify Write the letter of the rate that matches each ratio.

☐	\$7.50 : 3 pounds
☐	\$3.75 to 5 pounds
☐	\$6.00 : 4 pounds
☐	\$13.50 to 6 pounds

a. \$0.75 for every 1 pound

b. \$2.25 for each 1 pound

c. \$2.50 for every 1 pound

d. \$1.50 per 1 pound

15 Analyze Use the information on this nutrition label to write the unit rates described below. Show your work.

There are _____ calories in 1 cracker.

One cracker has a mass of _____ grams.

There are _____ fat calories in 1 cracker.

Nutrition Facts
Serving Size: 2 Crackers (14 grams)
Servings Per Container: About 20

Amount Per Serving

Calories 50	Calories From Fat 15

16 Compare Dawn earned \$97.50 for 10 hours of work. Amy earned \$120 for 12 hours of work. How much did each person earn per hour? How can you use this information to compare their earnings?

Understand Unit Rate

Name: _____

Prerequisite: How do you show division using fractions?

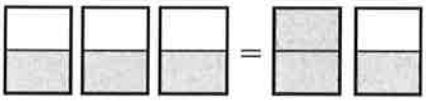
Study the example problem showing how to write a division problem as a fraction. Then solve problems 1–7.

Example

There are 3 bags of popcorn to divide equally among 2 students. How much popcorn will each student get?

There are 3 bags of popcorn for 2 students to share, which is $3 \div 2$.

Divide each of the 3 bags into 2 equal parts. Each student will get $\frac{1}{2}$ of each bag.



$\frac{1}{2} \times 3 = \frac{3}{2}$

$3 \div 2 = \frac{3}{2}$

Each student will get $\frac{3}{2}$ bags of popcorn.

1. How many whole bags plus how many one-half bags of popcorn would each student get?

   _____ whole bag(s) _____ one-half bag(s)

2. How can you combine your answers in problem 1 to write how many bags of popcorn each student will get as a mixed number?

3. Nine yards of ribbon are cut into 8 equal pieces. What is the length of each piece of ribbon? Write a division expression to represent the problem and solve.

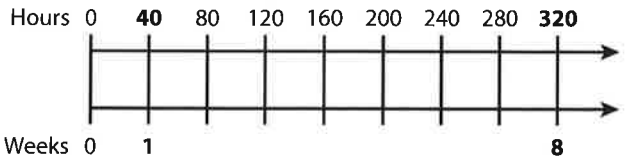
Name: _____

Find Unit Rates

Study the example showing how a double number line is used to find rate and unit rates. Then solve problems 1–6.

Example

The double number line below shows the relationship between the numbers of hours and weeks Linda works. Linda worked 320 hours in 8 weeks.



The ratio of hours to weeks is 320 to 8.

The rate is 40 hours to 1 week. The unit rate is 40.

1 Choose a corresponding pair of numbers from the top and bottom number lines. Write a multiplication equation to show how the number of weeks and hours are related.

_____

2 Use words to describe the relationship between corresponding numbers of hours and weeks.

_____

_____

_____

3 Explain how you can use the answer to problem 2 to verify the unit rate is 40.

_____

_____

_____

_____

Vocabulary

ratio compares two quantities.

rate compares the first quantity in a ratio to only one of the second quantity.

unit rate the number in a rate that is being compared to 1.

Name: _____

Reason and Write

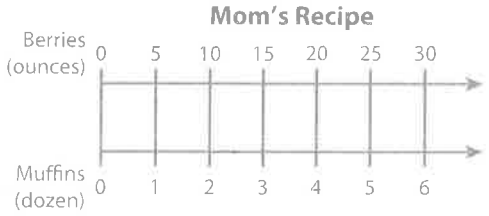
Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

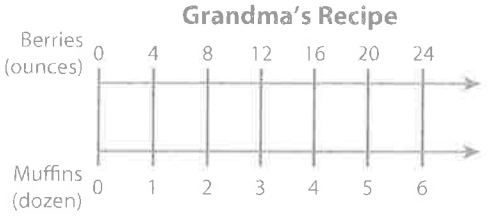
Mom's muffin recipe uses 10 ounces of berries for 2 dozen muffins. Grandma's muffin recipe uses 12 ounces of berries for 3 dozen muffins. Which recipe has more berries per dozen muffins? How many ounces of berries would you need to make 60 muffins of this recipe? (1 dozen = 12 muffins)

Show your work. Use ratios, unit rates, models, and words to explain your thinking.

