Geometry – 8th Grade

1. **2 points:** How many unique scalene triangles are there such that each triangle's sides have integral length and the triangle's perimeter is 15?

2. **2 points:** A wooden box, in the shape of a rectangular prism, is constructed with thin boards such that two of the boards each have an area of 48 square feet, two of the boards each have an area of 64 square feet, and two of the boards each have an area of 192 square feet. What is the greatest possible distance between two of the vertices of the box? **Express your answer to the nearest tenth of a foot.**

3. **2 points:** Suppose that there is a small shed with a triangular base, each side measuring 8 yards in length, with a pet cow attached to a 10-yard long rope on the outside of one vertex. What is the total area of the region in which the pet cow can "moo"ve around freely? **Express your answer to the nearest square yard.**

4. **3 points:** In a square of area 16, its two diagonals meet at the point with coordinates $(4, 2)$. One of the vertices of the square is the point with coordinates $(6, 0)$. If the y-intercepts of the two diagonals are at the points with coordinates (a, b) and (c, d), what is the value of $b + d$?

5. **3 points:** A rectangle has vertices at the points with coordinates $(-2, 4), (2, 4), (2, -4)$, and $(-2, -4)$. The rectangle is rotated 90 degrees about its center, and this rotated rectangle is called $\mathcal{R}$. A new rectangle is drawn with the following properties:

   - the new rectangle is similar to $\mathcal{R}$;
   - the new rectangle's sides are parallel to the sides of $\mathcal{R}$;
   - the new rectangle has the same center as $\mathcal{R}$;
   - the area of the new rectangle is double the area of $\mathcal{R}$.

   Given that the coordinates of the new rectangle are $(a, b), (c, d), (e, f)$, and (g, h), what is the value of $a + b + c + d + e + f + g + h$?

6. **3 points:** A right circular cone with a height of 10 units and a slant height of 26 units is chopped parallel to its base halfway between the base and the vertex, and the portion of the original cone below that is discarded. What is the volume of the new smaller cone? **Express your answer to the nearest tenth of a cubic unit.**

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7. **3 points:** Two go-karts are traveling around a figure-8 course designed as two circles that meet at one central point. The radius of each of the circles is $\frac{\pi}{4}$ miles. The first go-kart travels at a constant speed of 12 miles per hour, and the second go-kart travels at a constant speed of 10 miles per hour. Both start at the center of the figure-8 track and head off in the same direction. Assuming the two cars can travel side-by-side on the track, and a full lap is considered to be one trip around both circles, how many total combined laps will have been traveled by the two go-karts when they meet again at the center? **Express your answer as an exact decimal number of laps.**

8. **4 points:** A circle of radius 8 is centered at the point with coordinates $(0, 0)$. What is the area of the region within the circle bound by the inequalities $y > -x\sqrt{3}$ **and** $y > -x$? **Express your answer to the nearest square unit.**

9. **4 points:** Two congruent semicircles are drawn, one with the largest possible triangle inscribed within it and the other with the largest possible square inscribed within it. What is the ratio of the area of the triangle and the area of the square? Express your answer in the form $A : B$, where A and B are the least possible positive integers.

10. **4 points:** On the outside of each of the six sides of a regular hexagon, an equilateral triangle is placed, with the side of the hexagon as one of the triangle's sides. Then, another larger hexagon is formed by connecting the six new vertices in clockwise order. If the perimeter of the smaller hexagon is 36, what is the positive difference between the areas of the two hexagons? **Express your answer to the nearest tenth of a square unit.**