RAVALLI COUNTY CURRICULUM CONSORTIUM

Mathematics

CURRICULUM & STANDARDS

Developed June 2013

Darby School District
Hamilton School District
Lone Rock School District
Stevensville School District
Victor School District

Adopted: ________________________

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In June 2013, the committee met as the Ravalli Curriculum Consortium concentrating on Mathematics curriculum development. The committee aligned the curriculum with the Montana Mathematics K-12 Content Standards and Practices by reviewing current instructional practices, current research, curriculum from each district, and curriculum guides from other states. The curriculum guide which follows is the culmination of this work.

Gratitude and appreciation are extended to the individual committee members for their hard work and dedication.

Mathematical Goals of the RCCC

We believe the Montana Mathematics K-12 Content Standards and Practices support a vision to enhance all students' relationships with mathematics wherein:

- All students will confidently communicate their critical and independent thinking throughout life pursuits.
- All students will receive facilitated, focused instruction to achieve a deeper understanding of concepts needed for success.
- All students will become mathematically literate citizens who can collaboratively solve problems and apply math in meaningful ways.

As a consortium, we recognize that all students have a right to the opportunity to receive a comprehensive mathematics education.

Implementation and Transition Plan

Each school district in the Ravalli County Curriculum Consortium has developed their own implementation and transition plan. This document is available through the RCCC website and at the district office of each school.

Mathematics Document Structure

This document is organized in the following manner:

1. **Introduction to the Montana Mathematics K-12 Content Standards and Practices**

2. **Individual Grade Level Standards and Practices for K-12**

   Each grade level includes the following grade specific resources:
   - **Curriculum Organizers from OPI (K-8)**
   - **Example Pacing Guides/Year Long Plans**
   - **Sample Unit Organizers**
   - **Lesson Plan Examples**
   - **Assessment Examples- Performance Tasks & Constructed Response**

Introduction

Toward greater focus and coherence

> *Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.*
>
> —Mathematics Learning in Early Childhood, National Research Council, 2009

> *The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.*
>
> — Ginsburg, Leinwand and Decker, 2009

> *Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.*
>
> — Ginsburg et al., 2005

> *There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.*
>
> — Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge.

It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

> *articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organized and generated within that discipline. This implies that “to be coherent,” a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties).* (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.

In addition, the “sequence of topics and performances” that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing “sequenced obstacles and challenges for students…absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.” In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time.

Understanding mathematics

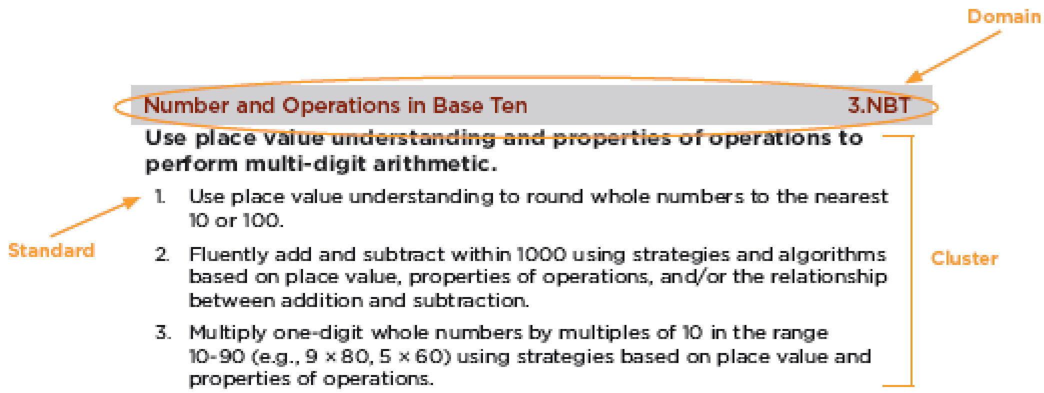
These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

The Standards begin with eight Standards for Mathematical Practice.

How to read the grade level standards

Standards define what students should understand and be able to do.
Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.



These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, “Students who already know A should next come to learn B.” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that these standards are not just promises to our children, but promises we intend to keep.

Montana Mathematics Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with long-standing importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. Building on the inherent problem-solving abilities of people over time, students can understand that mathematics is relevant when studied in a cultural context that applies to real-world situations and environments.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions within a cultural context, including those of Montana American Indians. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

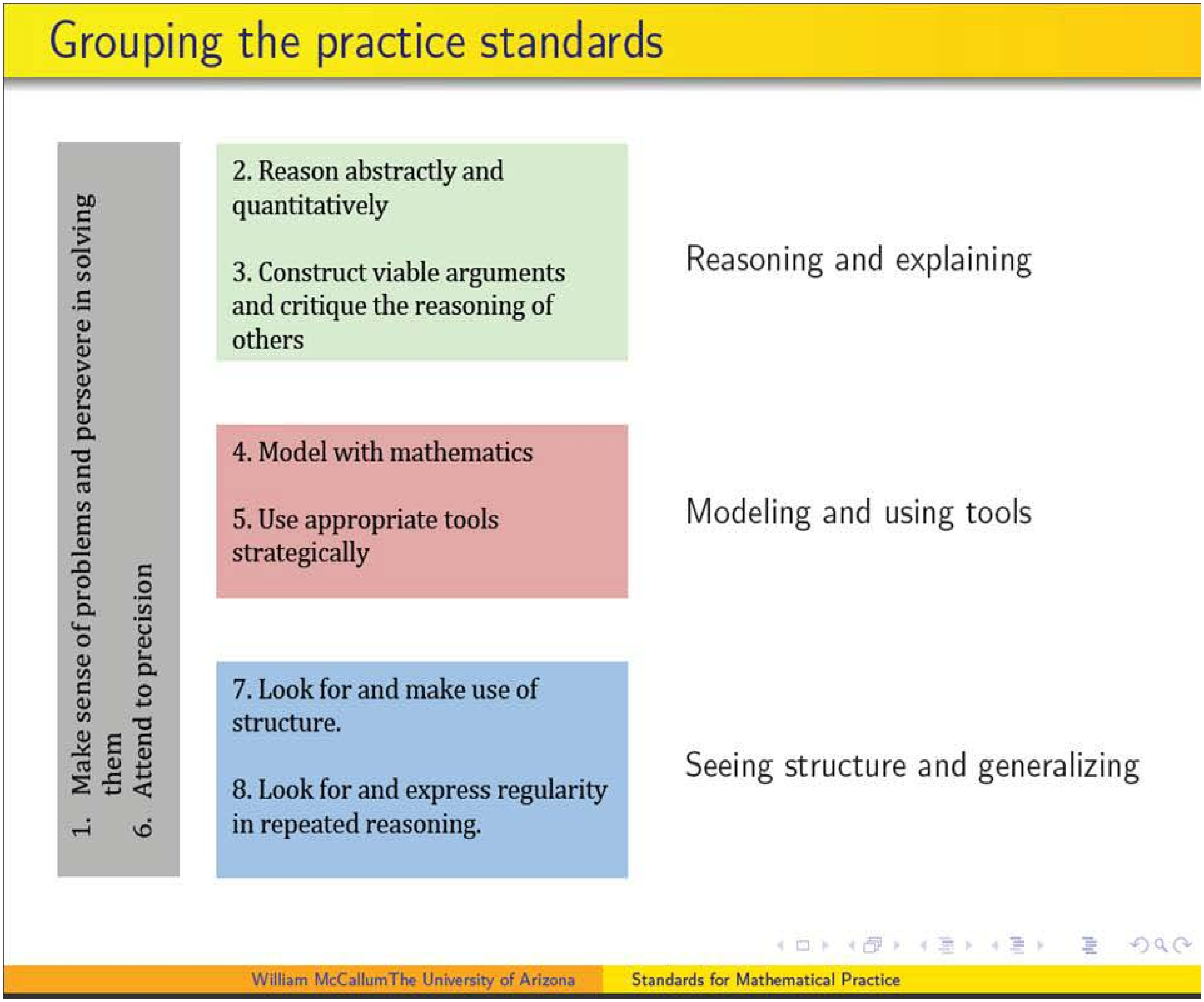
4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within a cultural context, including those of Montana American Indians. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.


Grouping the practice standards
1. Make sense of problems and persevere in solving them
6. Attend to precision
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
Reasoning and explaining
4. Model with mathematics
5. Use appropriate tools strategically
Modeling and using tools
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Seeing structure and generalizing
William McCallum The University of Arizona
Standards for Mathematical Practice

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Pursuant to Article X Sect 1(2) of the Constitution of the state of Montana and statutes §20-1-501 and §20-9-309 2(c) MCA, the implementation of these standards must incorporate the distinct and unique cultural heritage of Montana American Indians.

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or *–a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

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[3]Adapted from Wisconsin Department of Public Instruction, *op. cit.*
[4]To be more precise, this defines the *arithmetic mean*. one or more translations, reflections, and/or rotations. Rigid motions are here

assumed to preserve distances and angle measures.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[2]
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. **Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a – c = b – c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $–a < –b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Learning Progressions by Domain

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Kindergarten Overview

Domains	Counting and Cardinality	Operations and Algebraic Thinking	Number and Operations in Base Ten	Measurement and Data	Geometry
Clusters	• Know number names and the count sequence • Counting to tell the number of objects • Compare numbers	• Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	• Work with numbers 11 – 19 to gain foundations for place value	• Describe and compare measurable attributes • Classify objects and count the number of objects in each category	• Identify and describe shapes • Analyze, compare, create and compose shapes
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

In Kindergarten, instructional time should focus on two critical areas:

1. ***Representing and comparing whole numbers, initially with sets of objects***
    - Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.
2. ***Describing shapes and space***
    - Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

More learning time in Kindergarten should be devoted to number than to other topics.

First Grade Overview

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Measurement and Data	Geometry
Clusters	• Represent and solve problems involving addition and subtraction • Understand and apply properties of operations and the relationship between addition and subtraction • Add and subtract within 20 • Work with addition and subtraction equations	•Extend the counting sequence •Understand place value •Use place value understanding and properties of operations to add and subtract	• Measure lengths indirectly and by iterating length units • Tell and write time • Represent and interpret data	• Reason with shapes and their attributes
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

In Grade 1, instructional time should focus on four critical areas:

1. ***Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20***
   - Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
2. ***Developing understanding of whole number relationship and place value, including grouping in tens and ones***
   - Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
3. ***Developing understanding of linear measurement and measuring lengths as iterating length units***
   - Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. (Note: students should apply the principle of transitivity of measurement to make direct comparisons, but they need not use this technical term.)
4. ***Reasoning about attributes of, and composing and decomposing geometric shapes***
   - Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Second Grade Overview

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Measurement and Data	Geometry
Clusters	• Represent and solve problems involving addition and subtraction • Add and subtract within 20 • Work with equal groups of objects to gain foundations for multiplication	•Understand place value •Use place value understanding and properties of operations to add and subtract	• Measure and estimate lengths in standard units • Relate addition and subtraction to length • Work with time and money • Represent and interpret data	• Reason with shapes and their attributes
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

In Grade 2, instructional time should focus on four critical areas:

1. ***Extending understanding of base-ten notation***
    - Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
2. ***Building fluency with addition and subtraction***
    - Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
3. ***Using standard units of measure***
    - Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
4. ***Describing and analyzing shapes***
    - Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding attributes of two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Third Grade Overview

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Number & Operations: *Fractions*	Measurement and Data	Geometry
Clusters	• Represent and solve problems involving multiplication and division • Understand properties of multiplication and the relationship between multiplication and division • Multiply and divide within 100 • Solve problems involving the four operations, and identify and explain patterns in arithmetic	•Use place value understanding and properties of operations to perform multi-digit arithmetic	• Develop understanding of fractions as numbers	• Solve problems involving measurement and estimation of intervals of time, liquid, volumes and masses of objects • Represent and interpret data • Geometric measurement: understand concepts of area and relate area to multiplication and to addition • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures	• Reason with shapes and their attributes
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

In Grade 3, instructional time should focus on four critical areas (note: multiplication, division, and fractions are the most important developments):

1. ***Developing understanding of multiplication and division and strategies for multiplication and division within 100***
   - Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
2. ***Developing understanding of fractions, especially unit fractions (fractions with numerator 1)***
   - Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket; but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
3. ***Developing understanding of the structure of rectangular arrays and of area***
   - Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
4. ***Describing and analyzing two-dimensional shapes***
   - Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Fourth Grade Overview

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Number & Operations: *Fractions*	Measurement and Data	Geometry
Clusters	• Use the four operations with whole numbers to solve problems • Gain familiarity with factors and multiples • Generate and analyze patterns	•Generalize place value understanding for multi-digit whole numbers •Use place value understanding and properties of operations to perform multi-digit arithmetic	• Extend understanding of fraction equivalence and ordering • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers • Understand decimal notation for fractions, and compare decimal fractions	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit • Represent and interpret data • Geometric measurement: understand concepts of angle and measure angles	• Draw and identify lines and angles, and classify shapes by properties of their lines and angles
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.				

In Grade 4, instructional time should focus on three critical areas:

1. ***Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends***
   - Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
2. ***Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, multiplication of fractions by whole numbers***
   - Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
3. ***Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry***
   - Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Fifth Grade Overview

Domains	Operations and Algebraic Thinking	Number & Operations in Base Ten	Number & Operations: *Fractions*	Measurement and Data	Geometry
Clusters	• Write and interpret numerical expressions • Analyze patterns and relationships	•Understand the place value system •Perform operations with multi-digit whole numbers and with decimals to hundredths	• Use equivalent fractions as a strategy to add and subtract fractions • Apply and extend previous understandings of multiplication and division to multiply and divide fractions	• Convert like measurement units within a given measurement system • Represent and interpret data • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	• Graph points on the coordinate plane to solve real-world and mathematical problems • Classify two-dimensional figures into categories based on their properties
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

In Grade 5, instructional time should focus on three critical areas:

1. ***Developing fluency with addition and subtraction of fractions, developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)***
   - Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. ***Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operation***
   - Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
3. ***Developing understanding of volume***
   - Students recognize volume as an attribute of three-dimensional space. They understand that volume can be quantified by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to solve real world and mathematical problems.

Sixth Grade Overview

Domains	Ratios & Proportional Relationships	The Number System	Expressions and Equations	Geometry	Statistics and Probability
Clusters	• Understand ratio concepts and use ratio reasoning to solve problems	•Apply and extend previous understandings of multiplication and division to divide fractions by fractions •Compute fluently with multi-digit numbers and find common factors and multiples •Apply and extend previous understandings of numbers to the system of rational numbers	• Apply and extend previous understandings of arithmetic to algebraic expressions • Reason about and solve one-variable equations and inequalities • Represent and analyze quantitative relationships between dependent and independent variables	• Solve real-world and mathematical problems involving area, surface area, and volume	• Develop understanding of statistical variability • Summarize and describe distributions
Mathematical Practices	9. Make sense of problems and persevere in solving them. 10. Reason abstractly and quantitatively.	11. Construct viable arguments and critique the reasoning of others. 12. Model with mathematics.	13. Use appropriate tools strategically. 14. Attend to precision.	15. Look for and make use of structure. 16. Look for and express regularity in repeated reasoning.	

In Grade 6, instructional time should focus on four critical areas:

1. ***Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems***
   - Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.
2. ***Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers***
   - Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
3. ***Writing, interpreting, and using expressions and equations***
   - Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.
4. ***Developing understanding of statistical thinking***
   - Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Sixth Grade Overview (continued)

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Seventh Grade Overview

Domains	Ratios & Proportional Relationships	The Number System	Expressions and Equations	Geometry	Statistics and Probability
Clusters	• Analyze proportional relationships and use them to solve real-world and mathematical problems	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers	• Use properties of operations to generate equivalent expressions • Solve real-life and mathematical problems using numerical and algebraic expressions and equations	• Draw, construct and describe geometrical figures and describe the relationships between them • Solve real-life and mathematical problems involving angle measure, area, surface and volume	• Use random sampling to draw inferences about a population • Draw informal comparative inferences about two populations • Investigate chance processes and develop, use and evaluate probability models
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.				

In Grade 7, instructional time should focus on four critical areas:

1. ***Developing understanding of and applying proportional relationships***
   - Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.
2. ***Developing understanding of operations with rational numbers and working with expressions and linear equations***
   - Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.
3. ***Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume***
   - Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.
4. ***Drawing inferences about populations based on samples***
   - Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Eighth Grade Overview

Domains	The Number System	Expressions and Equations	Functions	Geometry	Statistics & Probability
Clusters	• Know that there are numbers that are not rational, and approximate them by rational numbers	• Work with radicals and integer exponents • Understand the connections between proportional relationships, lines, and linear equations • Analyze and solve linear equations and pairs of simultaneous linear equations	• Define, evaluate, and compare functions • Use functions to model relationships between quantities	• Understand congruence and similarity using physical models, transparencies, or geometry software • Understand and apply the Pythagorean Theorem • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres	• Investigate patterns of association in bivariate data
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

In Grade 8, instructional time should focus on three critical areas:

1. ***Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations***
   - Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.
   - Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.
2. ***Grasping the concept of a function and using functions to describe quantitative relationships***
   - Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.
3. ***Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem***
   - Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

HS Conceptual Category: Number and Quantity

Domains	The Real Number System	Quantities	The Complex Number System	Vector and Matrix Quantities
Clusters	• Extend the properties of exponents to rational exponents • Use properties of rational and irrational numbers.	• Reason quantitatively and use units to solve problems	• Perform arithmetic operations with complex Numbers • Represent complex numbers and their operations on the complex plane • Use complex numbers in polynomial identities and equations	• Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers .With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (51/3)3 should be 5(1/3)3 = 51 = 5 and that 51/3 should be the cube root of 5. Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

The Real Number System N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that $i^2 = –1$, and every complex number has the form $a + bi$ with *a* and *b* real.
2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.*
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.*
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities — N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
   d.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   b. Compute the magnitude of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|***v***. Compute the direction of *c**v*** knowing that when |*c*|***v*** ≠ 0, the direction of *c**v*** is either along ***v*** (for $c > 0$) or against ***v*** (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2 × 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.

HS Conceptual Category: Algebra

Domains	Seeing Structure in Expressions	Arithmetic with Polynomials and Rational Expressions	Creating Equations	Reasoning with Equations and Inequalities
Clusters	• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems	• Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions	• Create equations that describe numbers or relationships	• Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$shows that adding a tax is the same as multiplying the price by a constant factor. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x2 + 2 = 0$are complex numbers, not real numbers. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b1+b2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Seeing Structure in Expressions — A-SSE

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.★
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

Arithmetic with Polynomials and Rational Expressions — A-APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations	**A-CED**

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities	**A-REI**

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

[1]The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

HS Conceptual Category: Functions

Domains	Interpreting Functions	Building Functions	Linear, Quadratic, and Exponential Models	Trigonometric Functions
Clusters	• Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations	• Build a function that models a relationship between two quantities • Build new functions from existing functions	• Construct and compare linear, quadratic, and exponential models and solve problems • Interpret expressions for functions in terms of the situation they model	• Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions • Prove and apply trigonometric identities
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T. The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties. Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
    c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
    3. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.★

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x–1)$ *for* $x \neq 1$.
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g. science, history, and culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities.

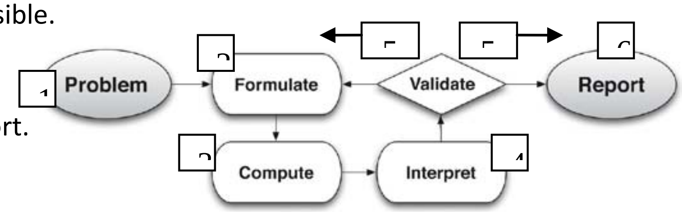
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

HS Conceptual Category: Modeling

Domains **Clusters**	*Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (☆).*
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. *4. Model with mathematics.* 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.



In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves

(1) identifying variables in the situation and selecting those that represent essential features,

(2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,

(3) analyzing and performing operations on these relationships to draw conclusions,

(4) interpreting the results of the mathematics in terms of the original situation,

(5a) validating the conclusions by comparing them with the situation, and then either (5b) improving the model or, if is acceptable,

(6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

HS Conceptual Category: Geometry

Domains	Congruence	Similarity, Right Triangles, and Trigonometry	Circles	Expressing Geometric Properties with Equations	Geometric Measurement and Dimension	Modeling with Geometry
Clusters	• Experiment with transformations in the plane Understand congruence in terms of rigid motions • Prove geometric theorems • Make geometric constructions	• Understand similarity in terms of similarity transformations • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles • Apply trigonometry to general triangles	• Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles	• Translate between the geometric description and the equation for a conic section • Use coordinates to prove simple geometric theorems algebraically	• Explain volume formulas and use them to solve problems • Visualize relationships between two dimensional and three-dimensional objects	• Apply geometric concepts in modeling situations
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.		5. Use appropriate tools strategically. 6. Attend to precision.		7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Congruence G-CO

Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions.

12. Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry — G-SRT

Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor:
    a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles.

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles G-C

Understand and apply theorems about circles.

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci and directrices.

Use coordinates to prove simple geometric theorems algebraically.

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Geometric Measurement and Dimension — G-GMD

Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry — G-MG

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

HS Conceptual Category: Statistics and Probability

Domains	Interpreting Categorical and Quantitative Data	Making Inferences and Justifying Conclusions	Conditional Probability and the Rules of Probability	Using Probability to Make Decisions
Clusters	• Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables Interpret linear models	• Understand and evaluate random processes underlying statistical experiments • Make inferences and justify conclusions from sample surveys, experiments and observational studies	• Understand independence and conditional probability and use them to interpret data • Use the rules of probability to compute probabilities of compound events in a uniform probability model	• Calculate expected values and use them to solve problems • Use probability to evaluate outcomes of decisions
Mathematical Practices	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range).Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Interpreting Categorical and Quantitative Data — S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables, and Montana American Indian data sources to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions — S-IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data including information from Montana American Indian data sources when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
    a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
    b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Notes on Courses and Transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the United States today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

KINDERGARTEN

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Kindergarten Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
K.MP.1. Make sense of problems and persevere in solving them.	In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
K.MP.2. Reason abstractly and quantitatively.	Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
K.MP.3. Construct viable arguments and critique the reasoning of others.	Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
K.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
K.MP.5. Use appropriate tools strategically.	Younger students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side.
K.MP.6. Attend to precision.	As kindergarteners begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.
K.MP.7. Look for and make use of structure.	Younger students begin to discern a pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that $3 + 2 = 5$ and $2 + 3 = 5$.
K.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). In addition, students continually check their work by asking themselves, "Does this make sense?"

Explanations and Examples Grade K
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Kindergarten Content Standards

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade K Overview

Counting and Cardinality
- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten
- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data
- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry
- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Counting and Cardinality — K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   - **a.** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians.
   - b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   - c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects from a variety of cultural contexts, including those of Montana American Indians.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking — K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems from a variety of cultural contexts, including those of Montana American Indians, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

Number and Operations in Base Ten K.NBT

Work with numbers 11-19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

3. Classify objects from a variety of cultural contexts, including those of Montana American Indians, into given categories; count the numbers of objects in each category and sort the categories by count.[3]

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects, including those of Montana American Indians, in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world from a variety of cultural contexts, including those of Montana American Indians, by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

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[1] Include groups with up to ten objects.
[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
[3] Limit category counts to be less than or equal to 10.

KINDERGARTEN

Domain	Cluster	Code	Common Core State Standard
Counting and Cardinality	Know number names and the count sequence.	K.CC.1	Count to 100 by ones and by tens.
		K.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
		K.CC.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
	Count to tell the number of objects.	K.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
		K.CC.5	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle; or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects from a variety of cultural contexts, including those of Montana American Indians.
	Compare numbers.	K.CC.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)
		K.CC.7	Compare two numbers between 1 and 10 presented as written numerals.
Operations and Algebraic Thinking	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings (drawings need not show details, but should show the mathematics in the problem), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
		K.OA.2	Solve addition and subtraction word problems from a variety of cultural contexts, including those of Montana American Indians, and add and subtract within 10, e.g., by using objects or drawings to represent the problem .
		K.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
		K.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
		K.OA.5	Fluently add and subtract within 5.
Number and Operations in Base Ten	Work with numbers 11-19 to gain foundations for place value.	K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

KINDERGARTEN

Domain	Cluster	Code	Common Core State Standard
Measurement and Data	Describe and compare measurable attributes.	K.MD.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
		K.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
	Classify objects and count the number of objects in each category.	K.MD.3	Classify objects from a variety of cultural contexts, including those of Montana American Indians, into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)
Geometry	Identify and describe shapes	K.G.1	Describe objects, including those of Montana American Indians, in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
		K.G.2	Correctly name shapes regardless of their orientations or overall size.
		K.G.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
	Analyze, compare, create, and compose shapes.	K.G.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
		K.G.5	Model shapes in the world from a variety of cultural contexts, including those of Montana American Indians, by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
		K.G.6	Compose simple shapes to form larger shapes. For example, "can you join these two triangles with full sides touching to make a rectangle?"

Getting Ready
Every Montana Student
College and Career

Montana Common Core
Standards and Assessments

Montana Curriculum Organizer

Grade K

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

Montana Curriculum Organizer: Grade K Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

Montana Curriculum Organizer: Grade K Mathematics

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HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade-level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 17 consist of tables organized into learning progressions that can function as units. The table for each learning progression, unit, includes: 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

<table>
<tr><th>LEARNING PROGRESSION</th><th>STANDARDS IN LEARNING PROGRESSION</th></tr>
<tr><td>Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.</td><td>Standards covered in this learning progression.</td></tr>
<tr><th colspan="2">UNDERSTAND:</th></tr>
<tr><td colspan="2">What students need to understand by the end of this learning progression.</td></tr>
<tr><th>KNOW:</th><th>DO:</th></tr>
<tr><td>What students need to know by the end of this learning progression.</td><td>What students need to be able to do by the end of this learning progression, organized by cluster and standard.</td></tr>
<tr><th colspan="2">KEY TERMS FOR THIS PROGRESSION:</th></tr>
<tr><td colspan="2">Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.</td></tr>
<tr><th colspan="2">INSTRUCTIONAL STRATEGIES AND RESOURCES:</th></tr>
<tr><td colspan="2">Cluster: Title
Strategies for this cluster

Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><td colspan="2">Cluster: Title
Strategies for this cluster

Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><th colspan="2">CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:</th></tr>
<tr><td colspan="2">Standards that connect to this learning progression are listed here, organized by cluster.</td></tr>
<tr><th colspan="2">STANDARDS FOR MATHEMATICAL PRACTICE:</th></tr>
<tr><td colspan="2">A quick reference guide to the eight standards for mathematical practice is listed here.</td></tr>
</table>

Montana Curriculum Organizer: Grade K Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Kindergarten Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
K.MP.1. Make sense of problems and persevere in solving them.	In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
K.MP.2. Reason abstractly and quantitatively.	Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
K.MP.3. Construct viable arguments and critique the reasoning of others.	Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
K.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
K.MP.5. Use appropriate tools strategically.	Younger students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side.
K.MP.6. Attend to precision.	As kindergarteners begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.
K.MP.7. Look for and make use of structure.	Younger students begin to discern a pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that 3 + 2 = 5 and 2 + 3 = 5.
K.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). In addition, students continually check their work by asking themselves, "Does this make sense?"
CRITICAL AREAS FOR GRADE K MATH	
In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.	

Montana Curriculum Organizer: Grade K Mathematics

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Counting & Cardinality	K.CC.1, K.CC.2, K.CC.3, K.CC.4a-c, K.CC.5, K.CC.6, K.CC.7

UNDERSTAND:

Counting strategies can be used to determine the number of objects.
Understand the relationship between numbers and quantities.

KNOW:

The number-word sequence, combined with the order inherent in the natural numbers, can be used as a foundation for counting.

The counting sequence by ones and tens.

Ten different digits can be used and sequenced to express any whole number (In K, write numbers 0-20).

The last number named when counting said tells the number of objects counted (cardinality).

The number of objects is the same regardless of their arrangement or the order in which they were counted.

When comparing two sets of objects or numbers, the one with the largest quantity is more or smallest quantity is less.

DO:

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one, and only one, number name and each number name with one, and only one, object from a variety of cultural contexts, including those of Montana American Indians.
b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.

K.CC.5 Count to answer "How many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle; or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects from a variety of cultural contexts, including those of Montana American Indians.

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (by using matching and counting strategies).[1]

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

KEY TERMS FOR THIS PROGRESSION:

Add, Compare, Count, Equal to, Greater than, Less, Less than, More, Number, Subtract

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Know number names and the count sequence.

The Counting and Cardinality domain in Kindergarten contains standard statements that are connected to one another. Examine the three samples in this domain at the same time to obtain a more holistic view of the content.

Provide settings that connect mathematical language and symbols to the everyday lives of kindergarteners. Support students' ability to make meaning and mathematize the real world. Help them see patterns, make connections and provide repeated experiences that give students time and opportunities to develop understandings and increase fluency. Encourage students to explain their reasoning by asking probing questions such as "How do you know?"

Students view counting as a mechanism used to land on a number. Young students mimic counting often with initial lack of purpose or meaning. Coordinating the number words, touching or moving objects in a one-to-one correspondence may

[1] Include groups with up to 10 objects.

be little more than a matching activity. However, saying number words as a chant or a rote procedure plays a part in students constructing meaning for the conceptual idea of counting. They will learn how to count before they understand cardinality (i.e. that the last count word is the amount of the set).

Counting on or counting from a given number conflicts with the learned strategy of counting from the beginning. In order to be successful in counting on, students must understand cardinality. Students often merge or separate two groups of objects and then re-count from the beginning to determine the final number of objects represented. For these students, counting is still a rote skill or the benefits of counting on have not been realized. Games that require students to add on to a previous count to reach a goal number encourage developing this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. These concepts emerge over time and cannot be forced.

Like counting to 100 by either ones or tens, writing numbers from 0 to 20 is a rote process. Initially, students mimic the actual formation of the written numerals while also assigning it a name. Over time, children create the understanding that number symbols signify the meaning of counting. Numerals are used to communicate across cultures, and through time, or a certain meaning. Numbers have meaning when children can see mental images of the number symbols and use those images with which to think. Practice count words and written numerals paired with pictures, representations of objects, and objects that represent quantities within the context of life experiences for kindergarteners. For example, dot cards, dominoes and number cubes all create different mental images for relating quantity to number words and numerals.

One way students can learn the left to right orientation of numbers is to use a finger to write numbers in air (sky writing). Children will see mathematics as something that is alive and that they are involved.

Students should study and write numbers 0 to 20 in this order: numbers 1 to 9, the number 0, then numbers 10 to 20. They need to know that 0 is the number items left after all items in a set are taken away. Do not accept “none” as the answer to “How many items are left?” for this situation.