In Mom's recipe, the ratio of berries (ounces) to muffins (dozens) is 10 : 2. There are 5 ounces of berries per 1 dozen muffins. So the unit rate is 5.



In Grandma's recipe, the ratio of berries (ounces) to muffins (dozens) is 12 : 3. There are 4 ounces of berries per 1 dozen muffins. So the unit rate is 4.



The unit rate of 5 is greater than the unit rate of 4, so Mom's recipe has more berries per dozen.

60 muffins = 5 dozen muffins, so I would need 5 × 5 or 25 ounces of berries for 5 dozen muffins of Mom's recipe.

Where does the example . . .

- *answer both parts of the problem?*
- *use words to explain?*
- *use numbers to explain?*
- *use models to explain?*
- *give details?*

Name _____ Date _____

Ready® Mathematics

Lesson 2 Quiz

Solve the problems.

1 Patrick paid $4.00 for 5 organic peaches. How much did he pay per peach?

A $2.00 per peach

B $1.25 per peach

C $0.80 per peach

D $0.20 per peach

2 Zoe watched 6 episodes of her favorite online video series in 1.5 hours. She spent the same amount of time watching each episode. What is the unit rate of hours to episodes?

Show your work.

Answer: _____

3 Deena drove 585 miles in 9 hours. Kristen drove 605 miles in 11 hours. Who drove at a faster rate?

Fill in the blanks to compare the rates of speed for Deena and Kristen.

_____'s rate is _____ miles per hour faster than _____'s rate.

Lesson 6 Introduction

Understand Division with Fractions

Think It Through

What does it mean to divide a fraction by a fraction?

You know how to divide a whole number by a unit fraction. For example, you can think of 6 divided by $\frac{1}{4}$ as "how many one-fourths are there in 6?" Using a number line, you can divide 6 into fourths and count to see there are 24 fourths in 6.

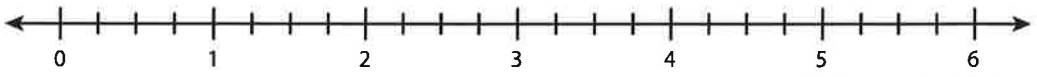
$6 \div \frac{1}{4} = 24$

You also learned that division and multiplication are inverse operations, and that you can solve division problems involving fractions by multiplying.

$6 \div \frac{1}{4}$ is the same as 6×4, or 24.

Think What does dividing a whole number by a fraction mean?

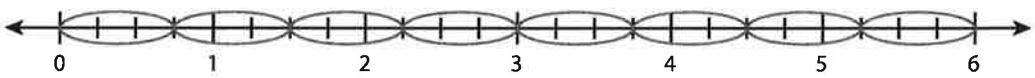
Madison has 6 yards of ribbon. She cuts the ribbon into pieces that are $\frac{3}{4}$ yard long. To figure out how many pieces Madison cut, think, "How many three-fourths are in 6?" You can draw the same number line to represent the 6 yards of ribbon and divide it into fourths.



You can circle three $\frac{1}{4}$ sections to represent $\frac{3}{4}$-yard pieces. You can see there are eight $\frac{3}{4}$-yard pieces in 6 yards.

Circle the multiplication expression that is the same as the division expression.

$6 \div \frac{3}{4} = 8$



$6 \times \frac{4}{3} = 8$

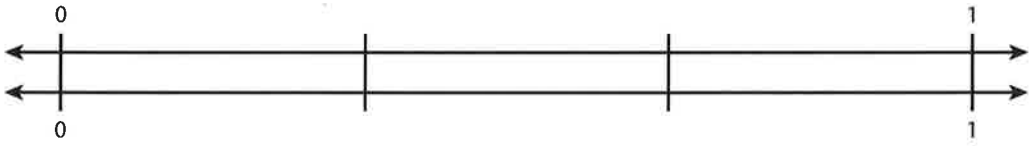
Think About Dividing by a Fraction

Let's Explore the Idea Explore dividing a fraction by a fraction with the problem below.

