Potpourri – 8$^{\text{th}}$ Grade

1. **2 points:** What is the ones digit in the sum $1^{2017} + 2^{2017} + 3^{2017} + \ldots + 8^{2017} + 9^{2017}$?

2. **2 points:** When the base-10 number 2017 is written as a base-9 number and a base-8 number, what is the sum of all of the digits in those two numbers?

3. **2 points:** Three standard, fair six-sided dice are stacked directly on top of each other on a solid opaque surface. What is the greatest possible sum of the dots on the sides of the dice that are still visible?

4. **3 points:** Define an "almost prime" as a positive integer that is one less than a prime number. Let $\mathcal{S}$ be the set of one-digit "almost primes". Two or more distinct elements of $\mathcal{S}$ are chosen, and their product is computed. What is the sum of all such possible products? (Note: It is possible that two products will have the same *numerical* values, but will be composed of *different* sets of "almost primes" as their factors. These repeated products **should** be included in your sum.)

5. **3 points:** If the base-ten numbers from 1 to 44 inclusive are written as base-5 numbers, how many times would the digit 4 be written?

6. **3 points:** An *Egyptian fraction* is an expression of a finite sum of distinct unit fractions (that is, fractions whose numerators are 1). For example, one possible Egyptian fraction expression for $\frac{5}{8}$ is $\frac{1}{2} + \frac{1}{8}$. What is the smallest possible sum of the denominators in an Egyptian fraction expression for $\frac{4}{5}$?

7. **3 points:** A librarian has 6 unique books relating to mathematics that he wants to use when creating a display for the school's math competition. If he chooses at least one book, and uniqueness is determined by the books chosen to display (rather than by the order in which they are placed), in how many unique ways can the librarian create a display?

8. **4 points:** The product of N and 36960 is a perfect square. What is the smallest positive integer value of N?

9. **4 points:** How many two-digit positive integers have exactly two **distinct** prime factors?

10. **4 points:** A *cryptarithm* is a puzzle in which each distinct letter represents a unique digit and the same letter represents the same digit throughout the puzzle. They can also sometimes appear surprising or amusing, depending on the letters chosen. For example, one silly-looking cryptarithm is: $TWO \times TWO = THREE$. What is the five-digit number represented by the letters that form $THREE$?