Instructional Resources/Tools
Board games that require counting

National Research Council. *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*. Washington, DC: The National Academies Press, 2009.

Winnipeg School Division. 2005-2006. *Numeracy Project: Dot Card and Ten Frame Activities*. (pp. 1-6, 12-17).

Cluster: Count to tell the number of objects.
One of the first major concepts in a student’s mathematical development is cardinality. Cardinality, knowing that the number word said tells the quantity you have and that the number you end on when counting, represents the entire amount counted. The big idea is that number means amount and, no matter how you arrange and rearrange the items, the amount is the same. Until this concept is developed, counting is merely a routine procedure done when a number is needed. To determine if students have the cardinality rule, listen to their responses when you discuss counting tasks with them. For example, ask, “How many are here?” The student counts correctly and says that there are seven. Then ask, “Are there seven?” Students may count or hesitate if they have not developed cardinality. Students with cardinality may emphasize the last count or explain that there are seven because they counted them. These students can now use counting to find a matching set.

Students develop the understanding of counting and cardinality from experience. Almost any activity or game that engages children in counting and comparing quantities, such as board games, will encourage the development of cardinality. Frequent opportunities to use and discuss counting as a means of solving problems relevant to kindergarteners is more beneficial than repeating the same routine day after day. For example, ask students questions that can be answered by counting up to 20 items before they change and as they change locations throughout the school building.

As students develop meaning for numerals, they also compare numerals to the quantities they represent. The models that can represent numbers, such as dot cards and dominoes, become tools for such comparisons. Students can concretely, pictorially or mentally look for similarities and differences in the representations of numbers. They begin to “see” the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities that are one larger. In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will build students' ability to break numbers into parts.

Provide a variety of experiences in which students connect count words or number words to the numerals that represent the quantities. Students will arrive at an understanding of a number when they acquire cardinality and can connect a number with the numerals and the number word for the quantity they all represent.

Instructional Resources/Tools
Winnipeg School Division. 2005-2006. *Numeracy Project: Dot Card and Ten Frame Activities*. (pp. 1-6, 12-17).

Cluster: Compare numbers.
As children develop meaning for numerals, they also compare these numerals to the quantities represented and their number words. The modeling numbers with manipulatives such as dot cards and five- and ten-frames become tools for such comparisons. Children can look for similarities and differences in these different representations of numbers. They begin to "see" the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities where one is larger. In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will greatly impact children's ability to break numbers into parts.

Children demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a "proof" by demonstrating a one-to-one match, by counting again or other similar means (concretely or pictorially) that makes sense. An ultimate level of understanding is reached when children can compare two numbers from 1 to10 represented as written numerals without counting.

Students need to explain their reasoning when they determine whether a number is greater than, less than, or equal to another number. Teachers need to ask probing questions such as "How do you know?" to elicit their thinking. For students, these comparisons increase in difficulty, from greater than to less than to equal. It is easier for students to identify differences than to find similarities.

Instructional Resources/Tools
Board games

Winnipeg School Division. 2005-2006. *Numeracy Project: Dot Card and Ten Frame Activities.*

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Describe and compare measurable attributes. (K.MD.1, K.MD.2)
Classify objects and count the number of objects in each category. (K.MD.3)
Analyze, compare, create, and compose shapes. (K.G.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade K Mathematics

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Operations & Algebraic Thinking (Addition & Subtraction)	K.OA.1, K.OA.2, K.OA.3, K.OA.4, K.OA.5

UNDERSTAND:

Understand addition as putting together and adding to.
Understand subtraction as taking apart and taking from.
Numbers can be decomposed into place value parts and represented in multiple ways.

KNOW:	DO:
There are different ways to show addition and subtraction solutions. Objects or drawings can be used to solve addition and subtraction word problems. Record equations to represent addition or subtraction problems. Addition and subtraction facts to 5. Quantities represented by numbers can be composed and decomposed into part/whole relationships (by place value up to 20 in K). Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.	***Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.*** **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.2** Solve addition and subtraction word problems from a variety of cultural contexts, including those of Montana American Indians, and add and subtract within 10 (e.g., by using objects or drawings to represent the problem). **K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., by using objects or drawings), and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **K.OA.4** For any number from 1 to 9, find the number that makes 10 when added to the given number (e.g., by using objects or drawings), and record the answer with a drawing or equation. **K.OA.5** Fluently add and subtract within 5.

KEY TERMS FOR THIS PROGRESSION:

Add, Addition, Equation, Subtract, Subtraction

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
Provide contextual situations for addition and subtraction that relate to the everyday lives of kindergarteners. A variety of situations can be found in children's literature books. Students then model the addition and subtraction using a variety of representations such as drawings, sounds, acting out situations, verbal explanations and numerical expressions. Manipulatives, like two-color counters, clothespins on hangers, connecting cubes and stickers can also be used for modeling these operations. Kindergarten students should see addition and subtraction equations written by the teacher. Although students might struggle at first, teachers should encourage them to try writing the equations. Students' writing of equations in Kindergarten is encouraged, but it is not required.

Create written addition or subtraction problems with sums and differences less than or equal to 10 using the numbers 0 to 10 and Table 1 on page 72 in the *Montana Common Core Standards for School Mathematics Grade-Band* for guidance. It is important to use a problem context that is relevant to kindergarteners. After the teacher reads the problem, students choose their own method to model the problem and find a solution. Students discuss their solution strategies while the teacher represents the situation with an equation written under the problem. The equation should be written by listing the numbers and symbols for the unknown quantities in the order that follows the meaning of the situation. The teacher and students should use the words *equal* and *is the same as* interchangeably.

Have students decompose numbers less than or equal to 5 during a variety of experiences to promote their fluency with sums and differences less than or equal to 5 that result from using the numbers 0 to 5. For example, ask students to use different models to decompose 5 and record their work with drawings or equations. Next, have students decompose 6, 7, 8, 9, and 10 in a similar fashion. As they come to understand the role and meaning of arithmetic operations in number systems, students gain computational fluency, using efficient and accurate methods for computing.

The teacher can use back mapping and scaffolding to teach students who show a need for more help with counting. For instance, ask students to build a tower of 5 using 2 green and 3 blue linking cubes while you discuss composing and decomposing 5. Have them identify and compare other ways to make a tower of 5. Repeat the activity for towers of 7 and 9. Help students use counting as they explore ways to compose 7 and 9.

Instructional Resources/Tools

Colored cubes
Linking cubes
Part-Part-Whole Mat and objects to model problem situations and find solutions. *(The mat is divided into three sections and the labels for the sections in order are Part, Part, and Whole.)*

Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band*: Table 1 on page 72 illustrates 12 addition and subtraction problem situations.

National Council of Teachers of Mathematics. 2000-2012.

- Exploring adding with sets: This lesson builds on the previous two lessons in the unit *Do It with Dominoes* and encourages students to explore another model for addition, the set model.
- Links Away: In this unit (lessons 2, 4, 5, and 7), students explore models of subtraction (counting, sets, balanced equations, and inverse of addition) and the relation between addition and subtraction using links. Students also write story problems in which subtraction is required.
- More and More Buttons: In this lesson, students use buttons to create, model, and record addition sentences.

Winnipeg School Division. 2005-2006. *Numeracy Project: Dot Card and Ten Frame Activities*

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Work with numbers 11-19 to gain foundations for place value. (K.NBT.1)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade K Mathematics

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Place Value	K.NBT.1

UNDERSTAND:

Teen numbers can be decomposed into place value parts and represented in multiple ways.

KNOW:	DO:
Quantities represented by numbers can be composed and decomposed into part/whole relationships (by place value up to 20 in K). The base-ten number system allows for a new place-value unit by grouping ten of the previous place-value units (and this process can be iterated to obtain larger and larger place-value units). The value of a digit in a written numeral depends on its place, or position, in a number. Each composition or decomposition can be recorded by a drawing or equation (e.g., 18 = 10 + 8). Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.	***Work with numbers 11-19 to gain foundations for place value.*** **K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones (e.g., by using objects or drawings), and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

KEY TERMS FOR THIS PROGRESSION:

Add, Subtract, Ones, Tens, Eleven, Twelve, Thirteen, Fourteen, Fifteen, Sixteen, Seventeen, Eighteen, Nineteen

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Work with numbers 11–19 to gain foundations for place value.

Kindergarteners need to understand the idea of *a ten* so they can develop the strategy of adding onto 10 to add within 20 in Grade 1. Students need to construct their own base-ten ideas about quantities and their symbols by connecting to counting by ones. They should use a variety of manipulatives to model and connect equivalent representations for the numbers 11 to19. For instance, to represent 13, students can count by ones and show 13 beans. They can anchor to five and show one group of 5 beans and 8 beans or anchor to ten and show one group of 10 beans and 3 beans. Students need to eventually see *a ten* as different from 10 ones.

After the students are familiar with counting up to 19 objects by ones, have them explore different ways to group the objects that will make counting easier. Have them estimate before they count and group. Discuss their groupings and lead students to conclude that grouping by ten is desirable. *10 ones make 1 ten* makes students wonder how something that means a lot of things can be one thing. They do not see that there are 10 single objects represented on the item for ten in pre-grouped materials, such as the rod in base-ten blocks. Students then attach words to materials and groups without knowing what they represent. Eventually they need to see the rod as *a ten* that they did not group themselves. Students need to first use groupable materials to represent numbers 11 to 19 because a group of ten such as a bundle of 10 straws or a cup of 10 beans makes more sense than *a ten* in pre-grouped materials.

Kindergarteners should use proportional base-ten models, where a group of ten is physically 10 times larger than the model for a one. Non-proportional models such as an abacus and money should not be used at this grade level.

Students should impose their base-ten concepts on a model made from groupable and pre-groupable materials (see Resources/Tools). Students can transition from groupable to pre-groupable materials by leaving a group of ten intact to be reused as a pre-grouped item. When using pre-grouped materials, students should reflect on the ten-to-one relationships in the materials, such as the “tenness” of the rod in base-ten blocks. After many experiences with pre-grouped materials, students can use dots and a stick (one tally mark) to record singles and a ten.

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Encourage students to use base-ten language to describe quantities between 11 and 19. At the beginning, students do not need to use *ones* for the singles. Some of the base-ten language that is acceptable for describing quantities such as18 includes *one ten and eight, a bundle and eight*, *a rod and 8 singles* and *ten and eight more.* Write the horizontal equation 18 = 10 + 8 and connect it to base-ten language. Encourage, but do not require, students to write equations to represent quantities.

Instructional Resources/Tools

Groupable models

- Dried beans and small cups for holding groups of 10 dried beans
- Linking cubes
- Plastic chain links

Pre-grouped materials

- Base-ten blocks
- Dried beans and bean sticks *(10 dried beans glued on a craft stick)*

Pearson Education, Inc. 2012:

- Five-frame and Ten-frame
- Place-value mat with ten-frames
- Strips (ten connected squares) and squares (singles)

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

None

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Measurement & Data (A strong connection to Number with counting and comparing)	K.MD.1, K.MD.2, K.MD.3

UNDERSTAND:

Objects can be sorted/classified by their attributes.
Objects can be described and compared by their attributes.

KNOW:	DO:
Objects have different attributes that can be measured or compared.	***Describe and compare measurable attributes.*** **K.MD.1** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.2** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* ***Classify objects and count the number of objects in each category.*** **K.MD.3** Classify objects from a variety of cultural contexts, including those of Montana American Indians, into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10).[3]

KEY TERMS FOR THIS PROGRESSION:

Attribute, Classify, Compare, Less than, Measure, More than, Sort

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Describe and compare measurable attributes.
It is critical for students to be able to identify and describe measureable attributes of objects. An object has different attributes that can be measured, like the height and weight of a can of food. When students compare shapes directly, the attribute becomes the focus. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: “Which box will hold the most?”, “Which box will hold least?”, and “Will they hold the same amount?” Students can decide to fill one box with dried beans then pour the beans into the other box to determine the answers to these questions.

Have students work in pairs to compare their arm spans. As they stand back-to-back with outstretched arms, compare the lengths of their spans, then determine who has the smallest arm span. Ask students to explain their reasoning. Then ask students to suggest other measureable attributes of their bodies that they could directly compare, such as their height or the length of their feet.

Connect to other subject areas. For example, suppose that the students have been collecting rocks for classroom observation and they wanted to know if they have collected typical or unusual rocks. Ask students to discuss the measurable attributes of rocks. Lead them to first comparing the weights of the rocks. Have the class chose a rock that seems to be a “typical” rock. Provide the categories: *Lighter Than Our Typical Rock* and *Heavier Than Our Typical Rock*. Students can take turns holding a different rock from the collection and directly comparing its weight to the weight of the typical rock and placing it in the appropriate category. Some rocks will be left over because they have about the same weight as the typical rock. As a class, they count the number of rocks in each category and use these counts to order the categories and discuss whether they collected “typical” rocks.

Instructional Resources/Tools
Dried beans
Rice
Two- and three-dimensional real-world objects

[3] Limit category counts to be less than or equal to 10.

National Council of Teachers of Mathematics. 2000-2012. Magnificent Measurement: The Weight of Things: This lesson introduces and provides practice with the measurable attribute of weight.

Cluster: Classify objects and count the number of objects in each category.

Provide categories for students to use to sort a collection of objects. Each category can relate to only one attribute, like *Red* and *Not Red* or *Hexagon* and *Not Hexagon,* and contain up to ten objects. Students count how many objects are in each category and then order the categories by the number of objects they contain.

Ask questions to initiate discussion about the attributes of shapes. Then have students sort a collection of two-dimensional and three-dimensional shapes by their attributes. Provide categories like *Circles* and *Not Circles* or *Flat* and *Not Flat*. Have students count the objects in each category and order the categories by the number of objects they contain.

Have students infer the classification of objects by guessing the rule for a sort. First, the teacher uses one attribute to sort objects into two loops or regions without labels. Then the students determine how the objects were sorted, suggest labels for the two categories and explain their reasoning.

Instructional Resources/Tools

Attribute blocks
Large paper to draw loops
Variety of objects to sort
Yarn for loops

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Count to tell the number of objects. (K.CC.4, K.CC.5)
Compare numbers. (K.CC.6, K.CC.7)
Analyze, compare, create, and compose shapes. (K.G.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Geometry (Shapes and Their Attributes)	K.G.1, K.G.2, K.G.3, K.G.4, K.G.5, K.G.6

UNDERSTAND:

Shapes can be described, compared, and sorted by their attributes.
Shapes can be joined together to make larger shapes.

KNOW:	DO:
A shape has the same name regardless of orientation or size. Shapes have attributes that allow them to be analyzed and compared. Shapes can be combined to form larger shapes. 2-D is "lying flat". 3-D is "solid".	***Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).*** **K.G.1** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. **K.G.2** Correctly name shapes regardless of their orientations or overall size. **K.G.3** Identify shapes as two-dimensional ("flat") or three-dimensional ("solid"). ***Analyze, compare, create, and compose shapes.*** **K.G.4** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/corners) and other attributes (e.g., having sides of equal length). **K.G.5** Model shapes in the world from a variety of cultural contexts, including those of Montana American Indians, by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **K.G.6** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

KEY TERMS FOR THIS PROGRESSION:

Above, Attributes, Below, Behind, Corner (vertices), Equal, Next to, Side

Shape names: **2D:, 3D:, Circles, Cones, Cubes, Cylinders, Hexagons, Rectangles, Spheres, Squares, Triangles**

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
Develop spatial sense by connecting geometric shapes to students' everyday lives. Initiate natural conversations about shapes in the environment. Have students identify and name two- and three-dimensional shapes in and outside of the classroom and describe their relative position.

Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see (e.g., *This rectangle (a poster) is over the sphere (globe))*. Teachers can use a digital camera to record these relationships.

Hide shapes around the room. Have students say where they found the shape using positional words (e.g*., I found a triangle UNDER the chair*.)

Have students create drawings involving shapes and positional words: (e.g., *Draw a window ON the door* or *Draw an apple UNDER a tree*.). Some students may be able to follow two- or three-step instructions to create their drawings.

Use a shape in different orientations and sizes along with non-examples of the shape so students can learn to focus on defining attributes of the shape.

Manipulatives used for shape identification actually have three dimensions. However, kindergartners need to think of these shapes as two-dimensional or "flat" and typical three-dimensional shapes as "solid." Students will identify two-dimensional shapes that form surfaces on three-dimensional objects. Students need to focus on noticing two and three dimensions, not on the words *two-dimensional* and *three-dimensional.*

Instructional Resources/Tools
Common two- and three-dimensional items
Pattern blocks
Die-cut shapes

Digital camera
Three-dimensional models

International Reading Association and the National Council of Teachers of English. 2012. *Going on a Shape Hunt: Integrating Math and Literacy:* In this unit, students are introduced to the idea of shapes through a read-aloud session with an appropriate book. They then use models to learn the names of shapes, work together and individually to locate shapes in their real-world environment.

Pearson Education, Inc. 2012:
- Assorted shapes
- Tangrams

Cluster: Analyze, compare, create and compose shapes.

Use shapes collected from students to begin the investigation into basic properties and characteristics of two- and three-dimensional shapes. Have students analyze and compare each shape with other objects in the classroom and describe the similarities and differences between the shapes. Ask students to describe the shapes while the teacher records key descriptive words in common student language. Students need to use the word *flat* to describe two-dimensional shapes and the word *solid* to describe three-dimensional shapes.

Use the sides, faces and vertices of shapes to practice counting and reinforce the concept of one-to-one correspondence.

The teacher and students orally describe and name the shapes found on a Shape Hunt. Students draw a shape and build it using materials regularly kept in the classroom such as construction paper, clay, wooden sticks or straws.

Students can use a variety of manipulatives and real-world objects to build larger shapes with these and other smaller shapes: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. Kindergarteners can manipulate cardboard shapes, paper plates, pattern blocks, tiles, canned food, and other common items.

Have students compose (build) a larger shape using only smaller shapes that have the same size and shape. The sides of the smaller shapes should touch and there should be no gaps or overlaps within the larger shape. For example, use one-inch squares to build a larger square with no gaps or overlaps. Have students also use different shapes to form a larger shape where the sides of the smaller shapes are touching and there are no gaps or overlaps. Ask students to describe the larger shape and the shapes that formed it.

Instructional Resources/Tools

Balls
Boxes that are cubes
Cans of food
Carpet squares or rectangles
Clay
Colored tiles
Construction paper
Cubes
Floor tiles
Paper plates
Pattern blocks
Straws
Tangrams
Three-dimensional models
Wooden sticks

National Council of Teachers of Mathematics. 2000-2012. What can you build with triangles?: The first lesson in this unit includes the *Just Two Triangles* activity worksheet where students are asked to form different larger shapes with two triangles.

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CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
Describe and compare measurable attributes. (K.MD.1) **Describe several measurable attributes of a single object.** (K.MD.2) **Classify objects and count the number of objects in each category.** (K.MD.3) **Count to tell the number of objects.** (K.CC.4, K.CC.5) **Compare numbers.** (K.CC.6, K.CC.7)

STANDARDS FOR MATHEMATICAL PRACTICE:	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

SAMPLE YEAR LONG PLAN

COURSE: KINDERGARTEN MATH

Unit	Knows number names & count sequence numbers 0-50	Count to tell the number of objects	Compare numbers & sets of objects 1-10	Describe and compare measurable attributes	Classifies & counts objects in categories	Identify & Describe Shapes	Analyze, compare,create, & compose shapes
(Time)	(10 days)	(10 days)	(10 days)	(5 days)	(5 days)	(10 days)	(10 days)
STANDARDS	K.CC.1 K.CC.2 K.CC.3	K.CC.4a K.CC.4b K.CC.4c K.CC.5	K.CC.6 K.CC.7	K.MD.1	K.MD.2	K.G.1 K.G.2 K.G.3	K.G.4 K.G.5 K.G.6

Major
Supporting
Additional

Count numbers 51-100		Count to tell the number of objects	Compare numbers & sets of objects 1-10		Understands addition to 5		Understands subtraction to 5		Decomposing Numbers less than or equal to 10		Work with numbers 11-19 for place value
(10 days)		(10 days)	(10 days)		(30 days)		(20 days)		(20 days)		(20 days)
K.CC.1 K.CC.2		K.CC.4a K.CC.4b K.CC.4c K.CC.5	K.CC.6 K.CC.7		K.OA.1 K.OA.2 K.OA.5		K.OA.1 K.OA.2 K.OA.5		K.OA.3 K.OA.4		K.NBT.1

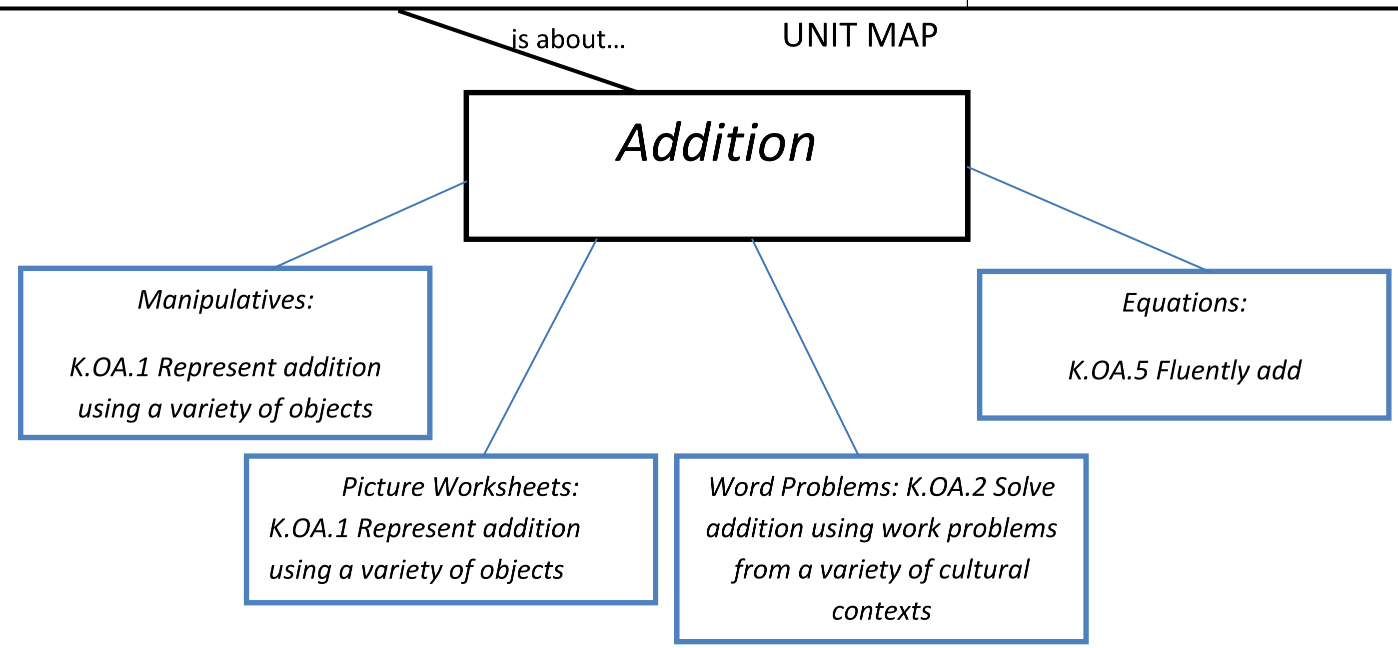
TEACHER: ____________________ SCHOOL DISTRICT/BUILDING: ____________________

COURSE: Math GRADE LEVEL(S): Kindergarten

LAST UNIT	CURRENT UNIT	NEXT UNIT
Compare numbers & sets of objects	**Addition**	Subtraction

	UNIT SCHEDULE
	Show examples using manipulatives
	Introduce symbols +,=
	I do, we do using manipulatives
	Manipulatives on own
	Picture worksheet w/help
	Do several similar picture worksheets whole group
	Picture worksheet on own
	Word problems together
	Word problems on own
	Equations whole group
	Equations on own
	Review concepts
	End of unit assessment

UNIT MAP



UNIT SELF TEST QUESTIONS

Use manipulatives and drawings to demonstrate an understanding of addition

Solve addition word problems

Solve addition equations up to five

MATH STANDARDS	
K.OA.1	
K.OA.2	
K.OA.5	

Kindergarten Lesson Example, from North Carolina Department of Public Instruction, www.ncdpi.wikispaces.net, retrieved 6/2013.

Lesson 2: Birds on the Tree
Overview and Background Information

Mathematical Goals	By the end of the lesson students will: • Join two groups of objects • Find the total quantity after joining two groups of objects • Explain their process of joining two groups of objects to find the quantity
Common Core State Standards	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **K.OA.5** Fluently add and subtract within 5. **Count to tell the number of objects.** **K.CC.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
Emphasized Standards for Mathematical Practice	1. Make Sense and Persevering while Solving Problems 2. Reason Abstractly and Quantitatively 4. Use Appropriate Tools Strategically 8. Look for and Express Regularity in Repeated Reasoning
Prior Knowledge Needed	making sets of objects (within 5), counting sets of objects consistently with accuracy (within 5)
Vocabulary	join
Materials	Whole class: tree work mats, counters, number cubes marked 1, 2 and 3, large picture cards Station 1: number cards, counters, ten frame Station 2: small picture cards, number cubes (1-3), tree work mat, counters Station 3: pom-poms, hoops Station 4: cubes Station 5: paper plate with a line drawn down the center, counters Recording sheets for Stations 3, 4, and 5

Tasks in the Lesson

Engage	3-5 minutes

During whole group time, read the following story off Picture Card A and show it to students:
There were 2 birds sitting in the tree.
The teacher will roll a number cube marked 1, 2, and 3
Suddenly __ more birds flew onto the tree. How many birds are on the tree now?

Ask students to come act out the story. The teacher might say:
Let's think about this problem. We have a tree. Here is our tree (points to the ground). We need two birds (calls 2 students to come and stand in front of the class. The teacher continues by asking __ more students to stand up but in a separate group from the first two students.)

The teacher can then ask the class, "How many students are on the tree now?"
As students give their solutions, it is acceptable to allow many students to share their solution, even if they all say the same solution.
The teacher might say:
How many birds are in the tree? Does anyone have a different solution?

Regardless of the solution, the teacher should not lead students to think whether their solution is correct or incorrect.

The teacher then asks students: *How can we find out how many birds are in the tree?*
Possible responses:
We can count the number of students (birds). I know that I start at 2 and then count on __ more.
I know that 2 and __ is __.

Explore and Explain	10-12 minutes

Pass out counters and tree work mats to every student.
I'm going to tell you a story. Use your counters and tree mat to solve the story problem.
Read the following story off Picture Card B:
There were 3 birds on the tree.
The teacher rolls the number cube.
__ more birds flew on the tree. How many birds are on the tree now?

Allow students to use their counters and tree mat however they want in order to represent the problem.
As students are working, observe:
- Do students place the correct number of counters on the tree?
- Can students tell you the total number of counters?
- Could any students immediately put counters out without modeling the problem?

After students have had a few minutes to model the problem, ask students:
How many birds are on the tree?

As students share their solution, ask students to share their strategies. Possible responses:
I put 2 counters on the tree and then I put __ counter(s) on the tree. Then I counted them.
There were 2 counters on the tree.
I knew that __ more is __.
I knew that 2 and __ is __.

Ask students to clear their work mat to get ready for a new story problem.
Read the following story off Picture Card C and show the card to students:
3 birds are on the tree. The teacher rolls the number cube.
__ more birds flew on the tree. How many birds are on the tree now?
Allow students to use their counters and tree mat however they want in order to represent the problem.
As students are working, observe:

- Do students place the correct number of counters on the tree?
- Can students tell you the total number of counters?
- Could any students immediately put counters out without modeling the problem?

After students have had a few minutes to model the problem, ask students:
How many birds are on the tree?
As students share their solution, follow up with students to share their strategies.
Possible responses:
I put 3 counters on the tree and I counted.
I put 3 counters on a tree and then counted on.
I knew that 3 and __ is __.

Tell students to clear their work mat and get ready for another story problem.
This time, choose a student to read the story card to the class. Follow the process above to have students act out and solve the problem. Do this with Picture Cards D, E, and F.

Elaborate 30-35 minutes

Students will spend the remainder of the lesson in independent work stations practicing concepts related to joining and number sense. The teacher's role is to scaffold and extend students' learning by interacting with students at each of the stations. The teacher should not be fixed at one station the entire time.
These stations are intended to focus on combinations of 3, 4, and 5 only. Each day a student should only be working on combinations of either 3, 4, or 5. This is determined by the teacher or the student. Stations 2, 3, 4, and 5 should be introduced prior to this lesson.

Here is an overview of the five stations:
Station 1: One or Two More Animals
Students will select a number card (0-5) and use that number as the start number in their story problem. Students make that number using counters. From the start number, students will determine how many there will be if there will be one more animal came. Students can also find "two more" if they need enrichment. No recording is needed at this station. Students continue to select different number cards.

Station 2: Adding to a Picture
Students will select a Picture Card and make the picture with counters. Students will roll a number cube marked 1, 2, and 3. Students will add that many counters to their picture. Students will determine how many total counters they have. No recording is needed at this station. Students continue to select different story cards.

Station 3: Pom-pom Toss
Students will toss pom-poms at a hoop on the floor. Students will count how many landed inside the hoop and how many landed outside the hoop. Students record their solutions on the sheet.

Station 4: Snap It
Students make a tower of cubes and place it behind their back. While their tower is behind their back, they snap it and bring one part of the tower in front so they can see it. Students determine how many cubes are still behind their back and record their results.

Station 5: Walk the line
Students will spill beans or counters onto a paper plate that has a line drawn down the center. Students will count the number of counters on the left and on the right and record their results.

Evaluation of Students
Formative: As students are working at Station 1: Do students accurately place the correct number of counters out? Are students able to join two sets and correctly identify the total? At Stations 2-5: Can students accurately determine the various parts and the whole while working? What strategies are students using to determine the various parts? (e.g., counting all, counting on, fluently identifying parts)
Plans for Individual Differences
Intervention: For struggling students, only work with combinations to 3 and 4. Extension: The number of objects at each station can be increased.

Not yet: Student shows evidence of misunderstanding, incorrect concept or procedure		**Got It:** Student essentially understands the target concept.	
NEEDS IMPROVEMENT (N)	**WITH ASSISTANCE (W)**		**INDEPENDENT (I)**
0 Unsatisfactory: **Little Accomplishment** The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little or no success. Further teaching is required.	**1Marginal:** **Partial Accomplishment** Part of the task is accomplished, but there is lack of evidence of understanding or evidence of not understanding. Further teaching is required.	**2 Proficient:** **Substantial Accomplishment** Student could work to full accomplishment with minimal feedback from teacher. Errors are minor. Teacher is confident that understanding is adequate to accomplish the objective with minimal assistance.	**3 Excellent:** **Full Accomplishment** Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas. May have minor errors.