Kate has $\frac{2}{3}$ yards of fabric to make small flags. Each flag requires $\frac{1}{6}$ yard of fabric. How many flags can Kate make?

2 You need to find out how many _____ are in _____.

3 The number lines below are divided into thirds. Label $\frac{2}{3}$ on the top number line to represent $\frac{2}{3}$ yards of fabric.



4 Each flag requires $\frac{1}{6}$ yard of fabric. Divide the bottom number line into sixths to show how many sixths are in $\frac{2}{3}$.

5 Look at the bottom number line. How many sixths are there in $\frac{2}{3}$? _____

6 How many flags can Kate make? _____

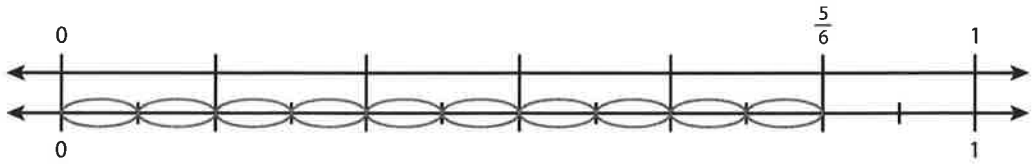
7 $\frac{2}{3} \div \frac{1}{6} =$ _____

8 $\frac{2}{3} \times$ _____ $= 4$

Connect

Talk through these problems as a class, then write your answers below.

17 **Explain** Look at the model below. Write the division equation that the model represents. Explain how to find the quotient using the model.



18 **Analyze** Sam said that $\frac{3}{2} \div \frac{1}{4}$ equals $\frac{3}{8}$. Draw a model and use words to explain why Sam's statement is not reasonable.

19 **Justify** Show that $2 \div \frac{4}{6} = 3$ by using a model. Explain why the answer is greater than the number you started with.

Understand Division with Fractions

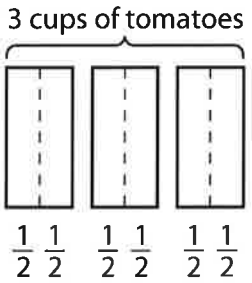
Name: _____

Prerequisite: How do you divide with unit fractions?

Study the example problem showing division of a whole number by a unit fraction. Then solve problems 1–7.

Example Problem

Karl puts $\frac{1}{2}$ cup of chopped tomatoes into each salad he makes. How many salads can he make with 3 cups of tomatoes?



The model represents the problem. You can use the model to write a division equation and a multiplication equation.

$3 \div \frac{1}{2} = 6$ $\quad\quad$ $3 \times 2 = 6$

Both equations show that Karl can make 6 salads with 3 cups of tomatoes.

1 Explain how the model represents $3 \div \frac{1}{2} = 6$.

_____

_____

2 Explain how the model represents $3 \times 2 = 6$.

_____

_____

3 Suppose Karl uses 5 cups of tomatoes. How many salads can he make? Write both a division equation and a multiplication equation to show your solution.

_____

_____

Vocabulary

unit fraction a fraction with a numerator of 1.

$\frac{1}{3}$, $\frac{1}{8}$, and $\frac{1}{12}$ are unit fractions.

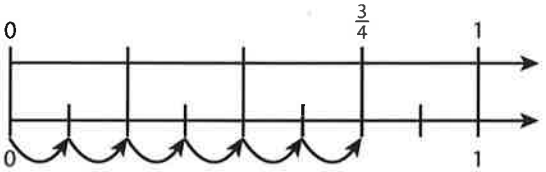
Name: _____

Divide by a Fraction

Study the example problem showing division of a fraction by a fraction. Then solve problems 1–10.

Example

Mr. Garcia has $\frac{3}{4}$ yard of ribbon to make badges for winners of the science fair. He uses $\frac{1}{8}$ yard of ribbon for each badge. How many badges can Mr. Garcia make?



