Individual – 8$^{\text{th}}$ Grade

Instructions: Problems 1–10 are multiple choice and count towards your team score. Answer by putting the appropriate letter in the blank on the answer sheet. Problems 11–30 count towards your individual score but not your team score.

1. In which one of the following equations is the slope the greatest?

    (A) $20y = 1 - 8x$
    (B) $2x - y = 18$
    (C) $3x - 14 = 15y$
    (D) $31 = 4x + 15y$
    (E) $1 + 12x + 35y = 811$

2. A group of 40 students were working together and were on pace to finish making 960 cupcakes in 4 hours. 20 more students joined to help when the 40 students were 25% done. If they worked at the same pace as the original 40 students, how many more hours will it take for all the students working together to finish the rest of the cupcakes?

    (A) 2   **(B)** 2.5   **(C)** 3   **(D)** 3.5   **(E)** 4

3. When 3.498×10^{2018} is divided by 9, what is the remainder?

    (A) 0   **(B)** 1   **(C)** 6   **(D)** 7   **(E)** 8

4. In the game of basketball, you can score either 1, 2, or 3 points per basket, depending on the situation. Over the course of one game, Michael scored M points by making 10 baskets. How many unique values are possible for M?

    (A) 3   **(B)** 10   **(C)** 21   **(D)** 28   **(E)** 30

5. What is the sum of all the three-digit positive integers?

    (A) 247,050   **(B)** 247,500   **(C)** 493,450   **(D)** 494,100   **(E)** 494,550

6. The infamous sock drawer has 6 red socks, 6 blue socks, and 6 white socks. The lights just happened to stop working, so Dr. Seuss had to choose two socks out at random. What is the probability that he drew a pair of same-colored socks?

    (A) $\dfrac{1}{9}$ **(B)** $\dfrac{5}{17}$ **(C)** $\dfrac{1}{3}$ **(D)** $\dfrac{2}{3}$ **(E)** $\dfrac{15}{17}$

7. How many unique non-empty subsets of the set $\{2, 0, 18, 3, 22\}$ contain only positive integers?

    (A) 4   **(B)** 5   **(C)** 12   **(D)** 15   **(E)** 16

8. What is the sum of the number of unique diagonals in a pentagon, a hexagon, and a cube?

    (A) 15   **(B)** 17   **(C)** 23   **(D)** 30   **(E)** 60

9. Mr. Ly Brarian is trying to arrange four series of books onto a shelf: the "Secret Coders" series of four books, the "Bravelands" series of two books, the "Wrinkle in Time" series of five books, and the "Lemonade War" series of three books. If each of the books within a series must be placed together, but within each series, the books can be in any order, how many different ways can he arrange the books on the shelf?

    (A) 24   **(B)** 2,880   **(C)** 34,560   **(D)** 207,360   **(E)** 829,440

10. A classroom of 27 students each have an identical box of 64 uniquely-colored crayons. One at a time, students choose one of their crayons at random. How many students must choose a crayon in order for there to be at least a 30% probability that there is a matching crayon?

    (A) 4   **(B)** 8   **(C)** 9   **(D)** 19   **(E)** 20

11. The ratio of Jim's age to Bob's age is currently $3 : 2$. In 21 years, the ratio of Jim's age to Bob's age will become $9 : 7$. What will the sum of their ages be in 21 years?

12. On a rather difficult math test that tested how well students could add two-digit numbers, six high school students scored 40 points, eleven scored 50 points, two scored 80 points, and only one scored 100 points. If M is equal to the median score and A is equal to the average score, what is the positive difference between M and A? **Express your answer as a decimal.**

13. The diameter of the top of a cylindrical can is half the measure of its height. If the volume of the soup can is 500π units, what is the surface area of the just the label that wraps around the can? **Express your answer to the nearest tenth of a square unit.**

14. How many unique six-digit odd numbers can be formed by the digits 1, 1, 2, 3, 3, and 3?

15. The American Mathematics Contest (AMC) has 25 questions. Contestants score 6 points for each correct answer, 1.5 points if they leave an answer blank, and 0 points for an incorrect answer. If your goal is to score at least 80 points, what is the minimum number of questions you must answer correctly?

16. Three of the vertices of an isosceles trapezoid are at the points with coordinates $(-5, 3)$, $(-4, 1)$, $(-3, 5)$, and (A, B). What is the sum of A and B?

17. What is the product of 2018_9 and 1010_2? **Express your answer as a base-10 number.**

18. Three fair six-sided dice are rolled. What is the probability that the product of the three numbers rolled will be even? **Express your answer as a reduced fraction.**

19. In how many unique ways can the letters in the word "ARRANGER" be arranged?

20. In an arithmetic sequence, the first term is 2 and the second term is 5. Term number N of the sequence has a value of M, such that M is the largest two-digit number in the sequence. What is the value of $M + N$?

21. A rectangular garden at a school is being constructed such that there is a walking path that extends out 2 feet from each side of the garden. If the original garden was 60 feet in length and 20 feet in width, what is the area, in square feet, of just the walking path?

22. Inside a pouch, there are six different colors of marbles. You know the following information about the marbles:

   - There are a total of 36 marbles.;
   - There is a unique, odd number of each color of marble.;
   - There are three times as many red marbles as yellow marbles.;
   - There are more yellow marbles than blue marbles.;
   - There are more orange marbles than red marbles.;
   - There are more brown marbles than green marbles.;

   How many red, blue, and yellow marbles are there in total?

23. A rather disruptive boy spent 24 straight hours being obnoxious by burping every 4 minutes, sneezing every 15 minutes, and making inappropriate noises with his armpits every 6 minutes. Over the course of half of a day, how many times was he obnoxious in at least two ways at once?

24. Last summer, Harry went on a shopping spree at the wizarding store, buying 9 wands, 14 potions, and 2 owls. His total cost was \$1,130. A wand costs 140% more than a potion costs, and an owl costs 100% more than a wand costs. If Ron were to visit that same store and buy 2 wands, 3 potions, and 1 owl, how much money, as a number of dollars, would he have to spend?

25. What is the value of the expression $H + A + T$ based on the following system of equations?

$$106 = 20H + A + 8T$$
$$31 = 3H - 14A - T$$
$$0 = 8H - 3A - 5T$$

26. $(2.5 \times 10^{92653}) \times (4.4 \times 10^{8282}) = A \times 10^C$, such that $1 < A < 10$, and C is a positive integer. What is the value of $A + C$?

27. Two schools, each of which have the same number of students graduating, give out awards at the end of each year. At Margaret Mead High School, 50% of students receive the award. At Inglemoor High School, 30% of students receive the award. If there were a total of 80 students who received the award, how many students graduated from the two schools combined that year?

28. A teacher is having his 6 students line up to walk to the nearby park. In his class, there are 2 girls and 4 boys. If he were to randomly choose the order in which the students line up, what is the probability that the girls will be separated by at least one boy? **Express your answer as a reduced fraction.**

29. What is the sum of all three-digit numbers that can be created using only the digits 2, 5, and 8?

30. If the least common multiple of the positive integer M and 840 is $420 \times M$, what is the smallest possible value of M?