Example: Kindergarten Formative Instructional and Assessment Tasks

OA Task 1a	
Domain	Operations and Algebraic Thinking
Cluster	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
Standard(s)	**K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, expressions, or equations. **K.OA.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. *Add to- Result Unknown*
Materials	Manipulatives such as chips/cubes, pencil, dice, picture cards that teacher will create
Task	Provide materials to the student. Read the problem to the student: *Sam saw two birds sitting in the tree. Suddenly, ___ more birds flew onto the tree How many birds are on the tree now? Show your thinking with objects, words, pictures or numbers*. (Dok3) Ask students to create his/her own problem and solve it.

Continuum of Understanding		
Developing Understanding	• Incorrectly solves the problem. • Includes an answer without an explanation of how the problem was solved.	Strategies Used: ❑ Counting All ❑ Counting On ❑ Knows Basic Fact ❑ Other
Complete Understanding	• Correctly solves the problem: • Uses objects, words, pictures, and/or numbers to show how the problem was solved.	Explained Thinking: ❑ Objects ❑ Pictures ❑ Numbers ❑ Words ❑ Other

Standards for Mathematical Practice
1. Makes sense and perseveres in solving problems.
2. Reasons abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.
4. Models with mathematics.
5. Uses appropriate tools strategically.
6. Attends to precision.
7. Looks for and makes use of structure.
8. Looks for and expresses regularity in repeated reasoning.

Available from www.ncpublicschools.org , Retrieved 6/2013.

FIRST GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grade 1 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
1.MP.1. Make sense of problems and persevere in solving them.	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
1.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
1.MP.3. Construct viable arguments and critique the reasoning of others.	First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions.
1.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
1.MP.5. Use appropriate tools strategically.	In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem.
1.MP.6. Attend to precision.	As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
1.MP.7. Look for and make use of structure.	First graders begin to discern a pattern or structure. For instance, if students recognize $12 + 3 = 15$, then they also know $3 + 12 = 15$. *(Commutative property of addition.)* To add $4 + 6 + 4$, *the first two numbers can be added to make a ten, so* $4 + 6 + 4 = 10 + 4 = 14$.
1.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten" and multiples of "ten" they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, "Does this make sense?"

Explanations and Examples Grade 1
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Grade 1 Content Standards

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 1 Overview

Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten
- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data
- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry
- Reason with shapes and their attributes.

Operations and Algebraic Thinking	1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems within a cultural context, including those of Montana American Indians, involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]
2. Solve word problems within a cultural context, including those of Montana American Indians, that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*
4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
8. Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.*

Number and Operations in Base Ten	1.NBT

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
    a. 10 can be thought of as a bundle of ten ones — called a “ten.”
    b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
    c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects from a variety of cultural contexts, including those of Montana American Indians, by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

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[1] Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term

[2] See Glossary, Table 1.

[3] Students need not use formal terms for these properties.

[4] Students do not need to learn formal names such as “right rectangular prism.”

GRADE 1

Domain	Cluster	Code	Common Core State Standard
Operations and Algebraic Thinking	Represent and solve problems involving addition and subtraction.	1.OA.1	Use addition and subtraction within 20 to solve word problems within a cultural context, including those of Montana American Indians, involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
		1.OA.2	Solve word problems within a cultural context, including those of Montana American Indians, that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem
		1.OA.3	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) (Students need not use formal terms for these properties.)
		1.OA.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.
	Add and subtract within 20.	1.OA.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
		1.OA.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
	Work with addition and subtraction equations.	1.OA.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
		1.OA.8	Determine the unknown number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? - 3, 6 + 6 = ?.
Number and Operations in Base Ten	Extend the counting sequence.	1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
	Understand place value.	1.NBT.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a tenand one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
		1.NBT.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
	Use place value understanding and properties of operations to add and subtract.	1.NBT.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
		1.NBT.5	1.NBT.5 Use place value understanding and properties of operations to add and subtract. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
		1.NBT.6	1.NBT.6 Use place value understanding and properties of operations to add and subtract. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

GRADE 1

Domain	Cluster	Code	Common Core State Standard
Measurement and Data	Measure lengths indirectly and by iterating length units.	1.MD.1	Order three objects from a variety of cultural contexts, including those of Montana American Indians, by length; compare the lengths of two objects indirectly by using a third object.
		1.MD.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
	Tell and write time.	1.MD.3	Tell and write time in hours and half-hours using analog and digital clocks.
	Represent and interpret data.	1.MD.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Geometry	Reason with shapes and their attributes.	1.G.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size) for a wide variety of shapes; build and draw shapes to possess defining attributes.
		1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.")
		1.G.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Getting Every Montana Student College and Career Ready

Montana Common Core
Standards and Assessments

Montana Curriculum Organizer

Grade 1

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

November 2012 Materials adapted from Arizona, Delaware and Ohio Departments of Education

Montana Curriculum Organizer: Grade 1 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

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Montana Curriculum Organizer: Grade 1 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 22 consists of tables organized into learning progressions that can function as units. The table for each learning progression unit includes: 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.	Standards covered in this learning progression.
UNDERSTAND:	
What students need to understand by the end of this learning progression.	
KNOW:	**DO:**
What students need to know by the end of this learning progression.	What students need to be able to do by the end of this learning progression, organized by cluster and standard.
KEY TERMS FOR THIS PROGRESSION:	
Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.	
INSTRUCTIONAL STRATEGIES AND RESOURCES:	
Cluster: Title Strategies for this cluster **Instructional Resources/Tools** Resources and tools for this cluster	
Cluster: Title Strategies for this cluster **Instructional Resources/Tools** Resources and tools for this cluster	
CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:	
Standards that connect to this learning progression are listed here, organized by cluster.	
STANDARDS FOR MATHEMATICAL PRACTICE:	
A quick reference guide to the eight standards for mathematical practice is listed here.	

Montana Curriculum Organizer: Grade 1 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Grade 1 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
1.MP.1. Make sense of problems and persevere in solving them.	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
1.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
1.MP.3. Construct viable arguments and critique the reasoning of others.	First-graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking." and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions.
1.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
1.MP.5. Use appropriate tools strategically.	In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first-graders decide it might be best to use colored chips to model an addition problem.
1.MP.6. Attend to precision.	As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
1.MP.7. Look for and make use of structure.	First graders begin to discern a pattern or structure. For instance, if students recognize $12 + 3 = 15$, then they also know $3 + 12 = 15$ (commutative property of addition). To add $4 + 6 + 4$, the first two numbers can be added to make a ten, so $4 + 6 + 4 = 10 + 4 = 14$.
1.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten", and multiples of "ten", they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves "Does this make sense?"

CRITICAL AREAS FOR GRADE 1 MATH

In Grade 1, instructional time should focus on four critical areas:

(1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;
(2) developing understanding of whole-number relationships and place value, including grouping in tens and ones;
(3) developing understanding of linear measurement and measuring lengths as iterating length units; and
(4) reasoning about attributes of, and composing and decomposing geometric shapes.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Operations & Algebraic Thinking (Adding & Subtracting within 20)	1.OA.1, 1.OA.2, 1.OA.3, 1.OA.4, 1.OA.5, 1.OA.6, 1.OA.7, 1.OA.8

UNDERSTAND:

There are multiple ways to represent and find sums/differences within 20 (story problems, pictures, equations, computational strategies, and manipulatives).
An equation must be balanced and the equal sign represents quantities on each side of the symbol as the same (equal).
The relationship between addition and subtraction can be used to solve problems.

KNOW:

Addition and subtraction are related operations.

Subtraction can be perceived as an unknown addend problem.

Addition and subtraction problems can be posed with the missing part being in different positions.

The commutative and associative properties of operations can be used to solve problems (but students do not need to know them by name).

Symbols can represent an unknown quantity in an equation.

Know combinations to 10 fluently.

Strategies: Counting on, Making Ten, Decomposing, Using Known Facts

DO:

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems within a cultural context, including those of Montana American Indians, involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).[1]

1.OA.2 Solve word problems within a cultural context, including those of Montana American Indians, that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).

Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA.3 Apply properties of operations as strategies to add and subtract.[2] *For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (associative property of addition).*

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

Footnote to 1.OA.7: These equations are purposeful in showing students how to determine if an equation is "balanced" (quantity on each side of the equation is the same).

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[1] See Glossary, Table 1 in the MCCSS document.
[2] Students need not use formal terms for these properties.

	1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = � - 3, 6 + 6 = �.*

KEY TERMS FOR THIS PROGRESSION:

Difference, Equation, Equivalent, Sum

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Represent and solve problems involving addition and subtraction.
Provide opportunities for students to participate in shared problem-solving activities to solve word problems.

Collaborate in small groups to develop problem-solving strategies using a variety of models such as drawings, words, and equations with symbols for the unknown numbers to find the solutions. Additionally, students need the opportunity to explain, write and reflect on their problem-solving strategies. The situations for the addition and subtraction story problems should involve sums and differences less than or equal to 20 using the numbers 0 to 20. They need to align with the 12 situations found in Table 1 on page 72 in the *Montana Common Core Standards for School Mathematics Grade-Band*.

Students need the opportunity of writing and solving story problems involving three addends with a sum that is less than or equal to 20. For example, each student writes or draws a problem in which three whole things are being combined. The students exchange their problems with other students, solving them individually and then discussing their models and solution strategies. Now both students work together to solve each problem using a different strategy.

Literature is a wonderful way to incorporate problem solving in a context that young students can understand. Many literature books that include mathematical ideas and concepts have been written in recent years. For Grade 1, the incorporation of books that contain a problem situation involving addition and subtraction with numbers 0 to 20 should be included in the curriculum. Use the situations found in Table 1 on page 72 in the *Montana Common Core Standards for School Mathematics Grade-Band* for guidance in selecting appropriate books. As the teacher reads the story, students use a variety of manipulatives, drawings, or equations to model and find the solution to problems from the story.

Instructional Resources/Tools
International Reading Association, National Council of Teachers of English. 2012. *Giant Story Problems: Reading Comprehension Through Math Problem-solving:* Using drawings, equations, and written responses, students work cooperatively in two class sessions to solve Giant Story Problems while they gain practice in reading for information.

Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band.* Table 1 on page 72, common addition and subtraction situations

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Cluster: Understand and apply properties of operations and the relationship between addition and subtraction.
One focus in this cluster is for students to discover and apply the commutative and associative properties as strategies for solving addition problems. Students do not need to learn the names for these properties. It is important for students to share, discuss and compare their strategies as a class. The second focus is using the relationship between addition and subtraction as a strategy to solve unknown-addend problems. Students naturally connect counting on to solving subtraction problems. For the problem “15 – 7 = ?”, they think about the number they have to add to 7 to get to 15. First-graders should be working with sums and differences less than or equal to 20 using the numbers 0 to 20.

Provide investigations that require students to identify and then apply a pattern or structure in mathematics. For example, pose a string of addition and subtraction problems involving the same three numbers chosen from the numbers 0 to 20, like 4 + 13 = 17 and 13 + 4 = 17. Students analyze number patterns and create conjectures or guesses. Have students choose other combinations of three numbers and explore to see if the patterns work for all numbers 0 to 20. Students then share and discuss their reasoning. Be sure to highlight students’ uses of the commutative and associative properties and the relationship between addition and subtraction.

Expand the student work to three or more addends to provide the opportunities to change the order and/or groupings to make tens. This will allow the connections between place-value models and the properties of operations for addition to be seen. Understanding the commutative and associative properties builds flexibility for computation and estimation, a key element of number sense.

Provide multiple opportunities for students to study the relationship between addition and subtraction in a variety of ways, including games, modeling and real-world situations. Students need to understand that addition and subtraction are related, and that subtraction can be used to solve problems where the addend is unknown.

Instructional Resources/Tools
A variety of objects for modeling and solving addition and subtraction problems

National Council of Teachers of Mathematics. 2000-2012.

> *How Many More Fish?: Balancing equations:* In this lesson, students imitate the action of a pan balance and record the modeled subtraction facts in equation form.
>
> *Macaroni Math: How many left?*: This lesson encourages the students to explore unknown-addend problems using the set model and the game *Guess How Many?*

Winnipeg School Division. 2005-2006. *Numeracy Project: Dot Card and Ten Frame Activities.* (pp. 9-11, 21-24, 26-30, 32-37)

Cluster: Add and subtract within 20.
Provide many experiences for students to construct strategies to solve the different problem types illustrated in Table 1 on page 72 in the *Montana Common Core Standards for School Mathematics Grade-Band*. These experiences should help students combine their procedural and conceptual understandings. Have students invent and refine their strategies for solving problems involving sums and differences less than or equal to 20 using the numbers 0 to 20. Ask them to explain and compare their strategies as a class.

Provide multiple and varied experiences that will help students develop a strong sense of numbers based on comprehension – not rules and procedures. Number sense is a blend of comprehension of numbers and operations and fluency with numbers and operations. Students gain computational fluency (using efficient and accurate methods for computing) as they come to understand the role and meaning of arithmetic operations in number systems.

Primary students come to understand addition and subtraction as they connect counting and number sequence to these operations. Addition and subtraction also involve part to whole relationships. Students' understanding that the whole is made up of parts is connected to decomposing and composing numbers.

Provide numerous opportunities for students to use the *counting on* strategy for solving addition and subtraction problems. For example, provide a ten frame showing 5 colored dots in one row. Students add 3 dots of a different color to the next row and write 5 + 3. Ask students to count on from 5 to find the total number of dots. Then have them add an equal sign and the number eight to 5 + 3 to form the equation 5 + 3 = 8. Ask students to verbally explain how counting on helps to add one part to another part to find a sum. Discourage students from inventing a counting back strategy for subtraction because it is difficult and leads to errors.

Instructional Resources/Tools
A variety of objects for counting
A variety of objects for modeling and solving addition and subtraction problems

Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band*:

National Council of Teachers of Mathematics: Illuminations:

> *Begin with Buttons: More and more buttons:* Students use buttons to create, model, and record addition sentences in this lesson. A *Sums to Ten* chart is provided for students to use.
>
> *Do It with Dominoes: Balancing discoveries*: This lesson encourages students to explore another model of addition, the balance model. The exploration also involves recording the modeled addition facts in equation form. Students begin to memorize the addition facts by playing the *Seven-Up* game.
>
> *Do It with Dominoes: Seeing doubles:* In this lesson, the students focus on dominoes with the same number of

spots on each side and on the related addition facts. They make triangle-shaped flash cards for the doubles facts.

Pearson Education, Inc. 2012:*Five-frame and Ten-frame.*

Cluster: Work with addition and subtraction equations.

Provide opportunities for students to use objects of equal weight and a number balance to model equations for sums and differences less than or equal to 20 using the numbers 0 to 20. Give students equations in a variety of forms that are true and false. Include equations that show the identity property, commutative property of addition, and associative property of addition. Students need not use formal terms for these properties.

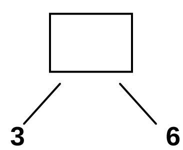
13 = 13	Identity Property
8 + 5 = 5 + 8	Commutative Property for Addition
3 + 7 + 4 = 10 + 4	Associative Property for Addition

Ask students to determine whether the equations are true or false and to record their work with drawings. Students then compare their answers as a class and discuss their reasoning.

Present equations recorded in a nontraditional way, like 13 = 16 – 3 and 9 + 4 = 18 – 5, then ask, “Is this true?” Have students decide if the equation is true or false. Then as a class, students discuss their thinking that supports their answers.

Provide situations relevant to first graders for these problem types illustrated in Table 1 on page 72 in the *Montana Common Core Standards for School Mathematics Grade-Band*: Add To / Result Unknown, Take From / Start Unknown, and Add To / Result Unknown. Demonstrate how students can use graphic organizers such as the Math Mountain to help them think about problems.

The Math Mountain shows a sum with diagonal lines going down to connect with the two addends, forming a triangular shape. It shows two known quantities and one unknown quantity. Use various symbols, such as a square, to represent an unknown sum or addend in a horizontal equation. For example, here is a Take From / Start Unknown problem situation such as: Some markers were in a box. Matt took 3 markers to use. There are now 6 markers in the box. How many markers were in the box before? The teacher draws a square to represent the unknown sum and diagonal lines to the numbers 3 and 6.



Have students practice using the Math Mountain to organize their solutions to problems involving sums and differences less than or equal to 20 with the numbers 0 to 20. Then ask them to share their reactions to using the Math Mountain.

Provide numerous experiences for students to compose and decompose numbers less than or equal to 20 using a variety of manipulatives. Have them represent their work with drawings, words, and numbers. Ask students to share their work and thinking with their classmates. Then ask the class to identify similarities and differences in the students' representations.

Instructional Resources/Tools

Number balances

Variety of objects that can be used for modeling and solving addition and subtraction problems

Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band*:

National Council of Teachers of Mathematics. 20111-2012.

Finding the Balance: This lesson encourages students to explore another model of subtraction, the balance. Students will use real and virtual balances. Students also explore recording the modeled subtraction facts in equation form.

Pan Balance – Numbers. This virtual tool can be used to strengthen students' understanding and computation of numerical expressions and equality.

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Pearson Education, Inc. 2012:

Double ten-frames
Five-frames and ten-frames

CONNECTIONS TO OTHER DOMAINS &/OR CLUSTERS:

Represent and interpret data. *(*1.MD.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Understanding Place Value	1.NBT.1, 1.NBT.2a-c, 1.NBT.3, 1.NBT.4, 1.NBT.5

UNDERSTAND:

The digits of a two-digit number represent tens and ones.

KNOW:

The two digits of a two-digit number represent amounts of tens and ones.

Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, or 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

"10 more" means one more group of tens and "ten less" means one less group of tens.

Counting can start with any number (not always with 1).

Numbers can be represented in many ways.

The placement of the numeral determines its place-value meaning (i.e., the 5 in 56 means 5 tens or 50, whereas the 5 in 15 means 5 ones).

DO:

Extend the counting sequence.

1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

1.NBT.2 The two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, or 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place-value understanding and properties of operations to add and subtract.

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

KEY TERMS FOR THIS PROGRESSION:

Compose, Decades, Decompose, Digit, Ones, Place value, Tens

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Extend the counting sequence.

In this grade, students build on their counting to 100 by ones and tens beginning with numbers other than 1 as they learned in Kindergarten. Students can start counting at any number less than 120 and continue to 120. It is important for students to connect different representations for the same quantity or number. Students use materials to count by ones and tens to a build models that represent a number, then they connect this model to the number word and its representation as a written numeral.

Students learn to use numerals to represent numbers by relating their place-value notation to their models. They build on their experiences with numbers 0 to 20 in Kindergarten to create models for 21 to 120 with groupable and pre-groupable materials. Students represent the quantities shown in the models by placing numerals in labeled hundreds, tens and ones columns. They eventually move to representing the numbers in standard form, where the group of hundreds, tens, then singles shown in the model matches the left-to-right order of digits in numbers.

Listen as students orally count to 120 and focus on their transitions between decades and the century number. These transitions will be signaled by a 9 and require new rules to be used to generate the next set of numbers. Students need to listen to their rhythm and pattern as they orally count so they can develop a strong number word list.

Extend hundreds charts by attaching a blank hundreds charts and writing the numbers 101 to 120 in the spaces following the same pattern as in the hundreds chart. Students can use these charts to connect the number symbols with their count words for numbers 1 to 120.
Post the number words in the classroom to help students read and write them.

Instructional Resources/Tools

<u>Groupable materials</u>

- Dried beans and a small cup for 10 beans
- Linking cubes
- Plastic chain links

<u>Pre-grouped materials</u>

- Base-ten blocks
- Dried beans and beans sticks (10 beans glued on a craft stick)

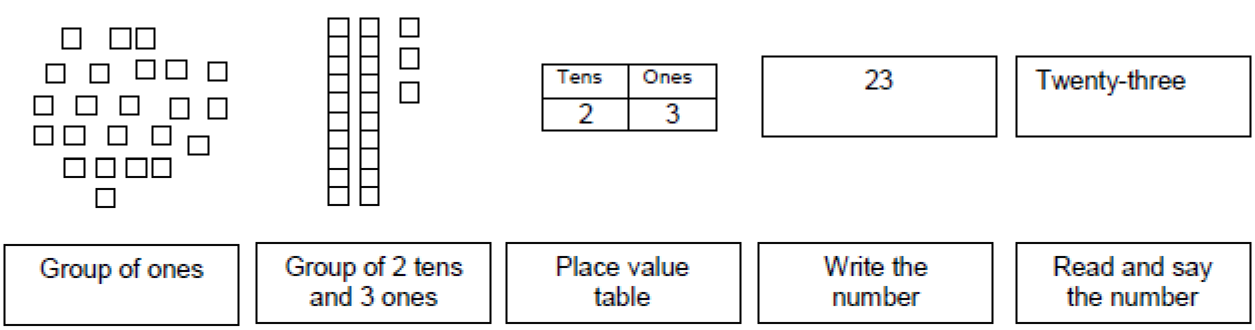
Pearson Education, Inc. 2012:

- *Hundreds chart and Blank hundreds chart*
- *Place-value mat with ten-frames*
- *Strips (ten connected squares) and squares (singles)*
- *Ten-frame*

Cluster: Understand place value.

Essential skills for students to develop include making tens (composing) and breaking a number into tens and ones (decomposing). Composing numbers by tens is foundational for representing numbers with numerals by writing the number of tens and the number of leftover ones. Decomposing numbers by tens builds number sense and the awareness that the order of the digits is important. Composing and decomposing numbers involves number relationships and promotes flexibility with mental computation.

The beginning concepts of place value are developed in Grade 1 with the understanding of ones and tens. The major concept is that putting ten ones together makes a ten and that there is a way to write that down so the same number is always understood. Students move from counting by ones, to creating groups and ones, to tens and ones. It is essential at this grade for students to see and use multiple representations of making tens using base-ten blocks, bundles of tens and ones, and ten-frames. Making the connections among the representations, the numerals and the words are very important. Students need to connect these different representations for the numbers 0 to 99.



Students need to move through a progression of representations to learn a concept. They start with a concrete model, move to a pictorial or representational model, then an abstract model. For example, ask students to place a handful of small objects in one region and a handful in another region. Next, have them draw a picture of the objects in each region. They can draw a likeness of the objects or use a symbol for the objects in their drawing. Now they count the physical objects or the objects in their drawings in each region and use numerals to represent the two counts. They also say and write the number word. Now students can compare the two numbers using an inequality symbol or an equal sign.

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Instructional Resources/Tools

Groupable materials

- Dried beans and a small cup for 10 dried beans
- Linking cubes
- Plastic chain links

Pre-grouped materials

- Base-ten blocks
- Dried beans and bean sticks (10 dried beans glued on a craft stick)

Pearson Education, Inc. 2012:

- *Five-frame and Ten-frame*
- *Place-value mat with ten-frames*
- *Strips (ten connected squares) and squares (singles)*

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Base Block.* (Adjust the application to only deal with ones and tens.)

CONNECTIONS TO OTHER DOMAINS &/OR CLUSTERS:

None

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Number & Operations in Base Ten (Adding & subtracting within 100, including Place Value)	1.NBT.4, 1.NBT.5, 1.NBT.6

UNDERSTAND:

When adding two-digit numbers, one adds tens and tens, and then the ones and ones.

KNOW:	DO:
Addition and subtraction are related operations. 10 more or 10 less of any number under 100 mentally. How to add and subtract multiples of 10 to any number (by 20, 30, 70, etc.). Place-value strategies can be used to add two-digit plus one-digit numbers. Place-value strategies for adding and subtracting (counting on, making 10's/100's, breaking apart and putting together, using known facts). Commutative and associative properties of operations can be used to solve problems (e.g., students know that if 20 + 40 = 60, then 40 + 20 = 60 without actually naming the commutative property). Models for adding and subtracting (e.g., number line, base-ten materials). Methods for recording addition and subtraction strategies using number lines and equations. Symbols can represent an unknown quantity in an equation.	***Use place-value understanding and properties of operations to add and subtract.*** **1.NBT.4** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **1.NBT.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **1.NBT.6** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

KEY TERMS FOR THIS PROGRESSION:

Difference, Digit, Mental math, Model, Ones, Place value, Sum, Tens, Two-digit

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Use place-value understanding and properties of operations to add and subtract.

Provide multiple and varied experiences that will help students develop a strong sense of numbers based on comprehension – not rules and procedures. Number sense is a blend of comprehension of numbers and operations and fluency with numbers and operations. Students gain computational fluency (using efficient and accurate methods for computing) as they come to understand the role and meaning of arithmetic operations in number systems.

Students should solve problems using concrete models and drawings to support and record their solutions. It is important for them to share the reasoning that supports their solution strategies with their classmates.

Students will usually move to using base-ten concepts, properties of operations, and the relationship between addition and subtraction to invent mental and written strategies for addition and subtraction. Help students share, explore, and record their invented strategies. Recording the expressions and equations in the strategies horizontally encourages students to think about the numbers and the quantities they represent. Encourage students to try the mental and written strategies created by their classmates. Students eventually need to choose efficient strategies to use to find accurate solutions.

Students should use and connect different representations when they solve a problem. They should start by building a concrete model to represent a problem. This will help them form a mental picture of the model. Now students move to using pictures and drawings to represent and solve the problem. If students skip the first step, building the concrete model, they might use finger counting to solve the problem. Finger counting is an inefficient strategy for adding within 100 and subtracting within multiples of 10 between10 and 90.

Have students connect a 0-99 chart or a 1-100 chart to their invented strategy.

Instructional Resources/Tools

Groupable materials

- Dried beans and a small cup for 10 dried beans
- Linking cubes
- Plastic chain links

Pregrouped materials

- Base-ten blocks
- Dried beans and bean sticks (10 dried beans glued on a craft stick)

Pearson Education, Inc. 2012:

- *Strips (ten connected squares) and squares (singles)*
- *Five-frame and Ten-frame*
- *Place-value mat with ten-frames*
- *Hundreds chart and Blank hundreds chart*

Utah State University. National Library of Virtual Manipulatives. 1999-2010.*Base Block.* (Adjust the application to only deal with ones and tens.)

CONNECTIONS TO OTHER DOMAINS &/OR CLUSTERS:

Represent and interpret data. (1.MD.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 1 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Measurement (Length & Time)	1.MD.1, 1.MD.2, 1.MD.3

UNDERSTAND:

Lengths of objects can be measured and compared using non-standard units.

KNOW:	DO:
Any object can be used as a length unit. The length of two objects can be compared by using the same unit of measure. Analog and digital clocks are used to measure time. Know how to tell time to the hour and half-hour (see Geometry 1.G.3 as a model for the idea that a half-hour is half of a circle.)	***Measure lengths indirectly and by iterating length units.*** **1.MD.1** Order three objects from a variety of cultural contexts, including those of Montana American Indians, by length; compare the lengths of two objects indirectly by using a third object. **1.MD.2** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* ***Tell and write time.*** **1.MD.3** Tell and write time in hours and half-hours using analog and digital clocks.

KEY TERMS FOR THIS PROGRESSION:

Analog, Compare, Digital, Length, Measure

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Measure lengths indirectly and by iterating length units.

The measure of an attribute is a count of how many units are needed to fill, cover or match the attribute of the object being measured. Students need to understand what a unit of measure is and how it is used to find a measurement. They need to predict the measurement, find the measurement and then discuss the estimates, errors and the measuring process. It is important for students to measure the same attribute of an object with differently sized units.

It is beneficial to use informal units for beginning measurement activities at all grade levels because they allow students to focus on the attributes being measured. The numbers for the measurements can be kept manageable by simply adjusting the size of the units. Experiences with informal or non-standard units promote the need for measuring with standard units.

Measurement units share the attribute being measured. Students need to use as many copies of the length unit as necessary to match the length being measured. For instance, use large footprints with the same size as length units. Place the footprints end to end, without gaps or overlaps, to measure the length of a room to the nearest whole footprint. Use language that reflects the approximate nature of measurement, such as the length of the room is about 19 footprints. Students need to also measure the lengths of curves and other distances that are not straight lines.

Students need to make their own measuring tools. For instance, they can place paper clips end to end along a piece of cardboard, make marks at the endpoints of the clips and color in the spaces. Students can now see that the spaces represent the unit of measure, not the marks or numbers on a ruler. Eventually they write numbers in the center of the spaces. Encourage students not to use the end of the ruler as a starting point. Compare and discuss two measurements of the same distance, one found by using a ruler and one found by aligning the actual units end to end, as in a chain of paper clips. Students should also measure lengths that are longer than a ruler.

Have students use reasoning to compare measurements indirectly. For example, to order the lengths of Objects A, B and C, examine then compare the lengths of Object A and Object B and the lengths of Object B and Object C. The results of these two comparisons allow students to use reasoning to determine how the length of Object A compares to the length of Object C. For example, to order three objects by their lengths, reason that if Object A is smaller than Object B and Object B is smaller than Object C, then Object A has to be smaller than Object C. The order of objects by their length from smallest to largest would be Object A - Object B - Object C.

Instructional Resources/Tools
Clothesline rope
Connecting cubes
Cuisenaire rods
Paper clips
Straws
Strips of tagboard or cardboard
Toothpicks
Variety of common two- and three-dimensional objects
Yarn

American Association for the Advancement of Science. 2012. *Estimation and Measurement*: In this lesson students will use non-standard units to estimate and measure distances.

National Council of Teachers of Mathematics. 2000-2012. *Magnificent Measurement: The Length of My Feet*: This lesson focuses students' attention on the attributes of length and develops their knowledge of and skill in using non-standard units of measurement.

Cluster: Tell and write time.
Students are likely to experience some difficulties learning about time. On an analog clock, the little hand indicates approximate time to the nearest hour and the focus is on where it is pointing. The big hand shows minutes before and after an hour and the focus is on distance that it has gone around the clock or the distance yet to go for the hand to get back to the top. It is easier for students to read times on digital clocks, but these do not relate times very well.

Students need to experience a progression of activities for learning how to tell time. Begin by using a one-handed clock to tell times in hour and half-hour intervals, then discuss what is happening to the unseen big hand. Next use two real clocks, one with the minute hand removed, and compare the hands on the clocks. Students can predict the position of the missing big hand to the nearest hour or half-hour and check their prediction using the two-handed clock. They can also predict the display on a digital clock given a time on a one- or two-handed analog clock and vice-versa.

Have students tell the time for events in their everyday lives to the nearest hour or half hour.

Make a variety of models for analog clocks. One model uses a strip of paper marked in half hours. Connect the ends with tape to form the strip into a circle.

Instructional Resources/Tools
National Council of Teachers of Mathematics. 2000-2012. *Magnificent Measurement: Grouchy Lessons of Time: This* lesson provides an introduction to and practice with the concept of time and hours.