6 eighths

Find the number of eighths in $\frac{3}{4}$. Use the number lines.

$\frac{3}{4} \div \frac{1}{8} = 6$

Mr. Garcia can make 6 badges.

1 What does $\frac{3}{4}$ on the top number line represent?

_____

2 What does each equal part on the bottom number line represent?

_____

3 How many eighths are in $\frac{3}{4}$? _____

4 Suppose Mr. Garcia is making badges using $\frac{3}{8}$ yard of ribbon for each badge. He starts with the same amount of ribbon, $\frac{3}{4}$ yard. How many badges can he make? Write a division equation that supports your answer.

_____

Name: _____

Reason and Write

Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

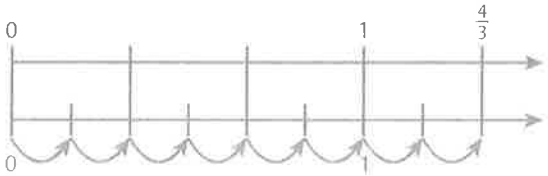
Example

Steve said that $\frac{4}{3} \div \frac{1}{6}$ equals $\frac{4}{6}$. How do you know without dividing whether Steve's statement is reasonable? Justify your answer by showing how to find the quotient.

Show your work. Use numbers, words, and models to explain your answer.

Steve's statement is not reasonable. The division $\frac{4}{3} \div \frac{1}{6}$ asks how many sixths are in $\frac{4}{3}$. $\frac{4}{3}$ is greater than 1, and there are 6 sixths in 1. So I know there are more than 6 sixths in $\frac{4}{3}$. That means the quotient must be greater than 1. It could not be a fraction less than 1, such as $\frac{4}{6}$.

I drew a number line model to find the quotient. The top number line is divided into thirds and shows $\frac{4}{3}$. The bottom number line is divided into sixths and shows that there are 8 sixths in $\frac{4}{3}$. So $\frac{4}{3} \div \frac{1}{6} = 8$.



8 sixths

Where does the example . . .

- use numbers to explain?
- use words to explain?
- use models to explain?
- give details?

Name _____ Date _____

Ready® Mathematics

Lesson 6 Quiz

Solve the problems.

1 Which problem does the number line model represent?

A $\frac{4}{6} \div \frac{1}{2}$

B $\frac{1}{2} \div \frac{4}{6}$

C $\frac{4}{6} \div \frac{2}{6}$

D $\frac{2}{6} \div \frac{4}{6}$

2 Fill in the blanks to explain how to solve $\frac{3}{5} \div \frac{1}{10}$.

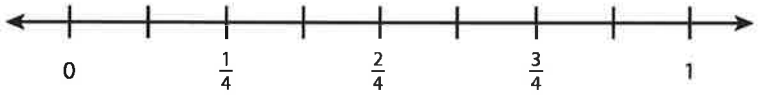
You can use a common denominator and rewrite $\frac{3}{5} \div \frac{1}{10}$ as _____ ÷ _____.

Then you can think, "How many _____s are in _____?"

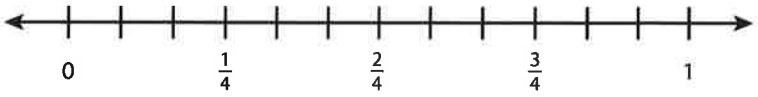
$\frac{3}{5} \div \frac{1}{10} =$ _____

3 Which number line would best model the quotient $\frac{3}{4} \div 6$?

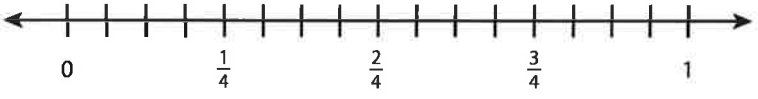
A



B



C



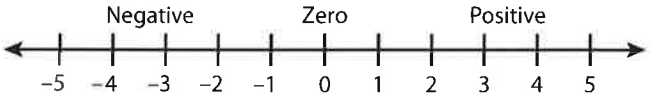
D

Understand Positive and Negative Numbers

Think It Through

What are positive and negative numbers?

Positive numbers are greater than 0 and located to the right of 0 on a number line. **Negative numbers** are less than 0 and located to the left of 0 on a number line. The number zero is neither positive nor negative.



Positive and negative numbers are sometimes called **signed numbers**.

a. Positive numbers can be written with or without a plus sign.

b. Negative numbers are always written with a negative sign.

When solving problems with positive and negative numbers, it is important to think about how far from 0 the number is and in what direction.

Think A thermometer shows positive and negative numbers.

Temperatures above 0 are positive.
Temperatures below 0 are negative.

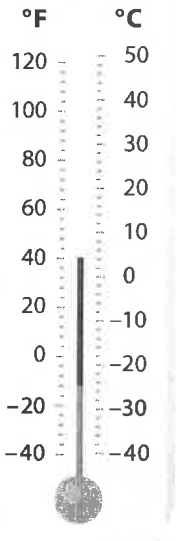
Look at the thermometer.

20°F is 20 degrees above 0°F.

−20°F is 20 degrees below 0°F.

−30°C is 30 degrees below 0°C.

30°C is 30 degrees above 0°C.



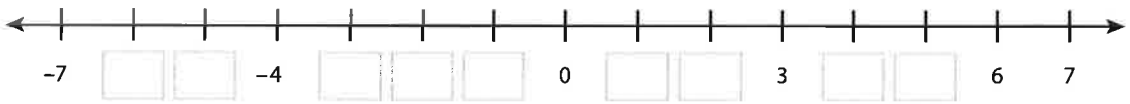
Circle the negative numbers labeled on the thermometer.

Think About

Identifying Positive and Negative Numbers

Let's Explore the Idea A number line can help you understand positive and negative numbers.

Jana and a friend are playing a game that shows a number line from −7 to 7. The game is played with 15 cards numbered with the integers from −7 to 7. Players draw a card from a pile. They earn points for correctly locating the number on the card on the number line and then identifying its opposite.



2 Finish labeling the number line.

3 Suppose Jana draws a card that shows −3. Draw a point at −3 on the number line.

4 What number is the opposite of −3? _____. Explain your reasoning.

_____

5 Jana's friend draws a card that shows a 0. Draw a point at 0. What is the opposite of 0?

_____ Explain. _____

6 The next card drawn is −6. How far from 0 is −6? _____ In which direction?

_____ Draw a point at −6.

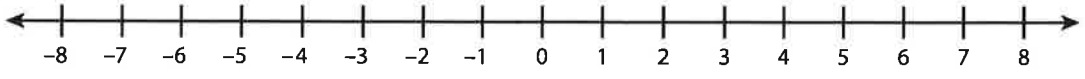
7 What number is the same distance from 0 as −6 but in the other direction? _____

8 Two numbers that are the same distance from 0 but on different sides of zero are called _____ numbers.

Now try this problem.

9 Graph each integer and its opposite on the number line below.

5 −1 4 −2



Connect

Talk through these problems as a class, then write your answers below.

14 Conclude What number is the opposite of the opposite of 5? What can you say about the opposite of the opposite of a number?

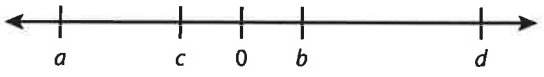
15 Interpret Positive and negative numbers can show an amount above or below zero. They can also be used to show an amount above or below a certain point.

Students at Taft Middle School have a goal of collecting 1,000 pounds of recycling materials each month. The following table shows their results over a 6-month time period. Complete the table. The first month is done for you.

Month	Pounds Collected	Compared to 1,000
January	985	−15
February	1,010	
March	995	
April	1,050	
May	975	
June	980	

←15 less than 1,000

16 Analyze Look at the number line below. The letters *a*, *b*, *c*, and *d* all represent integers.



Part A Which letters represent negative integers? How do you know?

Part B Which letters represent positive integers? How do you know?

Part C If *b* and *c* are the same distance from 0, how can you describe them?