CONNECTIONS TO OTHER DOMAINS &/OR CLUSTERS:

None

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Data (Represent & Interpret)	1.MD.4

UNDERSTAND:

Organizing, representing, and interpreting data allows for careful analysis and answering questions posed.
Data can be interpreted numerically or categorically.

KNOW:	DO:
Organizing data can help with interpreting and answering questions posed. Data can be represented in multiple ways (e.g., line plots, bar graphs/towers of cubes, Venn diagrams, tables). How we interpret data changes depending on the context of the question being asked.	***Represent and interpret data.*** **1.MD.4** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

KEY TERMS FOR THIS PROGRESSION:

Data, Interpret, Organize, Represent

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: *Represent and interpret data.*
Ask students to sort a collection of items in up to three categories. Then ask questions about the number of items in each category and the total number of items. Also ask students to compare the number of items in each category. The total number of items to be sorted should be less than or equal to 100 to allow for sums and differences less than or equal to 100 using the numbers 0 to 100.

Connect to the geometry content studied in Grade 1. Provide categories and have students sort identical collections of different geometric shapes. After the shapes have been sorted, ask these questions: "How many triangles are in the collection?", "How many rectangles are there?", "How many triangles and rectangles are there?", "Which category has the most items?", "How many more?", "Which category has the least?", and "How many less?"

Students can create real or cluster graphs after they have had multiple experiences with sorting objects according to given categories. The teacher should model a cluster graph several times before students make their own. A cluster graph in Grade 1 has two or three labeled loops or regions (categories). Students place items inside the regions that represent a category that they chose. Items that do not fit in a category are placed outside of the loops or regions. Students can place items in a region that overlaps the categories if they see a connection between categories. Ask questions that compare the number of items in each category and the total number of items inside and outside of the regions.

Instructional Resources/Tools
A variety of objects to sort
Geometric shapes
Yarn or large paper for loops

Charles A. Dana Center, University of Texas at Austin: Mathematics TEKS Toolkit. 2012. *Buttons, Buttons, Everywhere!*
In this lesson students use attributes such as shape, color, size, etc. to describe, compare, and sort buttons.

CONNECTIONS TO OTHER DOMAINS &/OR CLUSTERS:

Operations Algebraic Thinking and Number & Operations in Base Ten. (1.OA.1 – 8, and 1.NBT.4 – 6)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Geometry - Reason with Shapes & Their Attributes	1.G.1, 1.G.2, 1.G.3

UNDERSTAND:

Shapes have defining attributes that can be compared to other shapes.
Decomposing a shape into more equal shares creates smaller pieces.

KNOW:	DO:
Shapes can be sorted according to their defining geometric attributes such as the number of sides or closed/open figure (not by non-defining attributes such as color, size, orientation, etc.). For example, a triangle is a triangle no matter what color, size or orientation. Shapes can be composed and decomposed into other shapes. Equal partitions of shapes can be described as halves, fourths, as well as half of, fourth of. Distinguishing features of 2D and 3D shapes.	***Reason with shapes and their attributes.*** **1.G.1** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. **1.G.2** Compose two-dimensional shapes (i.e., rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (i.e., cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[3] **1.G.3** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

KEY TERMS FOR THIS PROGRESSION:

Attributes, Compose, Decompose, Equal shares, Fourth of, Fourths, Half-circles, Half of, Halves, Quarter-circles, Rectangle, Square, Trapezoid, Triangles

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Reason with shapes and their attributes.

Students can easily form shapes on geoboards using colored rubber bands to represent the sides of a shape. Ask students to create a shape with four sides on their geoboard, and then copy the shape on dot paper. Students can share and describe their shapes as a class while the teacher records the different defining attributes mentioned by the students.

Pattern-block pieces can be used to model defining attributes for shapes. Ask students to create their own rule for sorting pattern blocks. Students take turns sharing their sorting rules with their classmates and showing examples that support their rule. The classmates then draw a new shape that fits this same rule after it is shared.

Students can use a variety of manipulatives and real-world objects to build larger shapes. The manipulatives can include paper shapes, pattern blocks, color tiles, triangles cut from squares (isosceles right triangles), tangrams, canned food (right circular cylinders) and gift boxes (cubes or right rectangular prisms).

Folding shapes made from paper enables students to physically feel the shape and form the equal shares. Ask students to fold circles and rectangles first into halves and then into fourths. They should observe and then discuss the change in the size of the parts.

Instructional Resources/Tools

Canned food (right circular cylinders)
Color tiles
Gift boxes (cubes and right rectangular prisms)
Isosceles right triangles cut from squares
Paper shapes
Pattern blocks
Tangrams

[3] Students do not need to learn formal names such as "right rectangular prism."

Montana Curriculum Organizer: Grade 1 Mathematics

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CONNECTIONS TO OTHER DOMAINS &/OR CLUSTERS:	
None	
STANDARDS FOR MATHEMATICAL PRACTICE:	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 1 Mathematics

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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

SAMPLE YEAR LONG PLAN

First Grade Math

Math	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Unit	**Partners and Number Patterns Through 10**	**Add. & Sub. Strategies**	**Unknown Numbers in Addition and Subtraction**	**Place Value Concepts**	**Place Value Situations**	**Comparisons and Data**	**Geometry, Measurement and Equal Shares**	**Two Digit Addition**
(Time)	**15 days**	**30 days**	**29 days**	**24 days**	**19 days**	**15 days**	**23 days**	**20 days**
ANDARDS	1.OA.1	1.OA.1	1.OA.1	1.OA.1	1.OA.1	1.OA.1	1.MD.1	1.NBT.3
	1.OA.3	1.OA.3	1.OA.4	1.OA.3	1.OA.2	1.OA.2	1.MD.2	1.NBT.4
	1.OA.5	1.OA.5	1.OA.5	1.OA.5	1.OA.3	1.MD.4	1.MD.3	1.NBT.6
	1.OA.6	1.OA.6	1.OA.6	1.OA.6	1.OA.4		1.G.1	
	1.OA.8	1.OA.7	1.OA.7	1.OA.8	1.OA.5		1.G.2	
		1.OA.8	1.OA.8	1.NBT.1	1.OA.6		1.G.3	
				1.NBT.2	1.OA.8			
				1.NBT.2a	1.NBT.1			
				1.NBT.2b	1.NBT.2			
				1.NBT.2c	1.NBT.2c			
				1.NBT.3	1.NBT.4			
				1.NBT.4	1.NBT.5			
				1.NBT.5				

	Unit 1	Unit 2	Unit 3
Strategies	**Strategies:** Counting on, Making Ten, Decomposing, Use unknown facts	→	→
Key Terms	**Key Terms:** Difference, Equation, Equivalent, Sum	→	→
Connections	**Connections:** 1.MD.4 (Represent and interpret data)	→	→

Unit 4 – Key Terms: Compose, Decades, Decompose, Digit, Ones, Place Value, Tens

Unit 5 – Key Terms: Difference, Digit, Mental math, Ones, Place value, Sum, Tens, Two-digit

Connections: 1.MD.4 (Represent and interpret data) →

Unit 6 – Key Terms: Data, Interpret, Organize, Represent

Connections: 1.OA.1-8, 1.NBT.4-6

Unit 7 – Key Terms: Analog, Compare, Digital Length, Measure

Key Terms: Attributes, Compose, Decompose, Equal Shares, Fourth of, Fourths, Half-circles, Half of, Halves, Quarter-circles, Rectangle, Square, Trapezoid, Triangles

Unit 8 – Key Terms: Difference, Digit, Mental math, Ones, Compose, Tens, Decades, Decompose, Digit, Place Value, Sum, Two-digit

TEACHER: ____ SCHOOL DISTRICT/BUILDING: ____

COURSE: MATH ______________________________________ GRADE LEVEL(S): First Grade ______________________________

LAST UNIT Kindergarten Review	**CURRENT UNIT** **Partners and Number Patterns through 10**	NEXT UNIT Addition and Subtraction Strategies

Day	UNIT SCHEDULE
1	Engage: real life situations
2	Explore: manipulatives
3	Explain: vocabulary words
4	Elaborate: games/centers
5	Evaluate: performance assessment
6	Engage: real life situations
7	Explore: pictures
8	Explain: review vocabulary words
9	Elaborate: games/centers
10	Evaluate: performance assessment
11	Engage: real life situations
12	Explore: equations
13	Explain: demonstrate use of vocabulary words
14	Elaborate: games/centers
15	Evaluate: performance assessment

is about... UNIT MAP

Adding and subtracting within 20

Model :

1.0A.1: I can solve word problems with two numbers.

1.0A.3: I can apply properties of addition.

1.0A.5 and 1.0A.6: I can add and subtract within 20 using different strategies.

1.0A.8: I can determine the unknown number in all 3 positions.

UNIT SELF TEST QUESTIONS

How do you decompose or break apart a number?

Why would you need to break a number apart?

How do you determine the unknown number in all three positions?

Why do you think associative property is important?

Green: concrete Pink: pictoral Yellow: abstract

MATH STANDARDS

1.0A.1
1.0A.3
1.0A.5
1.OA.6
1.0A.8

First Grade Sample Lesson Plan

Grade Level

First Grade

Title

Buttons with Corduroy!

Content Standards

1.OA.1: Use addition and subtraction within 20 to solve word problems.

1.NBT.1: Count to 120, starting at any number less than 120.

Practice Standards

1. Make sense of problems and preserve in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

Engage

Read the story Corduroy, by Don Freeman or a similar book featuring buttons. Announce that the follow day will be Button Day, and send a note home to parents. Students should wear clothing with at least one button or more if at all possible. Discuss the difference between a button and a snap with the class and point out various pieces of clothing that may have buttons on them.

Explore

Allow students time to determine how many buttons everyone in the class is wearing. Provide students a variety of tools: paper, pencil, unifix cubes, and a variety of math manipulatives. Have students decide how they will collect and display their data. Provide class check list to aide in data collection. Student will move freely around the room to collect their individual data.

Explain

Students will model their data on their workspace. Students may present their data by making charts, unifix trains, grouping, etc. Students will share and compare their data with classmates.

Elaborate

Group discussion to include the following questions:

Who found the least/ most amount of buttons?

Why do we have different totals?

Have students represent the number of buttons they are personally wearing with unifix trains. Partner students and have them write an addition word problem and equation to figure out their total. Students will share and compare their results with the class.

Evaluate

Teacher will observe word problems, equations, unifix trains and strategies to make a formative assessment of the class.

Extend

Students will participate in a center activity to reinforce solving word problems and equations using addition. Ex: Students will draw 2 cards from a stack of cards that have basic word sentences on them. "I have 3 red buttons." and "I have 5 yellow buttons." will be represented in a number equation and can be modeled through drawing colored buttons.

Sources

https://sites.google.com/a/bryantschools.org/math-common-core-resource-site/home-1/1st-grade/10a1

https://www.**georgiastandards.org**

First Grade- Sample Assessment ~ Constructed Response

From Houghton Mifflin *Math Expressions.*

Story Problem Strategies

What Is Assessed

- Represent and solve addition story problems.
- Solve addition equations.
- Represent and solve subtraction story problems.
- Solve subtraction equations.

Explaining the Assessment

1. Show 10 counters, a bowl, and Number Cards 1–9. Ask a child to choose a Number Card. Count out that many counters and place them under the bowl. Ask:
   - How many counters are there altogether?
   - How many counters are hiding?
   - How many counters are left?

   Repeat with other numbers.
2. Read the activity aloud with the class. In Questions 1 and 2, emphasize that children can choose any number at the top of the page, but that they need to choose different numbers for each problem.

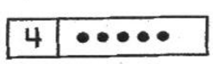
Materials

10 counters, bowl, Number Cards 1–9

Possible Responses

Questions 1 and 2: Answers will vary depending on which numbers the children choose. Solutions may include drawings, counting on, Math Mountains, or equations.

Sample solutions:

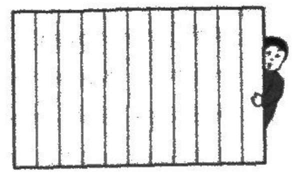


Question 3: Answers will vary.

Name ______________________________

ACTIVITY Hide and Seek

Some children are playing hide and seek.
Choose one of these numbers for each ◯.



3 4 5 7

Solve the story problems.

1. 10 children are playing. ◯ go and hide. How many are left? ☐ ______ label	
2. ◯ children are hiding. ◯ more children hide. How many children are hiding? ☐ ______ label	

3. Write the equations for Problems 1 and 2.

______________________ ______________________

Performance Assessment Rubric

An Exemplary Response (4 points)

- Uses a mathematical strategy to solve each problem efficiently
- Solves addition and subtraction story problems correctly
- Writes addition and subtraction as an equation and solves both correctly

A Proficient Response (3 points)

- Uses a mathematical strategy to solve each problem
- Solves addition or subtraction story problem correctly
- Writes addition and subtraction as an equation but solves one incorrectly

An Acceptable Response (2 points)

- Shows a strategy to solve each problem
- Solves addition or subtraction story problems correctly
- Writes addition or subtraction equation incorrectly

A Limited Response (1 point)

- Shows random or unidentifiable strategies to solve each problem
- Solves addition or subtraction story problems correctly
- Writes incomplete addition or subtraction equations

NBT Task 1i	
Domain	Number and Operations in Base Ten
Cluster	Extend the counting sequence.
Standard(s)	**1.NBT.1** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
Materials	Pencil, Paper
Task	Provide materials to the student. Read the problem to the student: *Mrs. Scrinzi is counting students as they enter the classroom. She has just counted the 8th student. What numbers will Mrs. Scrinzi say for the next 5 students?* 8, __, ___, ___, ___, ___. DOK 3: Teachers ask, "When have you had to count on?" "Why is it helpful to count on?"

Continuum of Understanding	
Developing Understanding	• Incorrectly states a number in the counting sequence. • Skips a number, but continues the counting sequence correctly.
Complete Understanding	• Correctly counts: 9, 10, 11, 12, 13.

Standards for Mathematical Practice
1. Makes sense and perseveres in solving problems.
2. Reasons abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.
4. Models with mathematics.
5. Uses appropriate tools strategically.
6. Attends to precision.
7. Looks for and makes use of structure.
8. Looks for and expresses regularity in repeated reasoning.

From www.ncpublicschools.org, Retrieved 6/2013.

Howard County Public School System • Draft 2011/2012
Adapted from Van de Walle, J. (2004) Elementary and Middle School Mathematics: Teaching Developmentally. Boston: Pearson Education, 65

Not yet: Student shows evidence of misunderstanding, incorrect concept or procedure		**Got It:** Student essentially understands the target concept.	
NEEDS IMPROVEMENT (N)	**WITH ASSISTANCE (W)**		**INDEPENDENT (I)**
0 Unsatisfactory: Little Accomplishment The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little or no success. Further teaching is required.	**1Marginal: Partial Accomplishment** Part of the task is accomplished, but there is lack of evidence of understanding or evidence of not understanding. Further teaching is required.	**2 Proficient: Substantial Accomplishment** Student could work to full accomplishment with minimal feedback from teacher. Errors are minor. Teacher is confident that understanding is adequate to accomplish the objective with minimal assistance.	**3 Excellent: Full Accomplishment** Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas. May have minor errors.

http://commoncoretasks.ncdpi.wikispaces.net/First+Grade+Tasks

SECOND GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grade 2 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
2.MP.1. Make sense of problems and persevere in solving them.	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach.
2.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length.
2.MP.3. Construct viable arguments and critique the reasoning of others.	Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask appropriate questions.
2.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
2.MP.5. Use appropriate tools strategically.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.
2.MP.6. Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
2.MP.7. Look for and make use of structure.	Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).
2.MP.8. Look for and express regularity in repeated reasoning.	Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, "Does this make sense?"

Explanations and Examples Grade 2
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Grade 2 Content Standards

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 2 Overview

Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data
- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry
- Reason with shapes and their attributes.

Operations and Algebraic Thinking — 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations within a cultural context, including those of Montana American Indians, of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten — 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

Measurement and Data — 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems within a cultural context, including those of Montana American Indians, involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set from a variety of cultural contexts, including those of Montana American Indians, with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

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Geometry 2.G

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[4] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. [5]

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[1] See Glossary, Table 1.
[2] See standard 1.OA.6 for a list of mental strategies.
[3] Explanations may be supported by drawings or objects
[4] See Glossary, Table 1
[5] Sizes are compared directly or visually, not compared by measuring.

GRADE 2

Domain	Cluster	Code	Common Core State Standard
Operations and Algebraic Thinking	Represent and solve problems involving addition and subtraction.	2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations within a cultural context, including those of Montana American Indians, of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	Add and subtract within 20.	2.OA.2	Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
	Work with equal groups of objects to gain foundations for multiplication.	2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
		2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Number and Operations in Base Ten	Understand place value.	2.NBT.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight or nine hundreds (and 0 tens and 0 ones).
		2.NBT.2	Count within 1000; skip-count by 5s, 10s, and 100s.
		2.NBT.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
		2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
	Use place value understanding and properties of operations to add and subtract.	2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
		2.NBT.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.
		2.NBT.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose
		2.NBT.8	Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
		2.NBT.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

GRADE 2

Domain	Cluster	Code	Common Core State Standard
Measurement and Data	Measure and estimate lengths in standard units.	2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
		2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
		2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.
		2.MD.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
	Relate addition and subtraction to length.	2.MD.5	Use addition and subtraction within 100 to solve word problems within a cultural context, including those of Montana American Indians, involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem
		2.MD.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ... , and represent whole-number sums and differences within 100 on a number line diagram.
	Work with time and money.	2.MD.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
		2.MD.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
	Represent and interpret data.	2.MD.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
		2.MD.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set from a variety of cultural contexts, including those of Montana American Indians, with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.
Geometry	Reason with shapes and their attributes.	2.G.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)
		2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
		2.G.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Getting Every Montana Student College and Career Ready

Montana Common Core Standards and Assessments

Montana Curriculum Organizer

Grade 2

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

November 2012 Materials adapted from Arizona, Delaware and Ohio Departments of Education

Montana Curriculum Organizer: Grade 2 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

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Montana Curriculum Organizer: Grade 2 Mathematics

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HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade-level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 20 consist of tables organized into learning progressions that can function as units. The table for each learning progression unit includes: 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.	Standards covered in this learning progression.
UNDERSTAND:	
What students need to understand by the end of this learning progression.	
KNOW:	**DO:**
What students need to know by the end of this learning progression.	What students need to be able to do by the end of this learning progression, organized by cluster and standard.
KEY TERMS FOR THIS PROGRESSION:	
Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.	
INSTRUCTIONAL STRATEGIES AND RESOURCES:	
Cluster: Title Strategies for this cluster **Instructional Resources/Tools** Resources and tools for this cluster	
Cluster: Title Strategies for this cluster **Instructional Resources/Tools** Resources and tools for this cluster	
CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:	
Standards that connect to this learning progression are listed here, organized by cluster.	
STANDARDS FOR MATHEMATICAL PRACTICE:	
A quick reference guide to the eight standards for mathematical practice is listed here.	

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Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Grade 2 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
2.MP.1. Make sense of problems and persevere in solving them.	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach.
2.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second-graders begin to know and use different properties of operations and relate addition and subtraction to length.
2.MP.3. Construct viable arguments and critique the reasoning of others.	Second-graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like: “How did you get that?”, “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions.
2.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
2.MP.5. Use appropriate tools strategically.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second-graders may decide to solve a problem by drawing a picture rather than writing an equation.
2.MP.6. Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
2.MP.7. Look for and make use of structure.	Second-graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).
2.MP.8. Look for and express regularity in repeated reasoning.	Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, “Does this make sense?”

CRITICAL AREAS FOR GRADE 2 MATH

In Grade 2, instructional time should focus on four critical areas:

(1) extending understanding of base-ten notation;
(2) building fluency with addition and subtraction;
(3) using standard units of measure; and
(4) describing and analyzing shapes.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Operations & Algebraic Thinking – Add/Sub within 100 & Foundations for Multiplication	2.OA.1, 2.OA.2, 2.OA.3, 2.OA.4

UNDERSTAND:

There are multiple ways to represent and find sums/differences within 100 (e.g., story problems, pictures, equations, computational strategies, manipulatives, and arrays).

KNOW:

Addition and subtraction are related operations.

Subtraction can be perceived as an unknown addend problem.

Addition and subtraction problems can be posed with the missing part being in different positions.

Word problems may require one or two computations to find a solution.

Mental strategies for adding single-digit numbers to know combinations to 20 fluently (e.g., doubles + 1, Make a Ten, Ten plus …, 9 + …).

The objects in an even number set can be paired or broken into two equal groups, and an odd number set of objects cannot.

Methods for recording addition and subtraction strategies using number lines and equations.

Symbols can represent an unknown quantity in an equation.

Rectangular arrays can represent the relationship between repeated addition and the foundations of multiplication.

DO:

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations within a cultural context, including those of Montana American Indians, of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).[1]

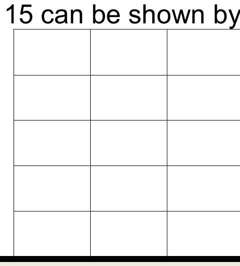
Add and subtract within 20.

2.OA.2 Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members (e.g., by pairing objects or counting them by 2's); write an equation to express an even number as a sum of two equal addends.

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. For example, 5 + 5 + 5 = 15 can be shown by a 3 x 5 rectangle:



KEY TERMS FOR THIS PROGRESSION:

Array, Difference, Equation, Even, Odd, Sum

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Represent and solve problems involving addition and subtraction.

Students now build on their work with one-step problems to solve two-step problems. Second-graders need to model and solve problems for all the situations shown in Table 1 on page 72 in the *Montana Common Core Standards for School Mathematics Grade-Band* and represent their solutions with equations. The problems should involve sums and differences less than or equal to 100 using the numbers 0 to 100. It is vital that students develop the habit of checking their answer to a problem to determine if it makes sense for the situation and the questions being asked.

Ask students to write word problems for their classmates to solve. Start by giving students the answer to a problem. Then tell students whether it is an addition or subtraction problem situation. Also let them know that the sums and differences can be less than or equal to 100 using the numbers 0 to 100. For example, ask students to write an addition word problem for their classmates to solve which requires adding four two-digit numbers with 100 as the answer. Students then share, discuss and compare their solution strategies after they solve the problems.

[1] See Glossary, Table 1 in the MCCS document.
[2] See standard 1.OA.6 for a list of mental strategies.

Instructional Resources/Tools
Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band*: Table 1 on page 72 illustrates 12 addition and subtraction problem situations.

National Council of Teachers of Mathematics. 2000-2012. *Mathematics and Football: Get the Picture — Get the Story.* In this lesson, students act as reporters at the Super Bowl. Students study four pictures of things that they would typically find at a football game then create problem situations that correspond to their interpretation of each of the pictures.

Cluster: Add and subtract within 20.
Provide many activities that will help students develop a strong understanding of number relationships, addition and subtraction so they can develop, share and use efficient strategies for mental computation. An efficient strategy is one that can be done mentally and quickly. Students gain computational fluency, using efficient and accurate methods for computing, as they come to understand the role and meaning of arithmetic operations in number systems. Efficient mental processes become automatic with use.

Provide activities in which students apply the commutative and associative properties to their mental strategies for sums less or equal to 20 using the numbers 0 to 20.

Have students study how numbers are related to 5 and 10 so they can apply these relationships to their strategies for knowing 5 + 4 or 8 + 3. Students might picture 5 + 4 on a ten-frame to mentally see 9 as the answer. For remembering 8 + 7, students might think: since 8 is 2 away from 10, take 2 away from 7 to make 10 + 5 = 15.

Provide simple word problems designed for students to invent and try a particular strategy as they solve it. Have students explain their strategies so that their classmates can understand it. Guide the discussion so that the focus is on the methods that are most useful. Encourage students to try the strategies that were shared so they can eventually adopt efficient strategies that work for them.

Make posters for student-developed mental strategies for addition and subtraction within 20. Use names for the strategies that make sense to the students and include examples of the strategies.

Present a particular strategy along with the specific addition and subtraction facts relevant to the strategy. Have students use objects and drawings to explore how these facts are alike.

Instructional Resources/Tools
National Council of Teachers of Mathematics. 2000-2012.

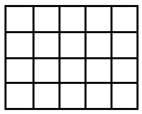
Comparing Connecting Cubes: Looking back and Moving Forward: In the game Race to Zero at the bottom of the page, students take turns rolling a number cube and subtracting the number they rolled each time from 20. The first person to reach 0 wins the round.

Do It with Dominoes: Finding Fact Families: In this lesson, the relationship of subtraction to addition is introduced with a book and with dominoes.

Pearson Education, Inc. 2012. *Five-frames and ten-frames*

Cluster: Work with equal groups of objects to gain foundations for multiplication.
Students need to understand that a collection of objects can be one thing (a group) and that a group contains a given number of objects. Investigate separating no more than 20 objects into two equal groups. Find the numbers (the total number of objects in collections up to 20 members) that will have some objects and no objects remaining after separating the collections into two equal groups. Odd numbers will have some objects remaining while even numbers will not. For an even number of objects in a collection, show the total as the sum of equal addends (repeated addition).

A rectangular array is an arrangement of objects in horizontal rows and vertical columns. Arrays can be made out of any number of objects that can be put into rows and columns. All rows contain the same number of items and all columns contain an equal number of items. Have students use objects to build all the arrays possible with no more than 25 objects. Their arrays should have up to five rows and up to five columns. Ask students to draw the arrays on grid paper and write two different equations under the arrays: one showing the total as a sum by rows and the other showing the total as a sum by columns. Both equations will show the total as a sum of equal addends.



The equation by rows: 20 = 5 + 5 + 5 +5

The equation by columns: 20 = 4 + 4 + 4 + 4 + 4

Build on knowledge of composing and decomposing numbers to investigate arrays with up to five rows and up to five columns in different orientations. For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends two different ways: by rows, 12 = 4 + 4 + 4; by columns, 12 = 3 + 3 + 3 + 3. Rotate the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, 12 = 3 + 3 + 3 + 3; by columns, 12 = 4 + 4 + 4. Have students discuss this statement and explain their reasoning: The two arrays are different and yet the same.

Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row. Students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2. Then students put two full ten-frame together end-to-end so they form 10 rows of 2 circles or 10 columns of 2 circles. They use this larger array to count by 2's up to 20 and write an equation that shows 20 equal to the sum of ten 2's.

Instructional Resources/Tools
Linking cubes
Tiles

Pearson Education, Inc. 2012.
- Five-frames and ten-frames
- Grid paper

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Reason with shapes and their attributes. (2.G.2)
Use place-value understanding and properties of operations to add and subtract. (2.NBT.5, 2.NBT.6, 2.NBT.9)
Relate addition and subtraction to length. (2.MD.5)
Work with time and money. (2.MD.8)
Represent and interpret data. (2.MD.10)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Understanding Place Value	2.NBT.1, 2.NBT.2, 2.NBT.3, 2.NBT.4

UNDERSTAND:

Three-digit numbers are composed of hundreds, tens, and ones.

KNOW:	DO:
2.NBT.1 The three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones). Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens – called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, or 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). The repeating patterns of the counting sequence up to 1,000. The meaning of recording symbols >, =, <.	***Understand place value.*** **2.NBT.2** Count within 1,000; skip-count by 5's, 10's, and 100's. **2.NBT.3** Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form. **2.NBT.4** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

KEY TERMS FOR THIS PROGRESSION:

Break apart, Digits, Hundreds, Ones, Place value, Put together, Tens

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Understand place value.

The understanding that 100 is 10 tens or 100 ones is critical to the understanding of place value. Using proportional models like base-ten blocks and bundles of tens along with numerals on place-value mats provides connections between physical and symbolic representations of a number. These models can be used to compare two numbers and identify the value of their digits.

Model three-digit numbers using base-ten blocks in multiple ways. For example, 236 can be 236 ones, or 23 tens and 6 ones, or 2 hundreds, 3 tens and 6 ones, or 20 tens and 36 ones. Use activities and games that have students match different representations of the same number.

Provide games and other situations that allow students to practice skip-counting. Students can use nickels, dimes and dollar bills to skip count by 5, 10 and 100. Pictures of the coins and bills can be attached to models familiar to students: a nickel on a five-frame with 5 dots or pennies and a dime on a ten-frame with 10 dots or pennies.

On a number line, have students use a clothespin or marker to identify the number that is ten more than a given number or five more than a given number.

Have students create and compare all the three-digit numbers that can be made using numbers from 0 to 9. For instance, using the numbers 1, 3, and 9, students will write the numbers 139, 193, 319, 391, 913 and 931. When students compare the numerals in the hundreds place, they should conclude that the two numbers with 9 hundreds would be greater than the numbers showing 1 hundred or 3 hundreds. When two numbers have the same digit in the hundreds place, students need to compare their digits in the tens place to determine which number is larger.

Instructional Resources/Tools

Base-ten blocks

Pictures of nickels and dimes

Pearson Education, Inc. 2012.

Base-ten grid paper

Five-frames and Ten-frames

Montana Curriculum Organizer: Grade 2 Mathematics

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Utah State University. National Library of Virtual Manipulatives. 1999-2010.

- *Base-ten blocks*
- *Hundreds chart (Use for counting by 5's and 10's.)*
- *Place-value number line*

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
Use place-value understanding and properties of operations to add and subtract. (2.NBT.7, 2.NBT.8, 2.NBT.9) **Work with time and money.** (2.MD.8)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Adding & Subtracting Within 1,000 including Place Value	2.NBT.5, 2.NBT.6, 2.NBT.7, 2.NBT.8, 2.NBT.9

UNDERSTAND:

Numbers can be composed and decomposed into place-value parts to add and subtract multi-digit numbers efficiently.

KNOW:	DO:
The strategy of mentally adding and subtracting 10 or a 100 to a given number. Addition and subtraction are related operations. Commutative and associative properties of operations can be used to solve problems. For example, students know that if 120 + 140 = 260, the 140 + 120 = 260 without actually naming the commutative property. Students know if 2 + 3 + 4 = 9, then they will know that 4 + 3 + 2 = 9 without actually naming the associative property. Place-value strategies for adding and subtracting (counting on, making 10's/100's, breaking apart and putting together, using known facts). Models for adding and subtracting (number line, base-ten materials). Methods for recording addition and subtraction strategies using number lines and equations. Symbols can represent an unknown quantity in an equation.	***Use place-value understanding and properties of operations to add and subtract.*** **2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **2.NBT.6** Add up to four two-digit numbers using strategies based on place value and properties of operations. **2.NBT.7** Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **2.NBT.8** Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. **2.NBT.9** Explain why addition and subtraction strategies work, using place value and the properties of operations.