Understand Positive and Negative Numbers

Name: _____

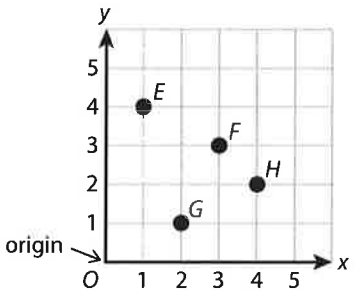
Prerequisite: How can you graph points on the coordinate plane?

Study the example showing how to name ordered pairs on the coordinate plane. Then solve problems 1–9.

Example

An ordered pair (x, y) describes the location of a point on the coordinate plane.

The first number in the ordered pair is the *x*-coordinate. It tells how many units the point is from the origin on the *x*-axis.



The second number is the *y*-coordinate. It tells how many units the point is from the origin on the *y*-axis.

The ordered pair for point *E* is (1, 4).

The ordered pair (0, 0) names the origin.

1 The *x*-coordinate of point *F* is ____ because it is ____ unit(s) to the right of the origin. The *y*-coordinate of point *F* is ____ because it is ____ unit(s) up from the origin. The ordered pair for point *F* is (____, ____).

2 Ray says that the ordered pair for point *G* is (1, 2). Is Ray correct? Why or why not?

_____

_____

3 Write the ordered pair for point *H*. Explain how you got your answer.

_____

_____

_____

4 Plot and label point *J* at (1, 2) on the coordinate plane.

Vocabulary

coordinate plane a two-dimensional space formed by two perpendicular number lines called axes.

origin the point (0, 0) where the *x*-axis and *y*-axis intersect.

ordered pair a pair of numbers (x, y) that describe the location of a point on the coordinate plane.

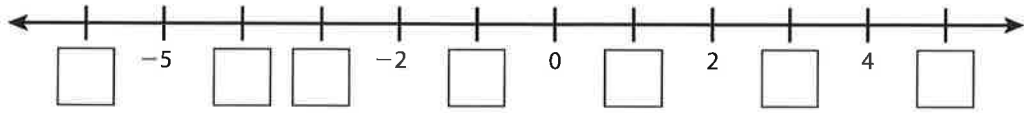
Name: _____

Identify Positive and Negative Numbers

Study the example showing positive and negative numbers on a number line. Then solve problems 1–10.

Example

Gareth is graphing some numbers and their opposites on the number line below. He has partially completed the number line as shown.



1. Fill in the missing numbers on Gareth's number line.

2. Choose a pair of numbers from the number line that you know are opposites. Explain how you know that the numbers are opposites.

_____

_____

3. Graph a point at 4 and at the opposite of 4 on the number line.

4. Mary says that the opposite of 0 is 0. Is she correct?

_____

_____

5. Name two numbers that are not integers but that are opposites. Explain how you know.

_____

_____

_____

Vocabulary

opposites numbers that are the same distance from 0 but on opposite sides of 0.

integers the set of whole numbers and their opposites.

Name: _____

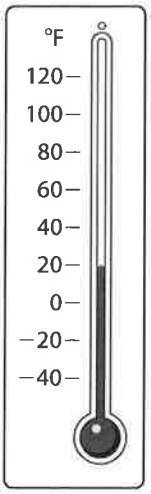
Reason and Write

Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

The thermometer shows the temperature outdoors at noon. The temperature at midnight was the opposite of the temperature at noon. Beth says that the temperature at midnight was −40°F. Is Beth correct? Explain your reasoning.

Show your work. Use a model, positive and negative numbers, and words to explain your answer.



The thermometer shows that the temperature at noon was 20°F.

I can draw a number line to model the temperature.

I know that the opposite of a number is the number that is the same distance from 0 in the opposite direction on a number line. So I can use the number line to see that the opposite of 20 is −20.

20 is 20 units to the right of 0 on the number line, and −20 is 20 units to the left of 0 on the number line.

So the temperature at midnight was −20°F. Beth was not correct. The temperature was not −40°F.

Where does the example . . .

- answer the question?
- use a model to explain?
- use positive and negative numbers to explain?
- use words to explain?