KEY TERMS FOR THIS PROGRESSION:

Difference, Digit, Mental math, Model, Strategy, Sum
Place-value words: **Ones, Tens, Hundreds, Thousands**

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Use place-value understanding and properties of operations to add and subtract.
Provide many activities that will help students develop a strong understanding of number relationships, addition and subtraction so they can develop, share and use efficient strategies for mental computation. An efficient strategy is one that can be done mentally and quickly. Students gain computational fluency, using efficient and accurate methods for computing, as they come to understand the role and meaning of arithmetic operations in number systems. Efficient mental processes become automatic with use.

Students need to build on their flexible strategies for adding within 100 in Grade 1 to fluently add and subtract within 100, add up to four two-digit numbers, and find sums and differences less than or equal to 1,000 using numbers 0 to 1,000.

Initially, students apply base-ten concepts and use direct modeling with physical objects or drawings to find different ways to solve problems. They move to inventing strategies that do not involve physical materials or counting by ones to solve problems. Student-invented strategies likely will be based on place-value concepts, the commutative and associative properties, and the relationship between addition and subtraction. These strategies should be done mentally or with a written record for support.

It is vital that student-invented strategies be shared, explored, recorded and tried by others. Recording the expressions and equations in the strategies horizontally encourages students to think about the numbers and the quantities they represent instead of the digits. Not every student will invent strategies, but all students can and will try strategies they have seen that make sense to them. Different students will prefer different strategies.

Students will decompose and compose tens and hundreds when they develop their own strategies for solving problems where regrouping is necessary. They might use the make-ten strategy (37 + 8 = 40 + 5 = 45, add 3 to 37 then 5) or (62 - 9 = 60 – 7 = 53, take off 2 to get 60, then 7 more) because no ones are exchanged for a ten or a ten for ones.

Have students analyze problems before they solve them. Present a variety of subtraction problems within 1,000. Ask students to identify the problems requiring them to decompose the tens or hundreds to find a solution and explain their reasoning.

Instructional Resources/Tools

Groupable materials

- Dried beans and small cups for groups of 10 beans
- Linking cubes
- Plastic chain links

Pre-grouped materials

- Base-ten blocks
- Dried beans and beans sticks (10 dried beans glued on a craft stick – 10 sticks can be bundled for 100)

Pearson Education, Inc. 2012:

- *Strips (ten connected squares) and squares (singles)*
- *Ten-frame*
- *Place-value mat with ten-frames*
- *Hundreds chart (numbers 1-100) and blank hundreds chart (Add numbers 101-120 and attach to hundreds chart.)*

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Relate addition and subtraction to length. (2.MD.5, 2.MD.6)
Work with time and money. (2.MD.8)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Measurement (Length, Time, Money)	2.MD.1, 2.MD.2, 2.MD.3, 2.MD.4, 2.MD.5, 2.MD.6, 2.MD.7, 2.MD.8

UNDERSTAND:

Tools that measure length, time, and money must have equal intervals between units (e.g.; clocks, number lines, coins).

KNOW:	DO:
The appropriate tool and unit of measure should be selected based on the context of the situation. Estimating strategies can be applied to measuring lengths to the closest standard unit of measure. Lengths of an object can be compared by using various units of measure. The value of the measurement of an object will be different depending on the size of the units used to measure it. When you compare two lengths, you are finding the difference. Strategies used for solving and representing addition/subtraction problems can be utilized to solve and represent measurement word problems involving length, money, and time. Methods for recording addition and subtraction strategies using number lines and equations. Symbols can represent an unknown quantity in an equation. Consecutive whole numbers are equidistant on a number line (e.g.; 0-10, 10-20, 20-30, etc.). The number line can be utilized as a model for adding and subtracting within 100. Time intervals on analog clock skip-count by 5 minutes. The time before 12 noon is a.m., and the time 12 noon and after is p.m.	***Measure and estimate lengths in standard units.*** **2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.MD.2** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **2.MD.3** Estimate lengths using units of inches, feet, centimeters, and meters. **2.MD.4** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. ***Relate addition and subtraction to length.*** **2.MD.5** Use addition and subtraction within 100 to solve word problems within a cultural context, including those of Montana American Indians, involving lengths that are given in the same units (e.g., by using drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ... and represent whole-number sums and differences within 100 on a number line diagram. ***Work with time and money.*** **2.MD.7** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **2.MD.8** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *For example, "If you have 2 dimes and 3 pennies, how many cents do you have?"*

KEY TERMS FOR THIS PROGRESSION:

Analog clock, Estimate, Length, Measure, Standards, Units

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Measure and estimate lengths in standard units.

Second-graders are transitioning from measuring lengths with informal or nonstandard units to measuring with these standard units: inches, feet, centimeters, and meters. The measure of length is a count of how many units are needed to match the length of the object or distance being measured. Students have to understand what a length unit is and how it is used to find a measurement. They need many experiences measuring lengths with appropriate tools so they can become very familiar with the standard units and estimate lengths. Use language that reflects the approximate nature of measurement, such as the length of the room is about 26 feet.

Have students measure the same length with different-sized units then discuss what they noticed. Ask questions to guide the discussion so students will see the relationship between the size of the units and measurement (i.e., the measurement made with the smaller unit is more than the measurement made with the larger unit and vice versa).

Insist that students always estimate lengths before they measure. Estimation helps them focus on the attribute to be measured, the length units, and the process. After they find measurements, have students discuss the estimates, their procedures for finding the measurements and the differences between their estimates and the measurements.

Instructional Resources/Tools
Centimeter rulers and tapes
Inch rulers and tapes
Meter sticks
Yardsticks

Cluster: Relate addition and subtraction to length.
Connect the whole-number units on rulers, yardsticks, meter sticks and measuring tapes to number lines showing whole-number units starting at 0. Use these measuring tools to model different representations for whole-number sums and differences less than or equal to 100 using the numbers 0 to 100.

Use the meter stick to view units of ten (10 cm) and hundred (100 cm), and to skip count by 5's and 10's.

Provide one- and two-step word problems that include different lengths measurement made with the same unit (inches, feet, centimeters, and meters). Students add and subtract within 100 to solve problems for these situations: adding to, taking from, putting together, taking apart, and comparing, and with unknowns in all positions. Students use drawings and write equations with a symbol for the unknown to solve the problems.

Have students represent their addition and subtraction within 100 on a number line. They can use notebook or grid paper to make their own number lines. First, they mark and label a line on paper with whole-number units that are equally spaced and relevant to the addition or subtraction problem. Then they show the addition or subtraction using curved lines segments above the number line and between the numbers marked on the number line. For 49 + 5, they start at 49 on the line and draw a curve to 50, then continue drawing curves to 54. Drawing the curves or making the hops between the numbers will help students focus on a space as the length of a unit and the sum or difference as a length.

Instructional Resources/Tools
Cash register tapes or paper strips
Measuring tapes
Meter sticks
Rulers
Yardsticks

National Council of Teachers of Mathematics. 2000-2012.

How Many More Fish? Hopping Backward to Solve Problems: In this lesson, students determine differences using the number line to compare lengths.
Macaroni Math: Where Will I Land? In this lesson, the students find differences using the number line, a continuous model for subtraction.

Cluster: Work with time and money.
Second-graders expand their work with telling time from analog and digital clocks to the nearest hour or half-hour in Grade 1 to telling time to the nearest five minutes using a.m. and p.m.

The topic of money begins at Grade 2 and builds on the work in other clusters in this and previous grades. Help students learn money concepts and solidify their understanding of other topics by providing activities where students make connections between them. For instance, link the value of a dollar bill as 100 cents to the concept of 100 and counting within 1,000. Use play money (i.e., nickels, dimes, and dollar bills) to skip count by 5's, 10's, and 100's. Reinforce place value concepts with the values of dollar bills, dimes, and pennies.

Students use the context of money to find sums and differences less than or equal to 100 using the numbers 0 to 100. They add and subtract to solve one- and two-step word problems involving money situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. Students use drawings and equations with a symbol for the unknown number to represent the problem. The dollar sign ($) is used for labeling whole-dollar amounts without decimals, such as $29.

Students need to learn the relationships between the values of a penny, nickel, dime, quarter and dollar bill.

Instructional Resources/Tools
Play money

National Council of Teachers of Mathematics. 2000-2012.

Coin Box: This game will help students learn how to count, collect, exchange and make change for coins.
Number Cents: In this unit, students explore the relationship between pennies, nickels, dimes, and quarters. They count sets of mixed coins, write story problems that involve money, and use coins to make patterns.

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Time – Match Clocks:* Students manipulate a digital clock to show the time given on an analog clock. They can also manipulate the hands on a face clock to show the time given on a digital clock. Times are given to the nearest five minutes.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Represent and interpret data. (2.MD.9)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Data – Represent & Interpret	2.MD.9, 2.MD.10

UNDERSTAND:

Data can be organized, represented, and interpreted in multiple ways for a variety of purposes.

KNOW:	DO:
Data can be organized and represented in multiple ways. Data presented in graphs can be interpreted and manipulated to solve problems.	***Represent and interpret data.*** **2.MD.9** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **2.MD.10** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set from a variety of cultural contexts, including those of Montana American Indians, with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

KEY TERMS FOR THIS PROGRESSION:

Bar graph, Line, Plot, Measure, Picture graph, Scale

INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Represent and interpret data.

Line plots are useful tools for collecting data because they show the number of things along a numeric scale. They are made by simply drawing a number line then placing an "X" above the corresponding value on the line that represents each piece of data. Line plots are essentially bar graphs with a potential bar for each value on the number line.

Pose a question related to the lengths of several objects. Measure the objects to the nearest whole inch, foot, centimeter or meter. Create a line plot with whole-number units (0, 1, 2, ...) on the number line to represent the measurements.

At first students should create real-object and picture graphs so each row or bar consists of countable parts. These graphs show items in a category and do not have a numerical scale. For example, a real-object graph could show the students' shoes (one shoe per student) lined end to end in horizontal or vertical rows by their color. Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to two to four rows or bars.

Students would then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale. Use the information in the graphs to pose and solve simple put-together, take-apart, and compare problems illustrated in Table 1 on page 72 in the *Montana Common Core Standards for School Mathematics Grade-Band*.

Instructional Resources/Tools

Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band*: Table 1 on page 72.

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Bar Chart:* This manipulative can be used to make a bar chart with 1 to 20 for the vertical axis and 1 to 12 bars on the horizontal axis. The colors for the bars are predetermined however, users can type in their own title for the graph and labels for the bars.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

None

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometry – Reason with Shapes & Their Attributes	2.G.1, 2.G.2, 2.G.3

UNDERSTAND:

Shapes have defining attributes that can be utilized for comparing and composing/constructing.
Rectangular arrays promote the connection between geometry and the foundations multiplication.
Decomposing shapes into equal-size pieces promotes the connection between geometry and fractional concepts.

KNOW:	DO:
Angles and sides are important specified attributes of 2D shapes. Faces, edges, and vertices are important specified attributes of 3D shapes. Distinguishing features of 2D and 3D shapes. Equal shares of identical wholes do not need to have the same shape. For example, ¼ of a square can look different for different equal squares. Rectangular arrays can represent the relationship between repeated addition and the foundations of multiplication.	**Reason with shapes and their attributes.** **2.G.1** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[3] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **2.G.2** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **2.G.3** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as *two halves, three thirds, four fourths*. Recognize that equal shares of identical wholes need not have the same shape.

KEY TERMS FOR THIS PROGRESSION:

Angles, Array, Columns, Cubes, Edges, Faces, Fourths, Halves, Hexagons, Pentagons, Polygons, Quadrilaterals, Regular, Rows, Sides, Thirds, Triangles

INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Reason with shapes and their attributes.

Modeling multiplication with partitioned rectangles promotes students' understanding of multiplication. Tell students that they will be drawing a square on grid paper. The length of each side is equal to 2 units. Ask them to guess how many 1 unit by 1 unit squares will be inside this 2 unit by 2 unit square. Students now draw this square and count the 1 by 1 unit squares inside it. They compare this number to their guess. Next, students draw a 2 unit by 3 unit rectangle and count how many 1 unit by 1 unit squares are inside. Now they choose the two dimensions for a rectangle, predict the number of 1 unit by 1 unit squares inside, draw the rectangle, count the number of 1 unit by 1 unit squares inside and compare this number to their guess. Students repeat this process for different-size rectangles. Finally, ask them to what they observed as they worked on the task.

It is vital that students understand different representations of fair shares. Provide a collection of different-size circles and rectangles cut from paper. Ask students to fold some shapes into halves, some into thirds, and some into fourths. They compare the locations of the folds in their shapes as a class and discuss the different representations for the fractional parts. To fold rectangles into thirds, ask students if they have ever seen how letters are folded to be placed in envelopes. Have them fold the paper very carefully to make sure the three parts are the same size. Ask them to discuss why the same process does not work to fold a circle into thirds.

Instructional Resources/Tools

Pearson Education, Inc. 2012. *Grid paper.*

Drexel University. The Math Forum. 1994-2012.

Equal Parts: Students learn to divide a circle into pieces of equal size.
Divide and Shade: Students shade equal parts to indicate fraction of the circle.
Extension Ideas: Introduction to fractions for primary students. This four-lesson unit introduces young children to fractions. Students learn to recognize equal parts of a whole as halves, thirds and fourths.

[3] Sizes are compared directly or visually, not compared by measuring.

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CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
Work with equal groups of objects to gain foundations for multiplication. (2.OA.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

SAMPLE YEAR LONG PLAN

COURSE: 2nd GRADE MATH

Unit	Add/ Subt within 100	Understanding Place Value within 100	Data- Represent & Interpret	Add/ Subt within 1000 including Place Value	Geometry Reason with shapes and their attributes
(Time)	30 days	30 days	10 days	30 days	15 days
STANDARDS	2 OA 2	2 NBT 1	2 MD 9	2 NBT 2	2 G 1
	2 OA 3	2 NBT 4	2 MD 10	2 NBT 3	2 G 2
	2 NBT 8	2 NBT 5		2 NBT 6	2 G 3
	2 NBT 9	2 OA 1		2 NBT 7	

SECOND GRADE MATH p. 2

Measurement Length	Measurement Time	Measurement Money	Foundations for Multiplication
15 days	10 days	10 days	10 days
2 MD 1	2 MD 7	2 MD 8	2 OA 4
2 MD 2			
2 MD 3			
2 MD 4			
2 MD 5			
2 MD 6			

TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: MATH GRADE LEVEL(S): 2nd Grade

LAST UNIT	**CURRENT UNIT** **Add/Subtract within 100**	NEXT UNIT

UNIT SCHEDULE

	Writing Addition Sentences
	Writing Subtraction Sentences
	Addition Word Problems
	Subtraction Word Problems
	Relating Addition & Subtraction
	Adding 0, 1, 2,
	Adding Doubles
	Adding in any Order
	Adding Three Numbers
	Thinking Addition to Subtract
	Models for Tens
	Models for Tens and Ones
	Reading and Writing Whole Numbers
	Using Models to Compare Numbers
	Using Symbols to Compare Numbers
	Ordering Numbers
	Number Patterns
	Even and Odd Numbers

is about… UNIT MAP

Represent and find sums/differences within 100

Fluently add/subtract within 20 and sums from memory of two one-digit numbers.

Explain why addition and subtraction strategies work using properties of operations and place value.

Mentally add and subtract 10 or 100 to/from a given number (100-900)

Determine odd/even numbers up to 20

UNIT SELF TEST QUESTIONS

1. What strategy did you use to find the sum? And why does it work?
2. What strategy did you use to find the difference? And why does it work?
3. How do you determine if a number is odd or even?
4. How can you model numbers in several ways?

MATH STANDARDS	
2 NBT 8	2 OA 2
2 NBT 9	2 OA 3

Second Grade Sample Lesson Plan
Odds and Evens

Common Core Standard:
Work with equal groups of objects to gain foundations for multiplication.
2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others..
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

Student Outcomes:
- I can write an equation to show that a number that is doubled has an even sum.
- I can explain why two even numbers have an even sum and why two odd numbers have an even sum and why an odd and even have an odd sum.

Materials:
- Odds and Evens gameboard (one for partners)
- Paperclip and pencil to use as spinner or a clear spinner to use on top of the gameboard
- Pencil to record on gameboard
- Color tiles or grid paper for students needing additional instruction
- Two of Everything by Lily Toy Hong
- Chart paper or a way to display the chart, marker
- Index cards with 1+1=, 2+2=, 3+3=, etc. to 10+10, one card for each set of partners
- Color tiles or grid paper to model

Engage
Give partners an index card with 1+1= or 2+2= or 3+3=, etc. Ask partners to find something or think of something in the real world that represents their equation. For example, 1+1= a pair of shoes, 4+4= the legs on an octopus (4 on each side), 5+5= the number of cents in a dime (nickel plus nickel)

Explore and Elaborate
Bring the cards back to the group and share the “doubles” found. Ask students about the sums. Do you notice what happens when you add two equal addends? Why do you think this happens? Brainstorm with the class and model with color tiles by creating rectangles to “prove” this concept.

Explain
Read Two of Everything to the class. Chart what happens when something is put in the pot. For example, if 3 of something goes in the pot, then how many come out? 3+3=6. Continue this with at least five examples.

Evaluate

Before:
What do you know about "doubles" facts?
How do we know if a number is odd or even?

During:
What have you noticed about the sums you are getting while playing the game?
What happens when you add two equal addends? Why do you think this happens?
Are you starting to notice what is happening when an even and an even are added together, odd and odd, even and odd?
If you played again would you like to be Even Steven or Odd Rod? Why?

After:
As a whole group discuss the questions listed above and focus on what student learned about odd and even addends.

Possible Misconceptions Suggestions

IF Students may think an odd and an odd will equal an odd.
THEN Show students a rectangle made with color tiles of an odd number and make another rectangle of an odd number then match the two odd tiles together so that it becomes even.

IFStudents may think an even and odd will equal an even.
THEN Repeat the task above using an odd and even number so students can see that you still have an odd tile left over.

EXTEND

Introduce the game Odds and Evens to the class by the teacher playing the game against the class. One player is Even Steven and one player is Odd Rod, each player spins one spinner and the two addends are added together. If the sum is even Steven records it by writing the equation on a blank sheet of paper or in their math journal, and then writing the sum in the box under Even Steven. If the sum is odd Rod records it by writing the equation on a blank sheet of paper or in their math journal, and then writing the sum in the box under Odd Rod and the number goes to Rod. The first player to fill all the blanks is the winner.
While the students are playing, the teacher should rotate around the room and see if students are starting to notice what is happening when an even and an even are added together, odd and odd, even and odd?
Ask students if they played again if they would like to be Even Steven or Odd Rod and why.
After playing discuss the game and the generalizations students were able to construct about even and odd numbers and what happens when you have two equal addends. As students share what they learned, the teacher could chart their ideas such as "odd + odd = even, odd + even=odd, even + even = even. "

Odd Todd and Even Steven game:
http://maccss.ncdpi.wikispaces.net/file/view/CCSSMathTasks-Grade2.pdf/376944368/CCSSMathTasks-Grade2.pdf

Unit 1 Assessment

1. Pick a number less than ten. What numbers are 2 more, 1 more, and o more than your number?

____

2. How are 2 less than 5 and 5 – 2 related?

____

3. What are two different ways you could add 3 + 4 + 7? Explain.

____

____

____

4. Sam scored 12 points in all. Write a number sentence for Sam’s points.

_________ + ________ + ________ =

43 34 61 58

1. Write two numbers from above in the boxes below. Choose an odd number for A. Choose an even number for B. Draw cubes to show the numbers.

Number A: _____

Number B: _____

2. Write the number of tens and ones for each of your numbers.

Number A has _____ tens and _____ ones.

Number B has _____ tens and _____ ones.

3. Use > or < to complete a sentence about your numbers.

_____ ◯ _____

Number A Number B

4. Use your two numbers. Write two even numbers that come between your numbers.

_____ _____ _____ _____

Number A Number B

Scoring Rubric Performance Task One Unit One-Second Grade

Standard to be achieved for performance of specified level

4-Point Answer The child correctly writes, and draws pictures for two selected numbers and identifies them as Numbers A and B; uses the inequality sign correctly to compare Numbers A and B; and writes two numbers that are between Numbers A and B.

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3 –Point Answer The child correctly writes, draws pictures, and states the numbers of tens and ones for two numbers, but numbers are not listed. The child uses the correct inequality sign to compare numbers. For Exercise 4, the child correctly writes two additional numbers that are between lesser and greater numbers written.

2-Point Answer The child correctly draws pictures for two numbers that are not listed at the top of the page, does not correctly state the numbers of tens and ones in the two numbers, and does not correctly compare the numbers. For exercise 4, the child writes Numbers A and B on the first two blanks and then incorrectly writes two additional numbers that are greater than or less than Numbers A and B.

1-Point Answer The child draws pictures for Numbers A and B that do not correspond to tens and ones in each number. The child incorrectly uses the inequality sign and is unable to write numbers that are between Numbers A and B.

- DOK Level 4: Remove given numbers and instruct students to choose their own numbers between 50 and 100.

THIRD GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grade 3 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
3.MP.1. Make sense of problems and persevere in solving them.	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
3.MP.2. Reason abstractly and quantitatively.	Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3.MP.3. Construct viable arguments and critique the reasoning of others.	In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
3.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
3.MP.5. Use appropriate tools strategically.	Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
3.MP.6. Attend to precision.	As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
3.MP.7. Look for and make use of structure.	In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
3.MP.8. Look for and express regularity in repeated reasoning.	Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Explanations and Examples Grade 3
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Grade 3 Content Standards

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 3 Overview

Operations and Algebraic Thinking
- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Geometry
- Reason with shapes and their attributes.

Number and Operations—Fractions
- Develop understanding of fractions as numbers.

Measurement and Data
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations within cultural contexts, including those of Montana American Indians. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when *a* whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
   b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories, within cultural contexts, including those of Montana American Indians. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
    b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
    a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
    b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
    c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
    d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems, including those of Montana American Indians.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry	**3.G**

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

---

[1] See Glossary, Table 2.
[2] Students need not use formal terms for these properties.
[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.
[4] A range of algorithms may be used.
[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.
[6] Excludes compound units such as cm3 and finding the geometric volume of a container.
[7] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

GRADE 3

Domain	Cluster	Code	Common Core State Standard
Operations and Algebraic Thinking	Represent and solve problems involving multiplication and division.	3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, , describe a context in which a total number of objects can be expressed as 5 × 7.
		3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
		3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
		3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = __÷ 3, 6 × 6 = ?.
	Understand properties of multiplication and the relationship between multiplication and division.	3.OA.5	Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by multiplying 3 × 5 = 15 then multiplying 15 × 2 = 30, or by multiplying 5 × 2 = 10 then multiplying 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 1 = 56. (Distributive property.) (Students need not use formal terms for these properties.)
		3.OA.6	Understand division as an unknown-factor problem. For example, divide 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
	Multiply and divide within 100.	3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By end of Grade 3, know from memory all products of one-digit numbers.
	Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.8	Solve two-step word problems using the four operations within cultural contexts, including those of Montana American Indians. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.)
		3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
Number and Operations in Base Ten	Use place value understanding and properties of operations to perform multi-digit arithmetic.	3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
		3.NBT.2	3.NBT.2 Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)
		3.NBT.3	3.NBT.3 Use place value understanding and properties of operations to perform multi-digit arithmetic. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (A range of algorithms may be used.)

GRADE 3

Domain	Cluster	Code	Common Core State Standard
Number and Operations: Fractions	Develop understanding of fractions as numbers.	3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
		3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
		3.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Measurement and Data	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
		3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes compound units such as cm^3 and finding the geometric volume of a container.)
	Represent and interpret data.	3.MD.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories, within cultural contexts, including those of Montana American Indians. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.
		3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters.
	Geometric measurement-- A. understand concepts of area and relate area to multiplication and to addition. B. recognize perimeter as an attribute of plane figures and distinguish between linear and area measures	3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
		3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
		3.MD.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems, including those of Montana American Indians.
		3.MD.8	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter.
Geometry	Reason with shapes and their attributes.	3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
		3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part is 1/4 of the area of the shape.

Getting Ready
Every Montana Student
College and Career

Montana Common Core
Standards and Assessments

Montana Curriculum Organizer

Grade 3

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

November 2012 Materials adapted from Arizona, Delaware and Ohio Departments of Education

Montana Curriculum Organizer: Grade 3 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

Montana Curriculum Organizer: Grade 3 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade-level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 26 consist of tables organized into learning progressions that can function as units. The table for each learning progression unit includes: 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

<table>
<tr><th>LEARNING PROGRESSION</th><th>STANDARDS IN LEARNING PROGRESSION</th></tr>
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<tr><th colspan="2">KEY TERMS FOR THIS PROGRESSION:</th></tr>
<tr><td colspan="2">Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.</td></tr>
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Montana Curriculum Organizer: Grade 3 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Grade 3 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
3.MP.1 Make sense of problems and persevere in solving them.	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third-graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
3.MP.2 Reason abstractly and quantitatively.	Third-graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3.MP.3 Construct viable arguments and critique the reasoning of others.	In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
3.MP.4 Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third-graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
3.MP.5 Use appropriate tools strategically.	Third-graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
3.MP.6 Attend to precision.	As third-graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
3.MP.7 Look for and make use of structure.	In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
3.MP.8 Look for and express regularity in repeated reasoning.	Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third-graders continually evaluate their work by asking themselves, "Does this make sense?"

CRITICAL AREAS FOR GRADE 3 MATH

In Grade 3, instructional time should focus on four critical areas:

(1) developing understanding of multiplication and division and strategies for multiplication and division within 100;
(2) developing understanding of fractions, especially unit fractions (fractions with numerator 1);
(3) developing understanding of the structure of rectangular arrays and of area; and
(4) describing and analyzing two-dimensional shapes.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Operations & Algebraic Thinking – Multiplication & Division	3.OA.1, 3.OA.2, 3.OA.3, 3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7, 3.OA.8, 3.OA9, 3.NBT.3

UNDERSTAND:

Multiplication and division situations involve equal-size groups, arrays, and/or area models.
Multiplication and division are inverse operations.
The commutative, associative, and distributive properties can be used to develop efficient strategies to multiply and divide. (Students do not need to know the names of these operations.)

KNOW:

Multiplication and division notation, including different division signs.

Methods of recording multiplication strategies include using equations and arrays.

A letter can be used to stand for an unknown quantity.

Division word problems can require finding the unknown number of groups or the unknown group size by grouping problems or by sharing problems.

Multiplication and division are inverse operations.

Fact families for multiplication and division.

DO:

Represent and solve problems involving multiplication and division.

3.OA.1 Interpret products of whole numbers (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each). *For example, describe a context in which a total number of objects can be expressed as 5 x 7.*

3.OA.2 Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each). *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).[1]

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48; 5 = � ÷ 3; and 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *For example, If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (commutative property of multiplication). 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (associative property of multiplication). Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (distributive property).*

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide by 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

[1] See Glossary, Table 2 in MCCS document.
[2] Students need not use formal terms for these properties.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations within cultural contexts, including those of Montana American Indians. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.)

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Use place value-understanding and properties of operations to perform multi-digit arithmetic.[3]

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

KEY TERMS FOR THIS PROGRESSION:

Area, Array, Equal, Estimation, Factor, Inverse operation, Multiple, Odd and even numbers

INSTRUCTIONAL STRATEGIES AND RESOURCES:

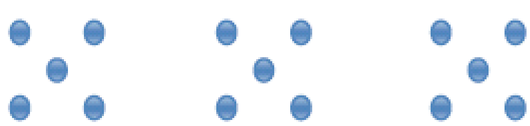
Cluster: Represent and solve problems involving multiplication and division.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This strategy is a foundation for multiplication because students should make a connection between repeated addition and multiplication.

Students need to experience problem-solving involving equal groups (whole unknown or size of group is unknown) and multiplicative comparison (unknown product, group size unknown or number of groups unknown) as shown in Table 2 on page 73 in the *Montana Common Core Standards for School Mathematics Grade-Band*. No attempt should be made to teach the abstract structure of these problems.

Encourage students to solve these problems in different ways to show the same idea and be able to explain their thinking verbally and in written expression. Allowing students to present several different strategies provides the opportunity for them to compare strategies.

Sets of counters, number lines to skip count and relate to multiplication and arrays/area models will aid students in solving problems involving multiplication and division. Allow students to model problems using these tools. They should represent the model used as a drawing or equation to find the solution.



This shows multiplication using grouping with 3 groups of 5 objects and can be written as 3 × 5.

Provide a variety of contexts and tasks so that students will have more opportunity to develop and use thinking strategies to support and reinforce learning of basic multiplication and division facts.

Have students create multiplication problem situations in which they interpret the product of whole numbers as the total

[3] A range of algorithms may be used.

number of objects in a group and write as an expression. Also, have students create division-problem situations in which they interpret the quotient of whole numbers as the number of shares.

Students can use known multiplication facts to determine the unknown fact in a multiplication or division problem. Have them write a multiplication or division equation and the related multiplication or division equation. For example, to determine the unknown whole number in 27 ÷ 9 = 3, students should use knowledge of the related multiplication fact of 3 × 9 = 27. They should ask themselves questions such as, “How many 3’s are in 27?” or “3 times what number is 27?” Have them justify their thinking with models or drawings.

Instructional Resources/Tools
Arrays
Number lines to skip count and relate to multiplication
Sets of counters

Montana Common Core Standards for School Mathematics Grade-Band.

National Council of Teachers of Mathematics. 2000-2012.

All About Multiplication: In this four-lesson unit, students explore several meanings and representation of multiplications and learn about properties of operations for multiplication.

All About Multiplication: Exploring equal sets: This four-part lesson encourages students to explore models for multiplication, the inverse of multiplication, and representing multiplication facts in equation form.

Utah State University. National Library of Virtual Manipulatives. 1999-2010.

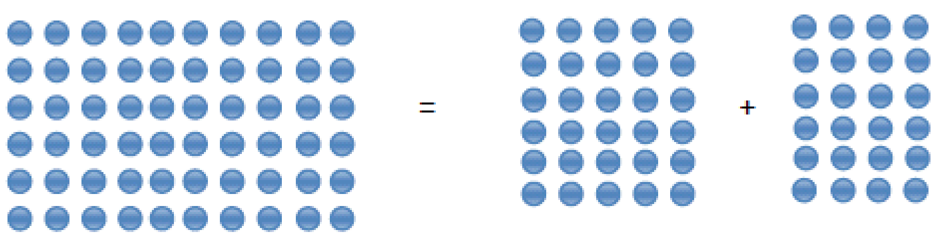
Number Line Arithmetic: This application illustrates arithmetic operations using a number line.

Number Line Bars: This application uses bars to show addition, subtraction, multiplication, and division on a number line.

Cluster: Understand properties of multiplication and the relationship between multiplication and division.
Students need to apply properties of operations (commutative, associative and distributive) as strategies to multiply and divide. Applying the concept involved is more important than students knowing the name of the property. Understanding the commutative property of multiplication is developed through the use of models as basic multiplication facts are learned. For example, the result of multiplying 3 x 5 = 15 is the same as the result of multiplying 5 x 3 = 15.

To find the product of three numbers, students can use what they know about the product of two of the factors and multiply this by the third factor. For example, to multiply 5 x 7 x 2, students know that 5 x 2 is 10. Then, they can use mental math to find the product of 10 x 7 = 70. Allow students to use their own strategies and share with the class when applying the associative property of multiplication.

Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty. For example, students can split a 6 x 9 array into 6 groups of 5 and 6 groups of 4; then, add the sums of the groups.



The 6 groups of 5 is 30 and the 6 groups of 4 is 24. Students can write 6 x 9 as 6 x 5 + 6 x 4.

Students’ understanding of the part/whole relationships is critical in understanding the connection between multiplication and division.

Instructional Resources/Tools
National Council of Teachers of Mathematics. 2010-2012.

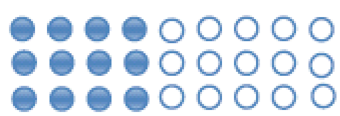
Multiplication: It's in the Cards: Looking for Calculator Patterns: Students use a web-based calculator to create and compare counting patterns using the constant function feature of the calculator. Making connections between multiple representations of counting patterns reinforces students understanding of this important idea and helps them recall these patterns as multiplication facts. From a chart, students notice that multiplication is commutative . *Multiplication: It's in the Cards: Looking for Patterns:* Students skip-count and examine multiplication patterns. They also explore the commutative property of multiplication.

Cluster: Multiply and divide within 100.
Students need to understand the part/whole relationships in order to understand the connection between multiplication and division. They need to develop efficient strategies that lead to the big ideas of multiplication and division. These big ideas include understanding the properties of operations, such as the commutative and associative properties of multiplication and the distributive property. The naming of the property is not necessary at this stage of learning.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This is called unitizing, and it requires students to count groups, not just objects. They see the whole as a number of groups of a number of objects. This strategy is a foundation for multiplication in that students should make a connection between repeated addition and multiplication.

As students create arrays for multiplication using objects or drawing on graph paper, they may discover that three groups of four and four groups of three yield the same results. They should observe that the arrays stay the same, although how they are viewed changes. Provide numerous situations for students to develop this understanding.

To develop an understanding of the distributive property, students need decompose the whole into groups. Arrays can be used to develop this understanding. To find the product of 3 × 9, students can decompose 9 into the sum of 4 and 5 and find 3 × (4 + 5).



The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers in Grade 5.

Once students have an understanding of multiplication using efficient strategies, they should make the connection to division.

Using various strategies to solve different contextual problems that use the same two one-digit whole numbers requiring multiplication allows for students to commit to memory all products of two one-digit numbers.

Instructional Resources/Tools
Grid or graph paper
Sets of counters
Unifix cubes or cubes

Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Students gain a full understanding of which operation to use in any given situation through contextual problems. Number skills and concepts are developed as students solve problems. Problems should be presented on a regular basis as students work with numbers and computations.

Researchers and mathematics educators advise against providing "key words" for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it. They should think through the problem and the meaning of the answer before attempting to solve it.

Encourage students to represent the problem situation in a drawing or with counters or blocks. Students should determine the reasonableness of the solution to all problems using mental computations and estimation strategies.

Students can use base-ten blocks on centimeter grid paper to construct rectangular arrays to represent problems.

Students are to identify and explain arithmetic patterns using properties of operations. They can explore patterns by determining likenesses, differences and changes. Use patterns in addition and multiplication tables.

Instructional Resources/Tools
National Council of Teachers of Mathematics. 2000-2012.

Multiplication: It's in the Cards: Looking for Calculator Patterns: Students use a web-based calculator to create and compare counting patterns using the constant function feature of the calculator. Making connections between multiple representations of counting patterns reinforces students understanding of this important idea and helps them recall these patterns as multiplication facts. From a chart, students notice that multiplication is commutative.
Multiplication: It's in the Cards: Looking for Patterns: Students skip-count and examine multiplication patterns. They also explore the commutative property of multiplication.
Multiplication: It's in the Cards: More Patterns with Products: Students look for patterns in the table.

Cluster: Use place-value understanding and properties of operations to perform multi-digit arithmetic.
Prior to implementing rules for rounding, students need to have opportunities to investigate place value. A strong understanding of place value is essential for the developed number sense and the subsequent work that involves rounding numbers.

Building on previous understandings of the place value of digits in multi-digit numbers, place value is used to round whole numbers. Dependence on learning rules can be eliminated with strategies such as the use of a number line to determine which multiple of 10 or of 100 a number is nearest (5 or more rounds up, less than 5 rounds down). As students' understanding of place value increases, the strategies for rounding are valuable for estimating, justifying and predicting the reasonableness of solutions in problem solving.

Strategies used to add and subtract two-digit numbers are now applied to fluently add and subtract whole numbers within 1,000. These strategies should be discussed so that students can make comparisons and move toward efficient methods.

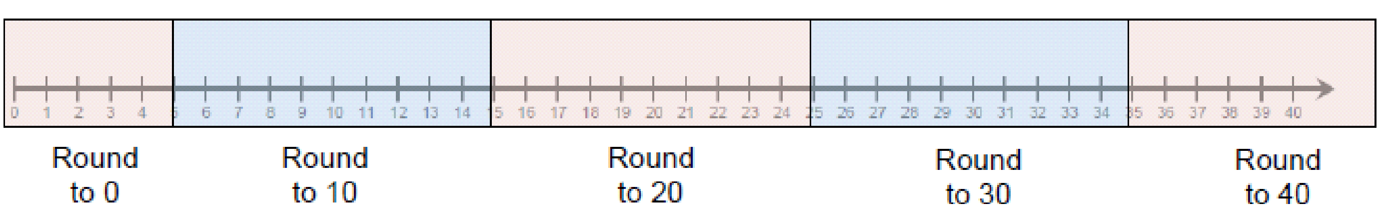
Number sense and computational understanding is built on a firm understanding of place value.

Understanding what each number in a multiplication expression represents is important. Multiplication problems need to be modeled with pictures, diagrams or concrete materials to help students understand what the factors and products represent. The effect of multiplying numbers needs to be examined and understood.

The use of area models is important in understanding the properties of operations of multiplication and the relationship of the factors and its product. Composing and decomposing area models is useful in the development and understanding of the distributive property in multiplication.

Continue to use manipulatives like hundreds charts and place-value charts. Have students use a number line or a roller coaster example to block off the numbers in different colors.

For example this chart show what numbers will round to the tens place.



Rounding can be expanded by having students identify all the numbers that will round to 30 or round to 200.

Instructional Resources/Tools
100's chart
Number lines

Charles A. Dana Center. University of Texas at Austin. Mathematics TEKS Toolkit. 2012. *Make a hundred:* Students roll a die seven times, each time determining whether to add that number of tens or that number of ones to make a sum as close to 100 as possible without going over.

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Rectangle Multiplication:* Visualize the multiplication of two numbers as area.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (3.MD.7)
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (3.MD.2)
Represent and interpret data. (3.MD.3)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Addition and Subtraction	3.NBT.1, 3.NBT.2

UNDERSTAND:

The value of a digit in our number system is determined by its place-value position.
Numbers can be decomposed and recomposed into component parts to add and subtract multi-digit numbers efficiently.

KNOW:	DO:
Using expanded notation for numbers up to 1,000. How to record addition and subtraction strategies using number lines and/or equations. A letter can be used to stand for an unknown quantity. Rounding is a formal way of estimating. When adding numbers, the order of the addends does not matter (e.g., 7 + 10 = 10 + 7. Commutative property). Numbers can be decomposed, recomposed, and re-ordered to make adding more efficient (e.g., 8 + 5 = 8 + (2 + 3) = (8 + 2) + 3 = 10 + 3 = 13 Associative property of addition). Addition and subtraction are related operations. A letter can be used to stand for an unknown quantity.	***Use place-value understanding and properties of operations to perform multi-digit arithmetic.***[4] **3.NBT.1** Use place-value understanding to round whole numbers to the nearest 10 or 100. **3.NBT.2** Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

KEY TERMS FOR THIS PROGRESSION:

Difference, Estimation, Reasonable (as in, "Is this answer reasonable?"), Rounding, Sum.

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Use place-value understanding and properties of operations to perform multi-digit arithmetic.
Prior to implementing rules for rounding, students need to have opportunities to investigate place value. A strong understanding of place value is essential for the developed number sense and the subsequent work that involves rounding numbers.

Building on previous understandings of the place value of digits in multi-digit numbers, place value is used to round whole numbers. Dependence on learning rules can be eliminated with strategies such as the use of a number line to determine which multiple of 10 or of 100, a number is nearest (5 or more rounds up, less than 5 rounds down). As students' understanding of place value increases, the strategies for rounding are valuable for estimating, justifying and predicting the reasonableness of solutions in problem-solving.

Strategies used to add and subtract two-digit numbers are now applied to fluently add and subtract whole numbers within 1,000. These strategies should be discussed so that students can make comparisons and move toward efficient methods.

Number sense and computational understanding is built on a firm understanding of place value.

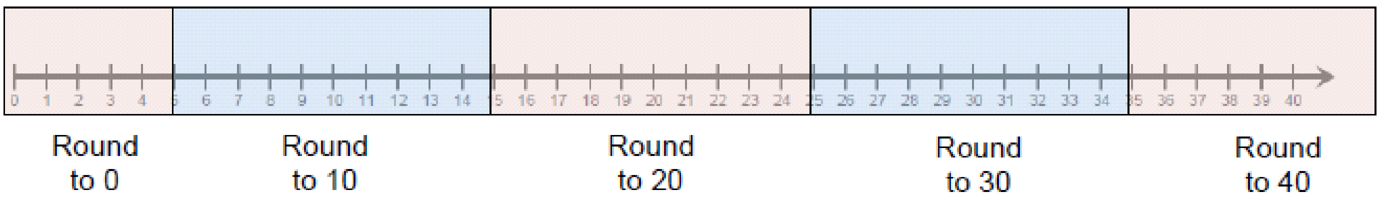
Understanding what each number in a multiplication expression represents is important. Multiplication problems need to be modeled with pictures, diagrams or concrete materials to help students understand what the factors and products represent. The effect of multiplying numbers needs to be examined and understood.

[4] A range of algorithms may be used.

The use of area models is important in understanding the properties of operations of multiplication and the relationship of the factors and its product. Composing and decomposing area models is useful in the development and understanding of the distributive property in multiplication.

Continue to use manipulatives like hundreds charts and place-value charts. Have students use a number line or a roller coaster example to block off the numbers in different colors.

For example this chart show what numbers will round to the tens place.



Instructional Resources/Tools
100's chart
Number lines

Charles A. Dana Center. University of Texas at Austin. Mathematics TEKS Toolkit. 2012. *Make a hundred:* Students roll a die seven times, each time determining whether to add that number of tens or that number of ones to make a sum as close to 100 as possible without going over.

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Rectangle Multiplication:* Visualize the multiplication of two numbers as area.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (3.MD.1, 3.MD.2)
Represent and interpret data. (3.MD.3)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Number & Operations – Fractions	3.G.2, 3.NF.1, 3.NF.2a-b, 3.NF.3a-d

UNDERSTAND:

Fractions are a special type of numbers.
- They refer to parts of wholes.
- They fall between whole numbers on a number line.

Unit fractions are the building blocks of all other fractions.
- A unit fraction is a quantity.
- Unit fractions refer to "1 out of ___ equal parts".
- Non-unit fractions are the sum of unit fractions (e.g., 3/5 = 1/5 + 1/5 + 1/5).

KNOW:

3.NF.1 Fractions are written as *a/b* where the denominator of the fraction indicates the *size of the parts* (the unit fraction it is made of) and the numerator indicates *how many* of those parts are being considered.

Fractions can be represented as equal areas of a region, or as points on a number line.

The whole on a number line is the interval or space between 0 and 1**.**

If the distance on a number line between 0 and 1 is divided into *b* equal intervals, then each interval has a size of 1/*b* (e.g., if the space on a number line is divided into 4 equal intervals, then each interval represents ¼ of the distance between 0 and 1).

When writing fractions on number lines, a fraction *a*/*b* should be placed *a*/*b* of the distance from 0 to 1 (e.g., ¼ should be placed on a number line at the point that is ¼ of the way from 0 to 1 and 2/3 should be placed at the point that is 2/3 of the distance from 0 to 1).)

Two fractions are equivalent (equal) if they are the same size or the same point on a number line.

Whole numbers can be written as fractions with a denominator of 1.

Fractions with the same numerator and denominator are equal to 1.

The size of a fractional part is relative to the size of the whole (e.g., 1/2 of a pizza is bigger than 1/2 of a cookie).

When comparing the size of two different fractions, one must assume that the wholes are the same size.

DO:

All work with fractions in 3rd grade is limited to fractions with the denominators of 2, 3, 4, 6, and 8.

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.

Develop understanding of fractions as numbers.

3.NF.2 Understand a fraction as a number on number line; represent fractions on a number line diagram.
- a. Represent a fraction 1/*b* on a number line diagram by defining the interval between 0 and 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number on the number line.
- b. Represent a fraction 1/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual-fraction model).
- c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers (e.g., *Express 3 in the form of 3 = 3/1; recognize that 6 = 6/1; locate 4/4 and 1 at the same point on a number line diagram).*
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <; and justify the conclusions (e.g., by using a visual-fraction model).

KEY TERMS FOR THIS PROGRESSION:

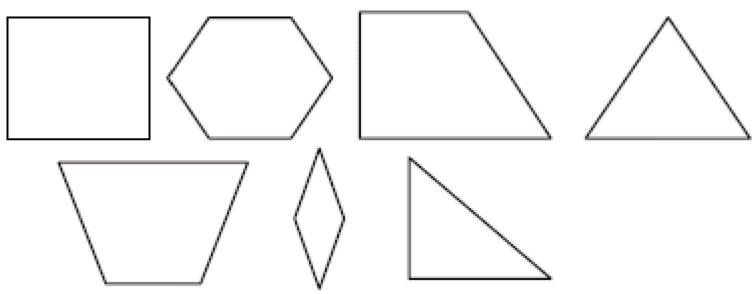
Denominator, Equivalent, Numerator, Whole

INSTRUCTIONAL STRATEGIES AND RESOURCES:

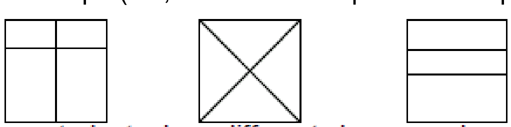
Cluster: Reason with shapes and their attributes.

In earlier grades, students have experiences with informal reasoning about particular shapes through sorting and classifying using their geometric attributes. Students have built and drawn shapes given the number of faces, number of angles and number of sides.

The focus now is on identifying and describing properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. These properties allow for generalizations of all shapes that fit a particular classification. Development in focusing on the identification and description of shapes' properties should include examples and non-examples, as well as drawings by students of shapes in a particular category. For example, students could start with identifying shapes with right angles. An explanation as to why the remaining shapes do not fit this category should be discussed. Students should determine common characteristics of the remaining shapes.



In Grade 2, students partitioned rectangles into two, three or four equal shares, recognizing that the equal shares need not have the same shape. They described the shares using words such as, halves, thirds, half of, a third of, etc., and described the whole as two halves, three thirds or four fourths. In Grade 4, students will partition shapes into parts with equal areas (the spaces in the whole of the shape). These equal areas need to be expressed as unit fractions of the whole shape (i.e., describe each part of a shape partitioned into four parts as 1/4 of the area of the shape.



Have students draw different shapes and see how many ways they can partition the shapes into parts with equal area.

Instructional Resources/Tools

National Council of Teachers of Mathematics. 2000-2012.

Exploring Properties of Rectangles and Parallelograms: Scroll down to find dynamic geometry software that provides an environment in which students can explore geometric relationships and make and test conjectures. In this example, properties of rectangles and parallelograms are examined. The emphasis is on identifying what distinguishes a rectangle from a more general parallelogram.

Rectangles and Parallelograms: While exploring properties of rectangles and parallelograms using dynamic software, students identify, compare, and analyze attributes of both shapes through physical and mental manipulation.

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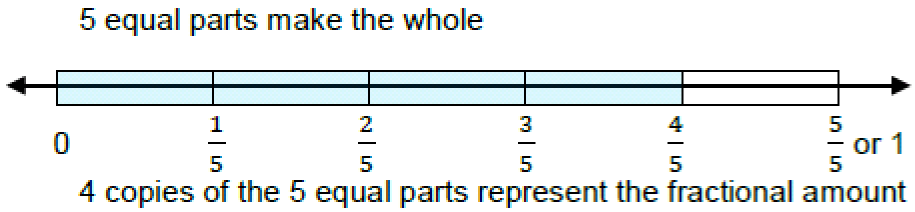
Cluster: Develop understanding of fractions as numbers.

This is the initial experience students will have with fractions and is best done over time. Students need many opportunities to discuss fractional parts using concrete models to develop familiarity and understanding of fractions. Expectations in this domain are limited to fractions with denominators of 2, 3, 4, 6 and 8.

Understanding that a fraction is a quantity formed by part of a whole is essential to number sense with fractions. Fractional parts are the building blocks for all fraction concepts. Students need to relate dividing a shape into equal parts and representing this relationship on a number line, where the equal parts are between two whole numbers. Help students plot fractions on a number line, by using the meaning of the fraction. For example, to plot 4/5 on a number line, there are 5 equal parts with 4 copies of the 5 equal parts.



As students counted with whole numbers, they should also count with fractions. Counting equal-sized parts helps students determine the number of parts it takes to make a whole and recognize fractions that are equivalent to whole numbers.

Students need to know how big a particular fraction is and can easily recognize which of two fractions is larger. The fractions must refer to parts of the same whole. Benchmarks such as 1/2 and 1 are also useful in comparing fractions.

Equivalent fractions can be recognized and generated using fraction models. Students should use different models and decide when to use a particular model. Make transparencies to show how equivalent fractions measure up on the number line.

Venn diagrams are useful in helping students organize and compare fractions to determine the relative size of the fractions, such as more than 1/2, exactly 1/2 or less than 1/2. Fraction bars showing the same sized whole can also be used as models to compare fractions. Students are to write the results of the comparisons with the symbols >, =, or <, and justify the conclusions with a model.

Instructional Resources/Tools

Fraction bars or strips
Geoboards
Grid or dot paper (draw pictures to explore fraction ideas)
Length or measurement models
Region or area models
Set models

Center for Research on Parallel Computation. 1997.
- *Introduction to Cuisenaire rods*
- *Learn fractions with Cuisenaire rods*

National Council of Teachers of Mathematics. 2000-2012.
- *Fun with Fractions:* In this unit, students explore relationships among fractions through work with the length model. This early work with fraction relationships helps students make sense of basic fraction concepts and facilitates work with comparing and ordering fractions and working with equivalency.
- *Fun with Fractions: Investigating Equivalent Fractions with Relationship Rods:* Students investigate the length model by working with relationship rods to find equivalent fractions. Students develop skills in reasoning and problem solving as they explain how two fractions are equivalent (the same length).

Utah State University. National Library of Virtual Manipulatives. 1999-2010

- *Fractions – Naming:* Write the fraction corresponding to the highlighted portion of a shape.
- *Fractions – Parts of a Whole:* Relates parts of a whole unit to written description and fraction.
- *Fractions – Visualizing:* Illustrate a fraction by dividing a shape and highlighting the appropriate parts.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Represent and interpret data. (3.MD.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Measurement and Data (time, liquid volume, mass, & graphing)	3.MD.1, 3.MD.2, 3.MD.3, 3.MD.4

UNDERSTAND:

Standard units enable people measure data in the same way.
Data can be organized, represented, and interpreted in multiple ways for a variety of purposes.

KNOW:	DO:
Line plots with whole numbers must include all the whole numbers in the range. Line plots with fractions must include all whole numbers and fractions within the range (e.g., 3, 3 ¼, 3 ½, 3 ¾, 4, 4 ¼, 4 ½, etc.). It is essential to include the unit when communicating measurement data. One interval on a scaled bar graph represents a larger quantity. One picture on a scaled picture graph represents a larger quantity. Bar graphs, picture graphs, and line plots provide opportunities to make comparisons.	***Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.*** **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., by representing the problem on a number line diagram). **3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[5] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings such as a beaker with a measurement scale) to represent the problem.[6] ***Represent and interpret data.*** **3.MD.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "How many more?" and "How many less?" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units — whole numbers, halves, or quarters.

KEY TERMS FOR THIS PROGRESSION:

Bar graph, Grams, Kilograms, Line plot, Liters, Mass, Picture graph, Scaled bar graph, Volume

INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
A clock is a common instrument for measuring time. Learning to tell time has much to do with learning to read a dial-type instrument and little with time measurement.

Students have experience in telling and writing time from analog and digital clocks to the hour and half-hour in Grade 1 and to the nearest five minutes, using a.m. and p.m. in Grade 2. Now students will tell and write time to the nearest minute and measure time intervals in minutes.

Provide analog clocks that allow students to move the minute hand.

Students need experience representing time from a digital clock to an analog clock and vice versa.

Provide word problems involving addition and subtraction of time intervals in minutes. Have students represent the problem on a number line. Students should relate using the number line with subtraction from Grade 2.

Provide opportunities for students to use appropriate tools to measure and estimate liquid volumes in liters only and masses of objects in grams and kilograms. Students need practice in reading the scales on measuring tools since the markings may not always be in intervals of one. The scales may be marked in intervals of two, five or ten.

[5] Excludes compound units such as cm^3 and finding the geometric volume of a container.
[6] Excludes multiplicative comparison problems involving notions of "times as much"; see Glossary, Table 2 in MCCS.

Allow students to hold gram and kilogram weights in their hand to use as a benchmark. Use water colored with food coloring so that the water can be seen in a beaker.

Students should estimate volumes and masses before actually finding the measuring. Show students a group containing the same kind of objects. Then, show them one of the objects and tell them its weight. Fill a container with more objects and ask students to estimate the weight of the objects.

Use similar strategies with liquid measures. Be sure that students have opportunities to pour liquids into different size containers to see how much liquid will be in certain whole liters. Show students containers and ask, “How many liters do you think will fill the container?”

Instructional Resources/Tools
Balance scales
Beakers with whole number measures
Food coloring
Graduated cylinders
Measuring cups with liter markings
Objects to weigh
Pan or bucket balances
Water
Weights in grams and kilograms

Public Broadcasting Service. 1995-2012. *It Takes Ten:* Students estimate and measure marbles to the nearest gram and squeeze water-saturated sponges to practice measuring in milliliters.

Time-for-Time. 2003. *Teaching Clock:* This site has interactive clocks, games, quizzes, worksheets, and reference materials, all related to time. Analog and digital clocks help students in grades K-3 tell time to the nearest hour, half hour, 5 minutes, and 1 minute.

Utah State University. National Library of Virtual Manipulatives. 1999-2010.
Time – Analog and Digital Clocks: Interactively set the time on a digital and analog clock.
Time - Match Clocks: Answer questions asking you to show a given time on digital and analog clocks.
Time - What Time Will It Be? Answer questions asking you to indicate what time it will be before or after a given time period.

Cluster: Represent and interpret data.
Representation of a data set is extended from picture graphs and bar graphs with single-unit scales to scaled picture graphs and scaled bar graphs. Intervals for the graphs should relate to multiplication and division with 100 (product is 100 or less and numbers used in division are 100 or less). In picture graphs, use values for the icons in which students are having difficulty with multiplication facts. For example, ☺ represents 7 people. If there are three ☺, students should use known facts to determine that the three icons represents 21 people. The intervals on the vertical scale in bar graphs should not exceed 100.

Students are to draw picture graphs in which a symbol or picture represents more than one object. Bar graphs are drawn with intervals greater than one. Ask questions that require students to compare quantities and use mathematical concepts and skills. Use symbols on picture graphs that students can easily represent half of, or know how many half of the symbol represents.

Students are to measure lengths using rulers marked with halves and fourths of an inch and record the data on a line plot. The horizontal scale of the line plot is marked off in whole numbers, halves or fourths. Students can create rulers with appropriate markings and use the ruler to create the line plots.

Instructional Resources/Tools
National Council of Teachers of Mathematics. 2000-2012.
All About Multiplication - Exploring equal sets: Students listen to the counting story, *What Comes in 2's, 3's, & 4's*, and then use counters to set up multiple sets of equal size. They fill in a table listing the number of sets, the number of objects in each set, and the total number in all. They study the table to find examples of the order

(commutative) property. Finally, they apply the equal sets model of multiplication by creating pictographs in which each icon represents several data points.
Data Grapher: This site contains a bar graph tool to create bar graphs.
What's in a Name? – Creating Pictographs: Students create pictographs and answer questions about the data set.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Represent and solve problems involving multiplication and division. (3.OA.1, 3.OA.3)
Develop understanding of fractions as numbers. (3.NF.2)

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometric Measurement – Area & Perimeter	3.MD.5, 3.MD.6, 3.MD.7a-d, 3.MD.8

UNDERSTAND:

Area and perimeter are attributes used to describe and measure 2D figures.
A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

KNOW:

Area covers a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Perimeter is the distance around a figure.

Strategies for finding area and perimeter use related to multiplication and addition.

Strategies for finding Area:
- Counting
- Repeated addition
- Multiplication of length by width
- Decomposing into more than one rectangle

Rectangles can have the same perimeter and different areas or the same areas and different perimeter.

DO:

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.6 Measure areas by counting unit squares (i.e., square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* x *b* and *a* x *c*. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems, including those of Montana American Indians.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

KEY TERMS FOR THIS PROGRESSION:

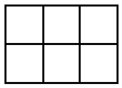
Area, Perimeter, Tiling, Unit squares

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
Students can cover rectangular shapes with tiles and count the number of units (tiles) to begin developing the idea that area is a measure of covering. Area describes the size of an object that is two-dimensional. The formulas should not be introduced before students discover the meaning of area.

The area of a rectangle can be determined by having students lay out unit squares and count how many square units it takes to completely cover the rectangle completely without overlaps or gaps. Students need to develop the meaning for computing the area of a rectangle. A connection needs to be made between the number of squares it takes to cover the rectangle and the dimensions of the rectangle. Ask questions such as: “What does the length of a rectangle describe about the squares covering it?” and “What does the width of a rectangle describe about the squares covering it?”

The concept of multiplication can be related to the area of rectangles using arrays. Students need to discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column. Ask questions about the dimensions if students do not make these discoveries. For example: “How do the squares covering a rectangle compare to an array?” or “How is multiplication used to count the number of objects in an array?”



Students should also make the connection of the area of a rectangle to the area model used to represent multiplication. This connection justifies the formula for the area of a rectangle.

Provide students with the area of a rectangle (e.g., 42 square inches and have them determine possible lengths and widths of the rectangle). Expect different lengths and widths such as, 6 inches by 7 inches or 3 inches by 14 inches.

Instructional Resources/Tools
Square tiles

Utah State University. Virtual Manulipative Library. 1999-2010. *Rectangle Multiplication:* Visualize the multiplication of two numbers as an area. This application allows student to create different size arrays and relate the array to the multiplication problem.

Cluster: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Students have created rectangles before when finding the area of rectangles and connecting them to using arrays in the multiplication of whole numbers. To explore finding the perimeter of a rectangle, have students use non-stretchy string. They should measure the string and create a rectangle before cutting it into four pieces. Then, have students use four pieces of the string to make a rectangle. Two pieces of the string should be of the same length and the other two pieces should have a different length that is the same. Students should be able to make the connection that perimeter is the total distance around the rectangle.

Geoboards can be used to find the perimeter of rectangles also. Provide students with different perimeters and have them create the rectangles on the geoboards. Have students share their rectangles with the class. Have discussions about how different rectangles can have the same perimeter with different side lengths.

Students experienced measuring lengths of inches and centimeters in Grade 2. They have also related addition to length and writing equations with a symbol for the unknown to represent a problem.

Once students know how to find the perimeter of a rectangle, they can find the perimeter of rectangular-shaped objects in their environment. They can use appropriate measuring tools to find lengths of rectangular-shaped objects in the classroom. Present problems situations involving perimeter, such as finding the amount of fencing needed to enclose a rectangular shaped park, or how much ribbon is needed to decorate the edges of a picture frame. Also present problem situations in which the perimeter and two or three of the side lengths are known, requiring students to find the unknown side length.

Students need to know when a problem situation requires them to know that the solution relates to the perimeter or the area. They should have experience with understanding area concepts when they recognize it as an attribute of plane figures. They also discovered that when plane figures are covered without gaps by n unit squares, the area of the figure is n square units.

Students need to explore how measurements are affected when one attribute to be measured is held constant and the other is changed. Using square tiles, students can discover that the area of rectangles may be the same, but the perimeter of the rectangles varies. Geoboards can also be used to explore this same concept.

Instructional Resources/Tools
1-inch or 1-centimeter grid paper
Geoboards
Non-stretchy string
Rubber bands

Square tiles

Marilyn Burns, *Spaghetti and Meatballs for All: A Mathematical Story.* (Scholastic Press, 1997). This book provides students with a real-life context for investigating variation in perimeter while area remains constant. In the story, small tables are pushed together to make one large table, until too many people show up, and the large table has to be subdivided into smaller arrangements to provide more seating. Activities and extensions are suggested at the back of the book.

National Council of Teachers of Mathematics. 2000-2010. *Junior Architects - Finding Perimeter and Area:* In this lesson, students develop strategies for finding the perimeter and area for rectangles and triangles using geoboards and graph paper. Students learn to appreciate how measurement is a critical component to planning their clubhouse design.

Public Broadcasting Service. 1995-2012. *For Real: Penned In*: Explore rectangles and perimeter in real-world applications. In this video clip from Cyberchase, Harry builds a rectangular fence with an assortment of different-size sections but forgets to add a gate to get out.

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Making a 5 Peg Triangle:* Use geoboards to illustrate area, perimeter, and rational number concepts.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Represent and solve problems involving multiplication and division. (3.OA.3)
Understand properties of multiplication and the relationship between multiplication and division. (3.OA.5)

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometry – Reason with Shapes & their Attributes	3.G.1, 3.G.2

UNDERSTAND:

Shapes in different categories may share attributes and the shared attributes can define a larger category.