Name _____ Date _____

Ready® Mathematics

Lesson 12 Quiz

Solve the problems.

1 Hiroki wrote a number that is the opposite of −3. What number did Hiroki write?

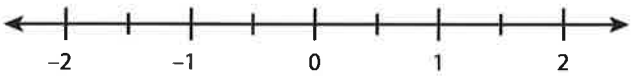
A $\frac{1}{3}$

B 3

C 0

D −3

2 Graph $\frac{1}{2}$ and its opposite on the number line below.



3 Write a positive or negative number to represent each situation.

a. you win $150 in a contest _____

b. a loss of 15 yards in a football game _____

c. 75 feet below sea level _____

d. 35 degrees above 0°F _____

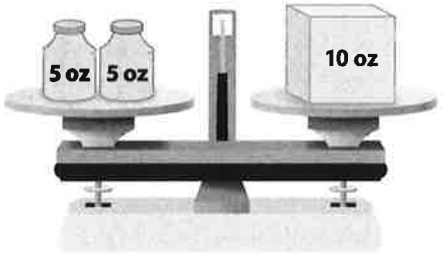
e. a debt of $24.75 _____

Lesson 18 Introduction
Understand Solutions to Equations

Think It Through

What does it mean to solve an equation?

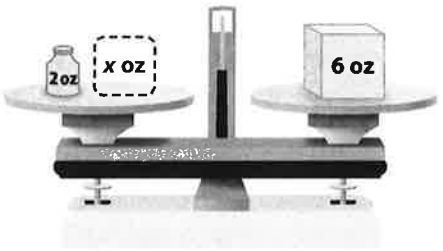
Have you ever seen a balance scale, or a balance? You put objects on both pans. If the objects' weights are the same, the balance beam below the pans is horizontal.



An equation is like a balance. A balance tells you two weights are equivalent. An equation tells you two expressions are equivalent.

Think **Solving an equation is like making the balance beam be horizontal.**

Imagine a balance like the one below.



Look at the picture. **Circle** the weights that must combine to equal 6 ounces.

The balance beam is horizontal, so the weight on each pan is the same. The pan on the right holds 6 ounces. The pan on the left must also hold 6 ounces.

How much weight do you have to add to 2 ounces to get a total of 6 ounces? 2 ounces and 4 ounces together are 6 ounces. Therefore, the unknown weight, x, must be 4 ounces.

Think About Writing and Solving Equations

Let's Explore the Idea Explore writing and solving equations with the problem below.

Andres buys 3 boxes of markers. Each box has the same number of markers. Andres now has 15 markers. Write and solve an equation to find how many markers are in one box.

2 Choose a variable to represent the number of markers in one box. _____

3 Write an expression to describe the total markers in 3 boxes. _____

4 How many markers does Andres have in all? _____

5 Write an equation that compares your answers from problems 3 and 4. _____

Use a bar model to help you solve the equation.

6 Draw a bar model to represent your equation from problem 5.

7 What number could you multiply by 3 to get 15? _____

8 What is the solution to your equation? _____

9 How many markers are in each box? _____

Now try these two problems.

10 At noon the temperature on Jessica's porch was 75° F. Then the temperature dropped d degrees. By midnight, the temperature on the porch was 63° F. Write an equation with an expression equivalent to the temperature at midnight. _____

11 By how many degrees did the temperature drop between noon and midnight? What is the solution to your equation?

_____

_____

Connect

Talk through these problems as a class, then write your answers below.

23 **Analyze** Explain why the solution to $3r = 2$ must be less than 1.

24 **Illustrate** Use a model to illustrate the equation $20 - x = 6$. Explain how you would solve the equation.

25 **Create** Write a real-world problem that you could represent with the equation $3 + x = 10$. Solve the equation to find the answer to your problem.

Understand Solutions to Equations

Name: _____

Prerequisite: How can you use the properties of operations to write equivalent expressions?

Study the example problem showing how to write equivalent expressions. Then solve problems 1–8.

Example

Gail plants 3 pots of roses and 2 pots of tulips. The number of flowers in each pot is the same. Write an expression for the total number of flowers. Simplify the expression to create an equivalent expression.