KNOW:	DO:
Shapes can be sorted according to their attributes. Quadrilaterals are polygons with four sides. Rectangles, rhombi, and squares are a particular type of quadrilateral (parallelograms).	***Reason with shapes and their attributes.*** **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, etc.) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

KEY TERMS FOR THIS PROGRESSION:

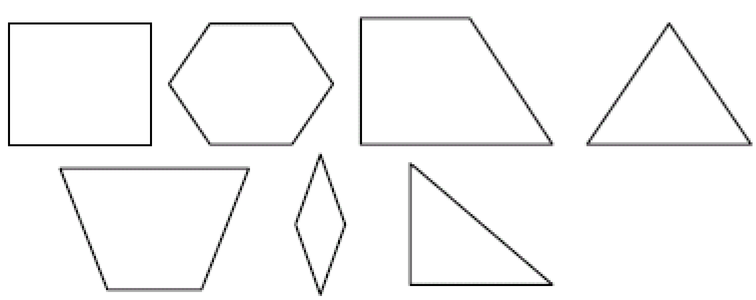
Attribute, Partition, Polygon, Quadrilateral, Rhombus, Rectangle, Square, Trapezoid, Unit fraction

INSTRUCTIONAL STRATEGIES AND RESOURCES:

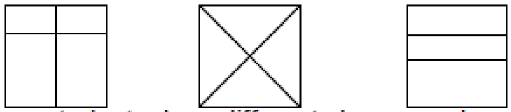
Cluster: Reason with shapes and their attributes.

In earlier grades, students have experiences with informal reasoning about particular shapes through sorting and classifying using their geometric attributes. Students have built and drawn shapes given the number of faces, number of angles and number of sides.

The focus now is on identifying and describing properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. These properties allow for generalizations of all shapes that fit a particular classification. Development in focusing on the identification and description of shapes' properties should include examples and non-examples, as well as examples and non-examples drawn by students of shapes in a particular category. For example, students could start with identifying shapes with right angles. An explanation as to why the remaining shapes do not fit this category should be discussed. Students should determine common characteristics of the remaining shapes.



In Grade 2, students partitioned rectangles into two, three or four equal shares, recognizing that the equal shares need not have the same shape. They described the shares using words such as, halves, thirds, half of, a third of, etc., and described the whole as two halves, three thirds or four fourths. In Grade 4, students will partition shapes into parts with equal areas (the spaces in the whole of the shape). These equal areas need to be expressed as unit fractions of the whole shape (i.e., describe each part of a shape partitioned into four parts as ¼ of the area of the shape.



Have students draw different shapes and see how many ways they can partition the shapes into parts with equal area.

Instructional Resources/Tools

National Council of Teachers of Mathematics. 1999-2012.

Exploring Properties of Rectangles and Parallelograms: Scroll down to find dynamic geometry software that provides an environment in which students can explore geometric relationships and make and test conjectures. In this example, properties of rectangles and parallelograms are examined. The emphasis is on identifying what distinguishes a rectangle from a more general parallelogram.

Rectangles and Parallelograms: While exploring properties of rectangles and parallelograms using dynamic software, students identify, compare, and analyze attributes of both shapes through physical and mental manipulation.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Develop understanding of fractions as numbers. (3.NF.1)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

COURSE: THIRD GRADE MATH

SAMPLE YEAR LONG PLAN

Unit	First Week establish routines and rapport	Place Value	Addition & Subtraction	Time	Multiplica-tion	Multiplica-tion cont.
(Time)	(4 days)	(10 days)	(22 days)	(10 days)	(15 days)	(16 days)
STANDARDS						
		3.NBT.1	3.NBT.2	3.MD.1	3.OA.1	3.OA.1
			3.OA.8	3.MD.3	3.OA.3	3.OA.3
				Graphing covered during Social Studies and science. MD3.3	3.OA.4	3.OA.4
					3.OA.8	3.OA.8
					3.NBT.3	

THIRD GRADE MATH P. 2

Division	Geometry & Measurement	Fractions	CRT Prep and Testing	Measurement	Multiply & divide numbers	Last Week of School Review
(12 days)	18 days	(10 days)	(10 days)	(20 days)	(26 days)	(5days)
3.OA.2	3.MD.2	3.NF.1		3.OA.8	3.OA.1	
3.OA.3	3.MD.5	3.NF.2		3.MD.4	3.OA.3	
3.OA.4	3.MD.6	3.NF.3		3.MD.2	3.OA.4	
3.OA.6	3.MD.7				3.OA.8	
	3.MD.2					
	3.G.1					
	G3..2					
	cm/m					
	area					
	qts/pints conversions					
	perimeter					
	(for CRT)					
*Without remainders	*Congruency & symmetry are not part of S.B.					

TEACHER: Donna, Dusty, Carolyn

SCHOOL DISTRICT/BUILDING: RCCC

COURSE: 3rd Grade Math

GRADE LEVEL(S): 3rd Grade

LAST UNIT	CURRENT UNIT	NEXT UNIT
Addition/Subtraction	Multiplication	Division

UNIT SCHEDULE	
Day 1	Manipulatives
Day 2	Repeated addition
Day 3	Arrays
Day 4	Multiplication stories
Day 5	Make a table
Day 6	2 as a factor
Day 7	5 as a factor
Day 8	10 as a factor
Day 9	Multi-step problem
Day 10	Multiply with 0 & 1
Day 11	9 as a factor
Day 12	Practice
Day 13	Games & review
Day 14	Assessment

UNIT MAP

is about...

Multiplication Concepts & Facts

Repeated Addition

Real-life Application

Array Model

Story Problems

UNIT SELF TEST QUESTIONS

1. How can making an array help with multiplication?
2. How does multiplication make repeated addition easier?
3. What patterns do you see in your table?
4. Can you count your multiples by 2, 5, 9 & 10?

MATH STANDARDS	
OA3.1	
OA3.3	
OA3.4	
OA3.8	
NBT3.3	

GRADE 3 MATH: WILD TURKEYS

UNIT OVERVIEW

The Wild Turkeys task is embedded in a 15-20 day unit focused on operations and algebraic thinking. Students demonstrate mastery by solving the Wild Turkeys task in one class period.

TASK DETAILS

Task Name: Wild Turkeys

Grade: 3

Subject: Math

Task Description: Students use a pattern to demonstrate number sense to 112, as well as knowledge of the days of the week and multiplication/addition.

Standards Assessed:
3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7.
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.

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The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through this year's Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

Entire lesson plan available at:

http://schools.nyc.gov/NR/rdonlyres/9C01DB60-A311-4B22-918B-309E65C433D4/0/NYCDOEG3MathWildTurkeysStudentWorkNoAnnotation.pdf

Acknowledgements: The unit outline was developed by Shenaz Hashim (CFN 109) and Haydee Santino with input from Curriculum Designers Alignment Review Team. The tasks were developed by the schools in the 2010-2011 NYC DOE Elementary School Performance Based Assessment Pilot, in collaboration with Exemplars, Inc. and Center for Assessment.

3rd Grade Sample Lesson Plan

Teacher: Dusty Schrock and Carolyn White

Date:

Subject area / course / grade level: 3rd grade math- multiplication

Materials: http://mathstory.com/youtubevids/countbytwo.aspx#.UbifjjcWTTo
Egg cartons, small manipulatives, small pieces of paper that will fit in egg carton labeled 1-12

TEKS/SEs: OA3.1, OA3.9
Students will:
Interpret products of whole numbers.
Identify arithmetic patterns.
Explain patterns using properties of operations.

Differentiation strategies to meet diverse learner needs:
Adjust manipulatives for individual student needs.

ENGAGEMENT

- Teacher will share the you tube video from above (materials list)
- Students should ask themselves what is the pattern we identify when counting by 2's in the video?

EXPLORATION

- Egg carton activity-
- Hand-out to each group an egg carton, manipulatives number cards.
- Explore making groups of two using manipulatives and the egg cartons.
- Write addition facts based on their results of the above sorting activity.
- Count by twos and relate this to the pattern identified by the video.
- "Using the small numbered pieces of paper show how many groups of 2 are possible."
- Ask Leading Questions to the group:
  - ✓ What is 1 group of 2 equal to?
  - ✓ What is 4 groups of 2 equal to?
  - ✓ Etc.
- Are you still able to identify the 2s pattern?
- Check throughout this activity that students are understanding equal grouping concepts.

EXPLANATION

  - What is 1 group of 2 equal to?
  - What is 4 groups of 2 equal to?
  - Etc.
- Higher Order Thinking: Ask students to count by 2s (give a starting number and ending number.) Ask why they know that they are accurate. (Answers should identify that they know and understand the pattern of 2s.)

ELABORATION

- Describe how students will develop a more sophisticated understanding of the concept.
- Vocabulary – equal groups
- When our artist friend comes in and each of you needs a large and a small paint brush, how many total brushes would we need for the whole class?
- When we are finding a partner to work with during class, how many groups will we have?

EVALUATION

- Have students answer essential questions (found under elaboration section)on student response boards.

Round Rock ISD – 5E Lesson Plan
www.roundrockisd.org

FOURTH GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grade 4 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
4.MP.1. Make sense of problems and persevere in solving them.	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
4.MP.2. Reason abstractly and quantitatively.	Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
4.MP.3. Construct viable arguments and critique the reasoning of others.	In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
4.MP.5. Use appropriate tools strategically.	Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
4.MP.6. Attend to precision.	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
4.MP.7. Look for and make use of structure.	In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
4.MP.8. Look for and express regularity in repeated reasoning.	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Explanations and Examples Grade 4
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Grade 4 Content Standards

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 4 Overview

Operations and Algebraic Thinking
- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten
- Generalize place value understanding for multidigit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Geometry
- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Number and Operations—Fractions
- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems within cultural contexts, including those of Montana American Indians, with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2] 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions[3] 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.
    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
    c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
    d. Solve word problems within cultural contexts, including those of Montana American Indians, involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
    a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
    b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

    c. Solve word problems within cultural contexts, including those of Montana American Indians, involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? As a contemporary American Indian example, for family/cultural gatherings the Canadian and Montana Cree bake bannock made from flour, salt, grease, and baking soda, in addition to ¾ cup water per pan. When making four pans, how much water will be needed?*

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)...*
2. Use the four operations to solve word problems within cultural contexts, including those of Montana American Indians, involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect or arrow/spearhead collection.*

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
   b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems. e.g., by using an equation with a symbol for the unknown angle measure.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure, including those found in Montana American Indian designs, as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

[1] See Glossary, Table 2.

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

GRADE 4

Domain	Cluster	Code	Common Core State Standard
Operations and Algebraic Thinking	Use the four operations with whole numbers to solve problems.	4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
		4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
		4.OA.3	Solve multistep word problems within cultural contexts, including those of Montana American Indians, posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
	Gain familiarity with factors and multiples.	4.OA.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.
	Generate and analyze patterns.	4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example: Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
Number and Operations in Base Ten	Generalize place value understanding for multi digit whole numbers.	4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)
		4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)
		4.NBT.3	Use place value understanding to round multi-digit whole numbers to any place. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)
	Use place value understanding and properties of operations to perform multi-digit arithmetic.	4.NBT.4	Fluently dd and subtract multi-digit whole numbers using the standard algorithm. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.)
		4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.)
		4.NBT.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.)

GRADE 4

Domain	Cluster	Code	Common Core State Standard
Number and Operations: Fractions	Extend understanding of fraction equivalence and ordering.	4.NF.1	Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
		4.NF.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	4.NF.3	Understand a fraction a/b with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems within cultural contexts, including those of Montana American Indians, involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
		4.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.) c. Solve word problems within cultural contexts, including those of Montana American Indians, involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
	Understand decimal notation for fractions, and compare decimal fractions.	4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100 and add 3/10 + 4/100 = 34/100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)
		4.NF.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 1 62/100 ; describe a length as 0.62 meters; locate 0.62 on a number line diagram. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
		4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons comparisons are valid only when two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)

GRADE 4

Domain	Cluster	Code	Common Core State Standard
Measurement and Data	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of smaller unit. Record measurement equivalents in a two-column table. For example: Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...
		4.MD.2	Use the four operations to solve word problems within cultural contexts, including those of Montana American Indians, involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
		4.MD.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.
	Represent and interpret data.	4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect or arrow/spearhead collection.
	Geometric measurement--understand concepts of angle and measure angles.	4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.
		4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
		4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
Geometry	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel line. Identify these in two-dimensional figures.
		4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of specified size. Recognize right triangles as a category, and identify right triangles.
		4.G.3	Recognize a line of symmetry for a two-dimensional figure, including those found in Montana American Indian designs, as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Getting Every Montana Student College and Career Ready

Montana Common Core
Standards and Assessments

Montana Curriculum Organizer

Grade 4

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

November 2012 Materials adapted from Arizona, Delaware and Ohio Departments of Education

Montana Curriculum Organizer: Grade 4 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

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HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 23 consist of tables organized into learning progressions that can function as units. The table for each learning progression unit includes: 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

<table>
<tr><th>LEARNING PROGRESSION</th><th>STANDARDS IN LEARNING PROGRESSION</th></tr>
<tr><td>Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.</td><td>Standards covered in this learning progression.</td></tr>
<tr><th colspan="2">UNDERSTAND:</th></tr>
<tr><td colspan="2">What students need to understand by the end of this learning progression.</td></tr>
<tr><th>KNOW:</th><th>DO:</th></tr>
<tr><td>What students need to know by the end of this learning progression.</td><td>What students need to be able to do by the end of this learning progression, organized by cluster and standard.</td></tr>
<tr><th colspan="2">KEY TERMS FOR THIS PROGRESSION:</th></tr>
<tr><td colspan="2">Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.</td></tr>
<tr><th colspan="2">INSTRUCTIONAL STRATEGIES AND RESOURCES:</th></tr>
<tr><td colspan="2">Cluster: Title
Strategies for this cluster

Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><td colspan="2">Cluster: Title
Strategies for this cluster

Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><th colspan="2">CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:</th></tr>
<tr><td colspan="2">Standards that connect to this learning progression are listed here, organized by cluster.</td></tr>
<tr><th colspan="2">STANDARDS FOR MATHEMATICAL PRACTICE:</th></tr>
<tr><td colspan="2">A quick reference guide to the eight standards for mathematical practice is listed here.</td></tr>
</table>

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Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

STANDARDS FOR MATHEMATICAL PRACTICE: GRADE 4 EXPLANATIONS AND EXAMPLES	
Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
4.MP.1. Make sense of problems and persevere in solving them.	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth-graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
4.MP.2. Reason abstractly and quantitatively.	Fourth-graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place-value concepts.
4.MP.3. Construct viable arguments and critique the reasoning of others.	In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.
4.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth-graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
4.MP.5. Use appropriate tools strategically.	Fourth-graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
4.MP.6. Attend to precision.	As fourth-graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
4.MP.7. Look for and make use of structure.	In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
4.MP.8. Look for and express regularity in repeated reasoning.	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

CRITICAL AREAS FOR GRADE 4 MATH

In Grade 4, instructional time should focus on three critical areas:

(1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends;

(2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and

(3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Operations & Algebraic Thinking	4.OA.1, 4.OA.2, 4.OA.3, 4.OA.4, 4.OA.5

UNDERSTAND:

Factors and multiples can be used to determine part/whole relationships.

By utilizing efficient methods of multiplication and division, more complex problem solving is possible.

KNOW:	DO:
Multiplication scenarios can be interpreted differently based on the context of the problem. (e.g., a “5 times greater than 7” problem is interpreted differently than “5 groups of 7” but both are derived from 5 x 7). Additive thinking is “How many more?” Multiplicative thinking is “How many times more?” Problems can be solved by writing the solution pathway in algebraic notation and then solving for the unknown. Estimation in multiplication and division can predict the size of the answer and help to assess the reasonableness of a solution.	***Use the four operations with whole numbers to solve problems.*** **4.OA.1** Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), distinguishing multiplicative comparison from additive comparison.[1] **4.OA.3** Solve multistep word problems within cultural contexts, including those of Montana American Indians, posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. ***Gain familiarity with factors and multiples.*** **4.OA.4** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. ***Generate and analyze patterns.*** **4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

KEY TERMS FOR THIS PROGRESSION:

Composite, Equation, Estimation, Factors, Multiples, Prime

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Use the four operations with whole numbers to solve problems.

Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in Table 2 on page 73 in the *Montana Common Core Standards for School Mathematics Grade-Band*. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or

[1] See Glossary, Table 2 in MCCS document.

additive comparison (solved when adding and subtracting in Grades 1 and 2).
Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.

Examples of multistep word problems can be accessed from the released questions on the National Assessment of Educational Progress. (NAEP) Assessment. For example, a constructed response question from the 2007 Grade 4 NAEP assessment reads, "Five classes are going on a bus trip and each class has 21 students. If each bus holds only 40 students, how many buses are needed for the trip?"

Instructional Resources/Tools
Institute of Education Sciences. National Center for Education Statistics. *National Assessment of Educational Progress* (NAEP) Assessments.

Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band.* Page 73, Table 2.

Cluster: Gain familiarity with factors and multiples.
Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counter-examples of factors.

Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

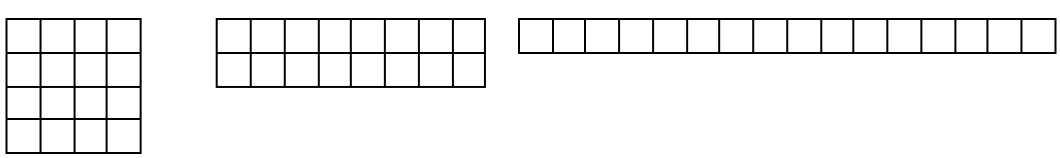
Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole-number factor that is not one or itself. Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart (e.g., 2 is prime, but 4, 6, 8, 10, 12, … are composite). Encourage the development of rules that can be used to aid in the determination of composite numbers (e.g., other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number).

Using area models will also enable students to analyze numbers and arrive at an understanding of whether a number is prime or composite. Have students construct rectangles with an area equal to a given number. They should see an association between the number of rectangles and the given number for the area as to whether this number is a prime or composite number.

Definitions of prime and composite numbers should not be provided, but determined after many strategies have been used in finding all possible factors of a number.

Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets (e.g., have them find several factors of 10, 14, 25 or 32, and write multiplication expressions for the numbers).

Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares. For example if you have 16 squares:



The idea that a product of any two whole numbers is a common multiple of those two numbers is a difficult concept to understand. For example, 5 x 8 is 40; the table below shows the multiples of each factor.

5	10	15	20	25	30	35	**40**	45
8	16	24	32	**40**	48	56	64	72

Ask students what they notice about the number 40 in each set of multiples; 40 is the 8th multiple of 5, and the 5th multiple of 8.
Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in Grade 6.

Writing multiplication expressions for numbers with several factors and for numbers with a few factors will help students in making conjectures about the numbers. Students need to look for commonalities among the numbers.

Instructional Resources/Tools
Calculators
Counters
Grid papers

Drexel University. The Math Forum. 1994-2012. *Understanding factoring through geometry*: Using square unit tiles, students work with a partner to construct all rectangles whose area is equal to a given number. After several examples, students see that prime numbers are associated with exactly two rectangles, whereas composite numbers are associated with more than two rectangles.

National Council of Teachers of Mathematics. 2000-2012.
- *Factor Game:* Engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors. Students are then guided through an analysis of game strategies and introduced to the definitions of prime and composite numbers.
- *Multiplication: It's in the Cards: More Patterns with Products:* Students practice multiplication facts and record their current level of mastery on their personal multiplication chart.
- *The Product Game:* Students start with factors and multiply to find the product.
- *The Product Game – Classifying Numbers:* Students construct Venn diagrams to show the relationships between the factors or products of two or more numbers in the Product Game.

Utah State University. 1999-2000.
- *National Library of Virtual Manipulatives:* The National Library of Virtual Manipulatives contains Java applets and activities for K-12 mathematics.
- *Sieve of Eratosthenes:* relate number patterns with visual patterns. Click on the link for *Activities* for directions on engaging students in finding all prime numbers 1-100.

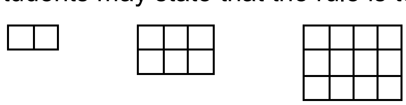
Cluster: Generate and analyze patterns.
In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking.

Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

As students generate numeric patterns for rules, they should be able to "undo" the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, "Add 4" starting with the number 1, the pattern 1, 5, 9, 13, 17, … is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, "How is a number in the sequence related to the one that came before it?", and "If they started at the end of the pattern, will this relationship be the same?" Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Students should also determine if there are other relationships in the patterns. In the numeric pattern generated above, students should observe that the numbers are all odd numbers.

Provide patterns that involve shapes so that students can determine the rule for the pattern. For example, students may state that the rule is to multiply the previous number of squares by 3.



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Instructional Resources/Tools

National Council of Teachers of Mathematics. 2000-2012.

> *Patterns that Grow: Looking Back and Moving Forward:* In this final lesson of the unit, students use logical thinking to create, identify, extend, and translate patterns. They make patterns with numbers and shapes and explore patterns in a variety of mathematical contexts.
>
> *Patterns that Grow: Growing Patterns:* Students use numbers to make growing patterns. They create, analyze, and describe growing patterns and then record them. They also analyze a special growing pattern called Pascal's triangle.

Public Broadcasting Service. 1995-2012. *Snake Patterns–s-s-s:* Students will use given rules to generate several stages of a pattern and will be able to predict the outcome for any stage.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (4.MD.1, 4.MD.2, 4.MD.3)
Generalize place-value understanding for multi-digit whole numbers. (4.NBT.1, 4.NBT.2)
Use place-value understanding and properties of operations to perform multi-digit arithmetic. (4.NBT.4)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (4.NF.4)
Understand decimal notation for fractions, and compare decimal fractions. (4.NF.5, 4.NF.7)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Number and Operations in Base Ten[2] - Place Value	4.NBT.1, 4.NBT.2, 4.NBT.3, 4.NBT.4, 4.NBT.5, 4.NBT.6

UNDERSTAND:

The number system is a repeated counting pattern based on tens and powers of ten.

Efficient strategies for multi-digit arithmetic are based on applying the properties of operations.

KNOW:	DO:
Expanded notation can be used to show order, values of each digit, and the powers of 10. The Distributive Property of Multiplication can be modeled in an array as well as with expanded notation. Rounding a number to the largest place value can be accomplished by answering: "Is this number closest to N-thousand or N+1 thousand?" Multiplication and division are inverse operations.	***Generalize place-value understanding for multi-digit whole numbers.*** **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **4.NBT.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.NBT.3** Use place-value understanding to round multi-digit whole numbers to any place. ***Use place-value understanding and properties of operations to perform multi-digit arithmetic.*** **4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm. **4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

KEY TERMS FOR THIS PROGRESSION:

Expanded Form (1,000 + 200 + 30 + 4), Place value, Standard Form (1,234), Word Form (One thousand, two hundred thirty-four)

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Generalize place-value understanding for multi-digit whole numbers.

Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers.

Students need to have opportunities to compare numbers with the same number of digits (e.g., compare 453, 698 and 215); numbers that have the same number in the leading digit position (e.g., compare 45, 495 and 41,223); and numbers that have different numbers of digits and different leading digits (e.g., compare 312, 95, 5, 245 and 10,002).

Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select four to six cards; then, using all the cards, make the largest number possible with the cards, the smallest number possible and the closest number to 5,000 that is greater than 5,000 or less than 5,000.

In Grade 4, rounding is not new, and students need to build on the Grade 3 skill of rounding to the nearest 10 or 100 to include larger numbers and place value. What is new for Grade 4 is rounding to digits other than the leading digit (e.g.,

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

round 23,960 to the nearest hundred). This requires greater sophistication than rounding to the nearest ten thousand because the digit in the hundreds place represents 900 and when rounded it becomes 1,000, not just zero.

Students should also begin to develop some rules for rounding, building off the basic strategy of “Is 48 closer to 40 or 50?” Since 48 is only 2 away from 50 and 8 away from 40, 48 would round to 50. Now students need to generalize the rule for much larger numbers and rounding to values that are not the leading digit.

Instructional Resources/Tools
Number cards
Place-value boxes
Place-value flip charts

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.
A crucial theme in multi-digit arithmetic is encouraging students to develop *strategies* that they understand, can explain, and can think about; rather than merely follow a sequence of directions that they don't understand.

It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to *understand* all the steps in the algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately.

Start with a student’s understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work.

It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtractions algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. “Think-Pair-Share” is a good protocol for all students.

When asking students to gain understanding about multiplying larger numbers, provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even *attempt* to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (e.g., multiples of 10) when doing mental math, and this promotes its understanding.

Use a variation of an area model. For example, to multiply 23 × 36, arrange the partial products as follows:

	20 + 3	
30 + 6	600	90
	120	18

Then add the four partial products to get 828.

As students developed an understanding of multiplying a whole number up to four digits by a one-digit whole number, and multiplying two two-digit numbers through various strategies, they should do the same when finding whole-number quotients and remainders. By relating division to multiplication and repeated subtraction, students can find partial quotients. An explanation of partial quotients can be viewed during this video on TeacherTube.com: Outline of partial quotients. This strategy will help them understand the division algorithm.

Students will have a better understanding of multiplication or division when problems are presented in context.

Students should be able to illustrate and explain multiplication and division calculations by using equations, rectangular arrays and the properties of operations. These strategies were used in Grade 3 as students developed an understanding of multiplication.

To give students an opportunity to communicate their understandings of various strategies, organize them into small groups and ask each group to create a poster to explain a particular strategy and then present it to the class.

Vocabulary is important. Students should have an understanding of terms such as, sum, difference, fewer, more, less, ones, tens, hundreds, thousands, digit, whole numbers, product, factors and multiples.

Instructional Resources/Tools

Base-ten blocks
Bound place-value flip books (so that the digit in a certain place can be switched)
Hundreds flats
Place-value mats
Smartboard
Tens frames

TeacherTube.com. *Outline of partial quotients*. An explanation of partial quotients can be viewed during this video.

Whitin, David. Read Any Good Math Lately? 1992. A resource book that makes a connection to literature. This will help to identify books related to certain math topics. Books can provide a 'hook' for learning, to activate background knowledge, and to build student interest.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Understand decimal notation for fractions, and compare decimal fractions. (4.NF.5, 4.NF.6, 4.NF.7)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Number and Operations – Fractions - Equivalence, Comparing Fractions & Decimals	4.NF.1, 4.NF.2, 4.NF.5, 4.NF.6, 4.NF.7

UNDERSTAND:

Equivalent fractions or decimal fractions represent the same quantity in multiple ways.

Using visual models and place value is helpful in comparing fractions and decimals.

KNOW:	DO:
Multiplying a fraction by one always results in an equivalent fraction (e.g., ¼ x 3/3 = 3/12). Equivalent fractions can be generated using area models, ratio models, number lines and fractions bars. Compare fractions using common denominator, common numerator, comparison to benchmark and distance to benchmark; as well as determining when each strategy is appropriate. Compare decimal fractions using 10 x 10 grid, a number line, and measurement such as metric system, money.	*Grade 4 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.* ***Extend understanding of fraction equivalence and ordering.*** **4.NF.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.2** Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators), or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model). ***Understand decimal notation for fractions, and compare decimal fractions.*** **4.NF.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[3] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **4.NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **4.NF.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, (e.g., by using a visual model).

KEY TERMS FOR THIS PROGRESSION:

Benchmark fractions, Denominators, Equivalent fractions, Hundredth, Numerators, Tenth

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Extend understanding of fractions equivalence and ordering.

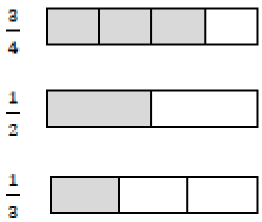
Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions.

Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators.

Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular

[3] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

models) so that the models represent the same whole. The models should be represented in drawings. Students should also use benchmark fractions such as 1/2 to compare two fractions. The result of the comparisons should be recorded using >, < and = symbols.



Instructional Resources/Tools
Fraction bars or strips
Pattern blocks

Cluster: Understand decimal notations for fractions, and compare decimal fractions.
The place-value system developed for whole numbers extends to fractional parts represented as decimals. This is a connection to the metric system. Decimals are another way to write fractions. The place-value system developed for whole numbers extends to decimals. The concept of one whole used in fractions is extended to models of decimals.

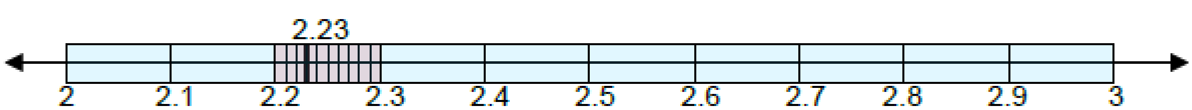
Students can use base-ten blocks to represent decimals. A 10 x 10 block can be assigned the value of one whole to allow other blocks to represent tenths and hundredths. They can show a decimal representation from the base-ten blocks by shading on a 10 x 10 grid.

Students need to make connections between fractions and decimals. They should be able to write decimals for fractions with denominators of 10 or 100. Have students say the fraction with denominators of 10 and 100 aloud. For example, 4/10 would be “four tenths” or 27/100 would be “twenty-seven hundredths.” Also, have students represent decimals in word form with digits and the decimal place value, such as 0.4 would be 4 tenths.

Students should be able to express decimals to the hundredths as the sum of two decimals or fractions. This is based on understanding of decimal place value. For example 0.32 would be the sum of 3 tenths and 2 hundredths. Using this understanding students can write 0.32 as the sum of two fractions (3/10 + 2/100).

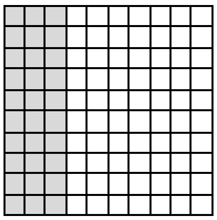
Students’ understanding of decimals to hundredths is important in preparation for performing operations with decimals to hundredths in Grade 5.

In decimal numbers, the value of each place is 10 times the value of the place to its immediate right. Students need an understanding of decimal notations before they try to do conversions in the metric system. Understanding of the decimal place-value system is important prior to the generalization of moving the decimal point when performing operations involving decimals.



Students extend fraction equivalence from Grade 3 with denominators of 2, 3, 4, 6 and 8 to fractions with a denominator of 10. Provide fraction models of tenths and hundredths so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.

When comparing two decimals, remind students that as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a 10 x 10 grid. The 10 x 10 grid is defined as one whole. The decimal must relate to the whole.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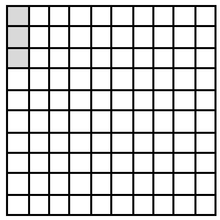
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Flexibility with converting fractions to decimals and decimals to fractions provides efficiency in solving problems involving all four operations in later grades.