You can use math tiles to represent the problem.

Roses	f	f	f
Tulips	f	f	

Add to find the total number of flowers. An expression for the total number of flowers is $3f + 2f$. Then simplify.

$3f + 2f = f(3 + 2) = 5f$

1 Look at the example. What does f represent?

_____

2 Tell what each expression below represents.

a. $3f$ _____

b. $2f$ _____

c. $3f + 2f$ _____

3 How was the distributive property used to create an expression that is equivalent to $3f + 2f$?

_____

Vocabulary

like terms terms in an expression that have the same variable raised to the same power. Constants are like terms.
x and $-4x$
1 and 1.5
x^2 and $8x^2$

Name: _____

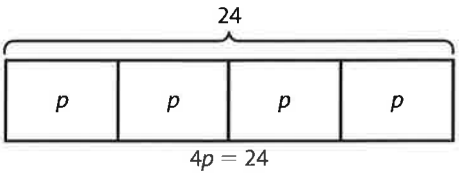
Writing and Solving Equations

Study the example problem showing how to write and solve equations. Then solve problems 1–9.

Example

Larry mows 4 lawns and earns $24. He is paid the same amount of money for each lawn. Write and solve an equation to find how much Larry is paid to mow one lawn.

You can draw a bar model to help you write and solve an equation that represents the problem. The equation $4p = 24$ represents the problem.



The equation is asking: What number could you multiply by 4 to get 24?

$4 \times 6 = 24$

Larry is paid $6 to mow one lawn.

1 What does p represent in the example?

_____

2 What does the expression $4p$ represent?

_____

3 What is the solution to the equation $4p = 24$?

$p =$ _____

4 Bev went to the grocery store with $45. She spent d dollars and came home with $21. Write and solve an equation to find how much Bev spent at the store.

Show your work.

Solution: _____

Name: _____

Reason and Write

Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

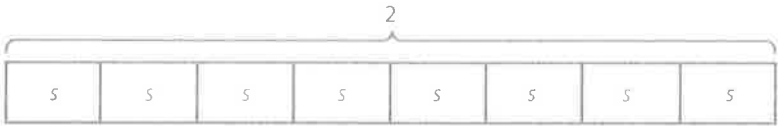
Example

Ling says that the solution to $8s = 2$ is that s must be greater than 1. Does Ling's solution make sense? Explain how you know whether or not Ling's solution makes sense without solving the equation. Then draw a model of the problem and solve the equation.

Show your work. Use numbers, words, and models to explain your answer.

Ling's solution does not make sense. The expression $8s$ means to multiply 8 by s. If I multiply 8 by 1, I get 8, which is greater than 2. So the solution must be less than 1.

I can draw a bar model to help me solve the problem.



The bar model shows that 8 times s gives me 2, so I can ask myself what number I could multiply by 8 to get 2. I know that the number is less than 1, so it must be a fraction.

The model shows that 8 bars represent 2, so 4 bars must represent 1. Therefore, each bar represents $\frac{1}{4}$.

The solution to $8s = 2$ is $s = \frac{1}{4}$.

Where does the example . . .

- *use numbers to explain?*
- *use words to explain?*
- *use models to explain?*
- *give details?*

Name _____ Date _____

***Ready*® Mathematics**

Lesson 18 Quiz

Solve the problems.

1 Which value makes each equation true?

Write a value for the variable that makes each equation true. Use the values in the box below. Not all values will be used.

$\frac{2}{7}$	$\frac{3}{7}$	$\frac{9}{7}$	$\frac{10}{7}$	43	71

$y + \frac{4}{7} = \frac{6}{7}$	$17 + b = 60$	$\frac{6}{7} = m + \frac{3}{7}$

2 Siera has 11.5 yards of yarn. She uses a certain amount for a project, leaving 5.25 yards of yarn. The equation $11.5 - x = 5.25$ represents this situation, where x is the amount of yarn Siera used for her project.

How much yarn did Siera use for her project?

A 5.25 yards

B 5.75 yards

C 6.25 yards

D 6.5 yards