Instructional Resources/Tools
10 x 10 square on a grid
Base-ten blocks
Decimal place-value mats
Length or area models
Number lines

National Council of Teachers of Mathematics. 2000-2012.*A Meter of Candy:* In this series of three hands-on activities, students develop and reinforce their understanding of hundredths as fractions, decimals and percentages. Students explore with candy pieces as they physically make and connect a set and linear model (meter) to produce area models (grids and pie graphs). At this time, students are not to do percents. The relationships among fractions, decimals and percents are developed in Grade 6.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (4.MD.1, 4.MD.2)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 4 Mathematics

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Number and Operations – Fractions - Operations	4.NF.3a-d, 4.NF.4a-c

UNDERSTAND:

Fractions are built from unit fractions through the process of addition and multiplication.
Visual fraction models and equations are tools for adding fractions, subtracting fractions, and multiplying a fraction by a whole number

KNOW:

A fraction *a/b* is a multiple of 1/*b* (i.e., *a* groups of 1/*b* = *a* x 1/b). For example, 5/4 is the same as 5 sets of 1/4 or 5 x ¼.

A mixed number is the sum of its decomposed fractional parts. For example, 2 ¼ = $^4/_4$ + $^4/_4$ + ¼.

Decomposing ¾ into ¼ + ¼ + ¼ allows for adding or subtracting fourths.

Either factor can be the multiplier when multiplying a fraction by a whole number. For example, ½ x 6 or 6 x ½

Visual Fraction models:
- Area model
- Array
- Clock model
- Fraction bars
- Number line

DO:

Grade 4 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, by using a visual fraction model (e.g., *3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 $^1/_8$ = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction), and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems within cultural contexts, including those of Montana American Indians, involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
c. Solve word problems within cultural contexts, including those of Montana American Indians, involving multiplication of a fraction by a whole number (e.g., by using visual-fraction models and equations to represent the problem). *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

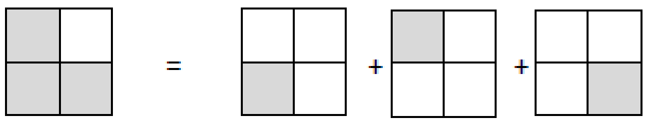
KEY TERMS FOR THIS PROGRESSION:

Decompose, Equivalence, Equivalent, Fractions, Justify, Mixed numbers

INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

In Grade 3, students added unit fractions with the same denominator. Now, they begin to represent a fraction by decomposing the fraction as the sum of unit fraction and justify with a fraction model. For example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.



Students also represented whole numbers as fractions. They use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem.

Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawings to show their understanding.

Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.

Instructional Resources/Tools

Circular fraction models
Fraction tiles/bars
Number lines
Rulers with markings of ½, 1/4, and 1/8.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (4.MD.2)
Represent and interpret data. (4.MD.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 4 Mathematics

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Measurement and Data – Measurement Systems, Area, Perimeter, Data	4.MD.1, 4.MD.2, 4.MD.3, 4.MD.4

UNDERSTAND:

Within a single system of measurement larger units are made from smaller units (i.e., 1 km = 1,000 meters). Smaller units are divisions of larger unit (i.e., 1 cm = 1/100 of a meter).

Formulas are an efficient way to solve for area and perimeter.

Line plots can be used to represent data.

KNOW:	DO:
Relative sizes of measurement units (i.e., km, cm, kg, g, lb., oz., liter, ml, min., sec., hr.). Equivalent measurements within a measurement system can be used to solve problems (e.g., 4 m = 400 cm, and 24 in = 2 ft.). An array model can justify the formulas: A = L x W and P = 2 L + 2 W. Line plots with whole numbers must include all the whole numbers in the range. Line plots with fractions must include all whole numbers and fractions within the range (e.g.; 3, 3 ½, 4, 4 ½, etc.). Consistent increments.	***Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.*** **4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm, kg, g, lb., oz., l, ml, hr., min., and sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), etc.* **4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* ***Represent and interpret data.*** **4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (e.g., 1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

KEY TERMS FOR THIS PROGRESSION:

Area, Line plot, Measurement, Perimeter

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit.

In order for students to have a better understanding of the relationships between units, they need to use measuring devices in class. The number of units needs to relate to the size of the unit. They need to discover that there are 12 inches in 1 foot and 3 feet in 1 yard. Allow students to use rulers and yardsticks to discover these relationships among these units of measurements. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers, which is the same as 3 feet since each ruler is 1 foot in length, are equivalent to one yardstick. Have students record the relationships in a two-column table or a T-chart. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters.

Present word problems as a source of students' understanding of the relationships among inches, feet and yards. Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

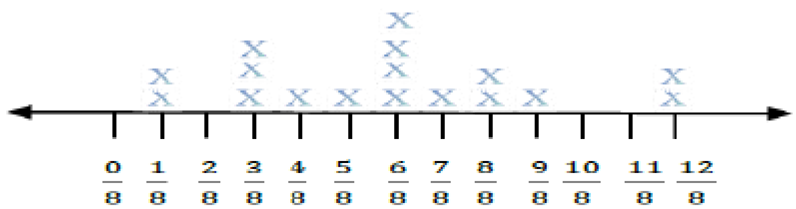
Present problems that involve multiplication of a fraction by a whole number (denominators are 2, 3, 4, 5 6, 8, 10, 12 and 100). Problems involving addition and subtraction of fractions should have the same denominators. Allow students to use strategies learned with these concepts.

Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems.

Instructional Resources/Tools
Graduated measuring cups (marked with customary and metric units)
Teaspoons and tablespoons
Yardsticks (meter sticks) and rulers (marked with customary and metric units)

Cluster: Represent and interpret data.
Data has been measured and represented on line plots in units of whole numbers, halves or quarters. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions.

Have students create line plots with fractions of a unit (1/2, 1/4, 1/8) and plot data showing multiple data points for each fraction.



Pose questions that students may answer, such as:

- "How many one-eighths are shown on the line plot?" Expect "two one-eighths" as the answer. Then ask, "What is the total of these two one-eighths?" Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.
- "What is the total number of inches for insects measuring 38 inches?" Students can use skip counting with fraction names to find the total, such as, "three-eighths, six-eighths, nine-eighths". The last fraction names the total. Students should notice that the denominator did not change when they were saying the fraction name. Have them make a statement about the result of adding fractions with the same denominator.
- "What is the total number of insects measuring 18 inch or 58 inches?" Have students write number sentences to represent the problem and solution such as, 18 + 18 + 58 = 78 inches.

Use visual-fraction strips and fraction bars to represent problems to solve problems involving addition and subtraction of fractions.

Instructional Resources/Tools
Fraction bars or strips

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (4.NF.3)
Understand decimal notation for fractions, and compare decimal fractions. (4.NF.6, 4.NF.7)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometry & Angle Measurement	4.MD.5a-b, 4.MD.6, 4.MD.7, 4.G.1, 4.G.2, 4.G.3

UNDERSTAND:

An angle is measured with reference to a circle and a circle is measured in terms of 360 degrees (i.e., full circle = 360 degrees).

Two-dimensional shapes can be classified based on properties of their angles (i.e., right, acute, obtuse,) and/or properties of their line segments (i.e., parallel, perpendicular).

KNOW:	DO:
An angle is a turn. Angles are measured in degrees (i.e., 1 full turn is 360 degrees, ½ turn = 180 degrees, ¼ turn = 90 degrees). A larger angle can be decomposed into smaller angles. Two or more angles can be combined to make a larger angle. 2-D shapes have angles at every vertex. Perpendicular lines intersect at a 90 degree angle. Parallel lines never intersect. A 2-D figure has line symmetry if it can be folded along the line into matching parts.	***Geometric measurement: understand concepts of angle and measure angles.*** **4.MD.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure). ***Draw and identify lines and angles, and classify shapes by properties of their lines and angles.*** **4.G.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **4.G.3** Recognize a line of symmetry for a two-dimensional figure, including those found in Montana American Indian designs, as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

KEY TERMS FOR THIS PROGRESSION:

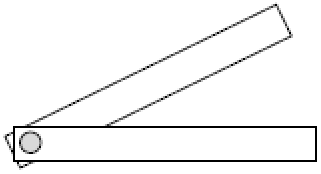
2-D shapes, Acute angle, Line, Line of symmetry, Line Segments, Obtuse angle, One-degree angle (1/360) Parallel lines, Perpendicular lines, Ray, Right triangle

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Geometric measurement: understand concepts of angle and measure angles.

Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger. Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles. Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures.

Students can also create an angle explorer (two strips of cardboard attached with a brass fastener) to learn about angles.



They can use the angle explorer to get a feel of the relative size of angles as they rotate the cardboard strips around.

Students can compare angles to determine whether an angle is acute or obtuse. This will allow them to have a benchmark reference for what an angle measure should be when using a tool such as a protractor or an angle ruler.

Provide students with four pieces of straw, two pieces of the same length to make one angle and another two pieces of the same length to make an angle with longer rays.

Another way to compare angles is to place one angle over the other angle. Provide students with a transparency to compare two angles to help them conceptualize the spread of the rays of an angle. Students can make this comparison by tracing one angle and placing it over another angle. The side lengths of the angles to be compared need to be different.

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a relationship between the number of degrees in an angle and circle which has a measure of 360 degrees. Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

Instructional Resources/Tools
Angle explorers
Angle ruler
Brass fasteners
Cardboard cut in strips to make an angle explorer
Protractor
Straws
Transparencies

National Council of Teachers of Mathematics. 2000-2012. *Figure This: What's My Angle?* Students can estimate the measures of the angles between their fingers when they spread out their hand.

Cindy Neuschwander. *Sir Cumference and the Great Knight of Angleland.* (Charlesbridge Publishing, 2001.) In this story, young Radius, son of Sir Cumference and Lady Di of Ameter, undertakes a quest, the successful completion of which will earn him his knighthood. With the help of a family heirloom that functions much like a protractor, he is able to locate the elusive King Lell and restore him to the throne of Angleland. In gratitude, King Lell bestows knighthood on Sir Radius.

Public Broadcasting Service. 1995-2012.*3rd Grade Measuring Game*: Identify acute, obtuse and right angles in this online interactive game

Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Angles:
Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as larger than, smaller than or the same size as a right angle.

Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

Symmetry:
When introducing line of symmetry, provide examples of geometric shapes with and without lines of symmetry. Shapes can be classified by the existence of lines of symmetry in sorting activities. This can be done informally by folding paper, tracing, creating designs with tiles or investigating reflections in mirrors.

With the use of a dynamic geometric program, students can easily construct points, lines and geometric figures. They can also draw lines perpendicular or parallel to other line segments.

Two-dimensional shapes:
Two-dimensional shapes are classified based on relationships by the angles and sides. Students can determine if the sides are parallel or perpendicular, and classify accordingly. Characteristics of rectangles (including squares) are used to develop the concept of parallel and perpendicular lines. The characteristics and understanding of parallel and perpendicular lines are used to draw rectangles. Repeated experiences in comparing and contrasting shapes enable students to gain a deeper understanding about shapes and their properties.

Informal understanding of the characteristics of triangles is developed through angle measures and side length relationships. Triangles are named according to their angle measures (right, acute or obtuse) and side lengths (scalene, isosceles or equilateral). These characteristics are used to draw triangles.

Instructional Resources/Tools
Geoboards
GeoGebra (free software for learning and teaching)
Mirrors

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (4.NF.4)

STANDARDS FOR MATHEMATICAL PRACTICE:

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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

SAMPLE YEAR LONG PLAN

COURSE: 4th GRADE MATH

Unit	Unit 1: Place Value	Unit 2: Addition/Subtraction of Large Numbers	Unit 6: Geometry	Unit 3: Multiplication	Unit 4: Division
(Time)	(15 days)	(20 days)	(5 days)	(30 days)	(30 days)
STANDARDS	4.NBT.1 4.NBT.2 4.NBT.3 4.OA.3	4.NBT.4	4.G.1 4.G.2 4.G.3 4.MD.3 4.MD.5 4.MD.6 4.MD.7	4.OA.1 4.OA.2 4.OA.3 4.OA.4 4.OA.5 4.NBT.5 4.NBT.6 4.MD.3	4.NBT.1 4.NBT.6 4.OA.5
Connections to other Domains and/or Clusters	4.NF.5, 4.NF.6, 4.NF.7	4.NF.3.c	4.NF.4	4.MD.1, 4.MD.2, 4.MD.3 4.NBT.1, 4.NBT.2, 4.NBT.4 4.NF.4, 4.NF.5, 4.NF.7	4.NF.4, 4.NF.5, 4.NF.7

4th GRADE MATH p. 2

Unit 6: Geometry	Unit 5: Fractions/Decimals	Unit 6: Geometry	Unit 7: Line Plots/Graphing	Unit 8: Measurement/ Probability
(5 days)	(35 days)	(5 days)	(15 days)	(15 days)
4.G.1 4.G.2 4.G.3 4.MD.3 4.MD.5 4.MD.6 4.MD.7	4.NF.1 4.NF.2 4.NF.3 4.NF.4 4.NF.5 4.NF.6 4.NF.7	4.G.1 4.G.2 4.G.3 4.MD.3 4.MD.5 4.MD.6 4.MD.7	4.MD.4 4.NF.1	4.MD.1 4.MD.2 4.MD.3
4.NF.4	4.MD.1, 4.MD.2, 4.MD.4	4.NF.4	4.MD.1, 4.MD.2 4.NF.3	4.NF.3, 4.NF.6, 4.NF.7

TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: MATH GRADE LEVEL(S): 4TH GRADE

LAST UNIT	CURRENT UNIT	NEXT UNIT
Basic Multiplication	**Factors/Multiples**	Multiplication of Larger Numbers

UNIT SCHEDULE	
Day 1	Review of basic multiplication facts
Day 2	Factor Patterns
Day 3	Rainbow Factors
Day 4	Factor Practice with Games
Day 5	Hands on Factor Activity
Day 6	Factor Assessment
Day 7	Multiple patterns (count-bys)
Day 8	Multiple practice with hundred grid and counters
Day 9	Multiple Games
Day 10	Multiple Towers
Day 11	Multiple Tower
Day 12	Multiples Assessment

is about…

Multiples

- *By analyzing patterns*
- *Using multiplication facts*
- *By comparing multiples of numbers 2-12*
- *Applying rules of divisibility*

UNIT SELF TEST QUESTIONS

1. Can I define multiples?
2. Can I compare multiples and factors?
3. Can I use multiples to solve problems?
4. Where would I apply this to solve real world problems?

MATH STANDARDS	
4.OA.4	

4th Grade Lesson Plan Sample

Teacher:

Date:

Subject area / course / grade level: ***Math/Multiples/4th***

Materials: ***scissors, pencils, tape, rulers, butcher paper, scratch paper***

Practice Standards: ***Make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, look for and express regularity in repeated reasoning.***

Content Standards: ***4.OA.4, 4.OA.5***

Lesson objective(s): ***to visually match common multiples in numbers and that multiples have common factors.***

ENGAGEMENT

- Buzz Game: Going around the room, teacher chooses a number to list multiples of. Students take turns counting starting at one, when they get to a multiple of the number the teacher chose they have to say BUZZ instead of the number or they are out and must sit down. Students continue counting until all multiples have been listed.
- Students should be recognizing patterns of multiples

EXPLORATION

- Students work in partners to create towers with unifix cubes. Students will be using two different colors of cubes. One color will represent the multiple. The other will represent all the other numbers. Students will build towers for the multiples of 2-12 for the time frame

EXPLANATION

- Students will explore each other's towers and make comparisons.
- Each group will bring one tower to the front of the room to place next to each other in order from 2-12
- Teacher will bring students together for discussion and students will share their findings with the group.

ELABORATION

- Students will work in groups with a specific multiple. They will be using rulers to create centimeter units for their given multiple (example: 3 would be 3 x 3). Students will cut out squares of paper for the first twelve multiples of their given number. They will label these with the multiples and glue together as a tower. Students will create a classroom skyscraper mural on butcher paper with their towers to demonstrate their knowledge of multiples.

EVALUATION

- Students will individually be given two long strips of paper at their seat. The class will be assigned two random numbers to write the multiples on their strips of paper. Students will evaluate their own understanding when teacher reviews the answers. Students will staple together and hand in for teacher review.

Constructed Response Sample
Grade: 04

Communicating Reasoning
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Concepts and Procedures:
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Primary Content Domain:
Operations and Algebraic Thinking.
Gain familiarity with factors and multiples.

Standard: 4.OA.4

Mathematical Practice: 1, 2, 3, 8

DOK: 2

Task:
Peter made the statement shown below.
"The number 32 is a multiple of 8. That means all of the factors of 8 are also factors of 32."

Is Peter's statement correct? In the space below, use numbers and words to explain why or why not.

Sample Response:
Peter's statement is correct. The factors of 8 are 1, 2, 4, and 8. The factors of 32 are 1, 2, 4, 8, 16, and 32.

Score: 2

Scoring Rubric:
Responses to this item will receive 0–2 points, based on the following:

2 points: The student has a thorough understanding of the relationship between factors and multiples of numbers. The student correctly answers both parts and provides an explanation of reasoning that is thorough and correct for each part.

1 point: The student has a partial understanding of the relationship between factors and multiples of numbers. The student indicates that Peter's statement is correct, but provides an explanation of reasoning that is incomplete or contains a flaw.

0 points: The student has no understanding of the relationship between factors and multiples of numbers. The student does not complete any part correctly. Identifying Peter's statement as correct is not sufficient, by itself, to earn any points.

SAMPLE CONSTRUCTED RESPONSE
Grade: 04

Communicating Reasoning
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Concepts and Procedures:
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Primary Content Domain:
Operations and Algebraic Thinking.
Solving problems with equal groups and arrays.

Standard: 4.OA.4

Mathematical Practice: 1, 2, 3, 4, 8

DOK: 3

Task:
Students at Creek Elementary are going to an assembly. Each class arranges its chairs in a rectangular form. What are all the possible arrangements for the following classes?

Miss Franklin 30 students
Mr. Clark 27 students
Ms. Rodriguez 31 students
Mrs. Smith 13 students

Help the students as they work through the task by asking questions to help them use multiple strategies; drawing pictures, using manipulatives, etc... Then debrief, talking about the characteristics of the above numbers. Which ones had equal rows, which did not and why (Odd vs. Even Numbers)? Have the students write in their math journals how they solved the task and why their answer was correct or incorrect.

Sample Response:
Score:

Scoring Rubric:
Responses to this item will receive 0–2 points, based on the following:

2 points: The student has a thorough understanding of the relationship between equal groups, arrays, factors, and multiples of numbers. The student correctly answers both parts and provides an explanation of reasoning that is thorough and correct for the task in their journal.

1 point: The student has a partial understanding of the relationship between equal groups, arrays, factors and multiples of numbers. The student indicates an explanation of reasoning that is incomplete or contains a flaw in their journal.

0 points: The student has no understanding of the relationship between equal groups, arrays, factors, and multiples of numbers. The student does not complete any part correctly. The student does not explain in a coherent manner how they solved the task in their journal.

Howard County Public School System • Draft 2012/2013
Adapted from Van de Walle, J. (2004) Elementary and Middle School Mathematics: Teaching Developmentally. Boston: Pearson Education, 65

Not yet: Student shows evidence of misunderstanding, incorrect concept or procedure		**Got It:** Student essentially understands the target concept.	
0 Unsatisfactory: **Little Accomplishment** The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little or no success. Further teaching is required.	**1 Marginal:** **Partial Accomplishment** Part of the task is accomplished, but there is lack of evidence of understanding or evidence of not understanding. Further teaching is required.	**2 Proficient:** **Substantial Accomplishment** Student could work to full accomplishment with minimal feedback from teacher. Errors are minor. Teacher is confident that understanding is adequate to accomplish the objective with minimal assistance.	**3 Excellent:** **Full Accomplishment** Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas. May have minor errors.

FIFTH GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grade 5 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
5.MP.1. Make sense of problems and persevere in solving them.	Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
5.MP.2. Reason abstractly and quantitatively.	Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
5.MP.3. Construct viable arguments and critique the reasoning of others.	In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
5.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5.MP.5. Use appropriate tools strategically.	Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.
5.MP.6. Attend to precision.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
5.MP.7. Look for and make use of structure.	In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
5.MP.8. Look for and express regularity in repeated reasoning.	Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Explanations and Examples Grade 1
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Grade 5 Content Standards

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 5 Overview

Operations and Algebraic Thinking
- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten
- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions
- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data
- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry
- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Operations and Algebraic Thinking — 5.OA

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as* $2 \times (8 + 7)$*. Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$*, without having to calculate the indicated sum or product.*

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten — 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations—Fractions — 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $2/3 + 5/4 = 8/12 + 15/12 = 23/12$*. (In general,* $a/b + c/d = (ad + bc)/bd$*.)*
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $2/5 + 1/2 = 3/7$*, by observing that* $3/7 < 1/2$*.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation within cultural contexts, including those of Montana American Indians. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem within cultural contexts, including those of Montana American Indians.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
   b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*
   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems within a cultural context, including those of Montana American Indians.

Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
   b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume within cultural contexts, including those of Montana American Indians.
    a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
    b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
    c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.
1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation including those found in Montana American Indian designs.

Classify two-dimensional figures into categories based on their properties.
3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
4. Classify two-dimensional figures in a hierarchy based on properties.

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

GRADE 5

Domain	Cluster	Code	Common Core State Standard
Operations and Algebraic Thinking	Write and interpret numerical expressions.	5.OA.1	Use parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols.
		5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.
	Analyze patterns and relationships.	5.OA.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.
Number and Operations in Base Ten	Understand the place value system.	5.NBT.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
		5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use positive integer exponents to denote powers of 10.
		5.NBT.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
		5.NBT.4	Use place value understanding to round decimals to any place.
	Perform operations with multi-digit whole numbers and with decimals to hundredths.	5.NBT.5	Fluently multiply multi-digit whole numbers using the standard algorithm.
		5.NBT.6	Find whole-number quotients with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
		5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

GRADE 5

Domain	Cluster	Code	Common Core State Standard
Number and Operations: Fractions	Use equivalent fractions as a strategy to add and subtract fractions.	5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)
		5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7 by observing that 3/7 < 1/2.
	Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	5.NF.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
		5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation within cultural contexts, including those of Montana American Indians. . Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
		5.NF.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.
		5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem within cultural contexts, including those of Montana American Indians.
		5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Note: Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context within cultural contexts, including those of Montana American Indians, for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication & division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context within cultural contexts, including those of Montana American Indians, for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

GRADE 5

Domain	Cluster	Code	Common Core State Standard
Measurement and Data	Convert like measurement units within a given measurement system.	5.MD.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step real world problems within a cultural context, including those of Montana American Indians.
	Represent and interpret data.	5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.
	Geometric measurement--understand concepts of volume and relate volume to multiplication and to addition.	5.MD.3	Recognize volume as an attribute of solid figures and understand concepts of volume measuremen a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
		5.MD.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
		5.MD.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume within cultural contexts, including those of Montana American Indians. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
Geometry	Graph points on the coordinate plane to solve real-world and mathematical problems.	5.G.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate)
		5.G.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation including those found in Montana American Indian designs.
	Classify two-dimensional figures into categories based on their properties.	5.G.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.
		5.G.4	Classify two-dimensional figures in a hierarchy based on properties.

Getting Ready
Every Montana Student
College and Career

Montana Common Core
Standards and Assessments

Montana Curriculum Organizer

Grade 5

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

November 2012 Materials adapted from Arizona, Delaware and Ohio Departments of Education

Montana Curriculum Organizer: Grade 5 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

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HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 24 consist of tables organized into learning progressions that can function as units. The table for each learning progression unit includes: 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.	Standards covered in this learning progression.
UNDERSTAND:	
What students need to understand by the end of this learning progression.	
KNOW:	**DO:**
What students need to know by the end of this learning progression.	What students need to be able to do by the end of this learning progression, organized by cluster and standard.
KEY TERMS FOR THIS PROGRESSION:	
Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.	
INSTRUCTIONAL STRATEGIES AND RESOURCES:	
Cluster: Title Strategies for this cluster **Instructional Resources/Tools** Resources and tools for this cluster	
Cluster: Title Strategies for this cluster **Instructional Resources/Tools** Resources and tools for this cluster	
CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:	
Standards that connect to this learning progression are listed here, organized by cluster.	
STANDARDS FOR MATHEMATICAL PRACTICE:	
A quick reference guide to the eight standards for mathematical practice is listed here.	

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Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Grade 5 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
5.MP.1. Make sense of problems and persevere in solving them.	Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions, including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?" and "Can I solve the problem in a different way?"
5.MP.2. Reason abstractly and quantitatively.	Fifth-graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place-value concepts.
5.MP.3. Construct viable arguments and critique the reasoning of others.	In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
5.MP.4. Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth-graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5.MP.5. Use appropriate tools strategically.	Fifth-graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real-world data.
5.MP.6. Attend to precision.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism, they record their answers in cubic units.
5.MP.7. Look for and make use of structure.	In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
5.MP.8. Look for and express regularity in repeated reasoning.	Fifth-graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

CRITICAL AREAS FOR GRADE 5

In Grade 5, instructional time should focus on three critical areas:

(1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions);

(2) extending division to two-digit divisors, integrating decimal fractions into the place-value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole-number and decimal operations; and

(3) developing an understanding of volume.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Operations & Algebraic Thinking (Expressions & Relationships)	5.OA.1, 5.OA.2, 5.OA.3

UNDERSTAND:

Mathematical rules and expressions depict mathematical relationships.

KNOW:	DO:
Mathematical computations are performed following a given order: the order of operations. Functions of mathematical symbols. Numerical patterns can be generated based on a rule. Ordered pairs form a relationship that generate a pattern and can be represented in multiple ways (e.g., tables, graphs, etc.). Numerical patterns (i.e., 0, 3, 6, 9, etc.). Parts of ordered pairs.	***Write and interpret numerical expressions.*** **5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.* ***Analyze patterns and relationships.*** **5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

KEY TERMS FOR THIS PROGRESSION:

Braces, Brackets, Calculations, Coordinate plane, Numerical expressions, Numerical patterns, Order of Operations, Ordered pair, Parenthesis

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Write and interpret numerical expressions.
Students should be given ample opportunities to explore numerical expressions with mixed operations. This is the foundation for evaluating numerical and algebraic expressions that will include whole-number exponents in Grade 6.

There are conventions (rules) determined by mathematicians that must be learned with no conceptual basis. For example, multiplication and division are always done before addition and subtraction.

Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Using the same digits, with the operations in a different order, have students evaluate the expressions and discuss why the value of the expression is different. For example, have students evaluate 5 × 3 + 6 and 5 + 3 × 6. Discuss the rules that must be followed. Have students insert parentheses around the multiplication or division part in an expression. A discussion should focus on the similarities and differences in the problems and the results. This leads to students being able to solve problem situations which require that they know the order in which operations should take place.

After students have evaluated expressions without grouping symbols, present problems with one grouping symbol, beginning with parentheses, then in combination with brackets and/or braces.

Have students write numerical expressions in words without calculating the value. This is the foundation for writing algebraic expressions. Then, have students write numerical expressions from phrases without calculating them.

Instructional Resources/Tools
Calculators (scientific or four-function)

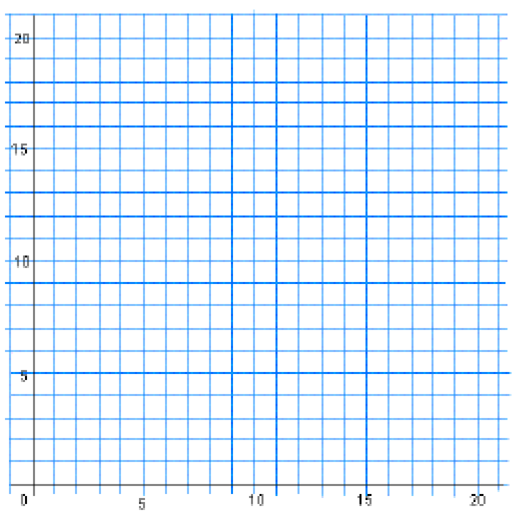
National Council of Teachers of Mathematics. 2000-2012. *Order of Operations Bingo:* Instead of calling numbers to play Bingo, you call (and write) numerical expressions to be evaluated for the numbers on the Bingo cards. The operations in this lesson are addition, subtraction, multiplication, and division; the numbers are all single-digit whole numbers.

Cluster: Analyze patterns and relationships.
Students should have experienced generating and analyzing numerical patterns using a given rule in Grade 4.

Given two rules with an apparent relationship, students should be able to identify the relationship between the resulting sequences of the terms in one sequence to the corresponding terms in the other sequence. For example, starting with 0, multiply by 4 and starting with 0, multiply by 8 and generate each sequence of numbers (0, 4, 8, 12, 16, …) and (0, 8, 16, 24, 32, …). Students should see that the terms in the second sequence are double the terms in the first sequence, or that the terms in the first sequence are half the terms in the second sequence.

Have students form ordered pairs and graph them on a coordinate plane. Patterns can be also discerned in graphs.

Graphing ordered pairs on a coordinate plane is introduced to students in the geometry domain where students solve real-world and mathematical problems. For the purpose of this cluster, only use the first quadrant of the coordinate plane, which contains positive numbers only. Provide coordinate grids for the students, but also have them make coordinate grids. In Grade 6, students will position pairs of integers on a coordinate plane.



The graph of both sequences of numbers is a visual representation that will show the relationship between the two sequences of numbers.

Encourage students to represent the sequences in T-charts so that they can see a connection between the graph and the sequences.

0	0
1	4
2	8
3	12
4	16

0	0
1	8
2	16
3	24
4	32

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
Graph points on the coordinate plane to solve real-world and mathematical problems. (5.G.1, 5.G.2)

STANDARDS FOR MATHEMATICAL PRACTICE:	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Number & Operations in Base Ten – Understanding Place Value	5.NBT.1, 5.NBT.2, 5.NBT.3a-b, 5.NBT.4, 5.NBT.5, 5.NBT.6, 5.NBT.7

UNDERSTAND:

The value of a digit in our number system is determined by its place-value position.
Place-value patterns are continued in decimal numbers.
Computational strategies with whole numbers can be applied to decimals.

KNOW:	DO:
Place value is based on multiples of ten. A digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. When multiplying/dividing whole numbers and decimals by powers of 10, the digits move based on place value, not the decimal point. Multiplying and dividing whole numbers and decimals by 10 results in a pattern of zeros. Decimal numbers fall between whole numbers. Proximity of a decimal to the nearest whole number. Rounding is a formal way of estimating. Process of standard algorithm for multiplication. There are multiple standard algorithms: partial products/distributed multiplication, traditional. Strategies for dividing whole numbers. Strategies to perform all operations. Properties of operations (i.e., distributive property). Addition and subtraction are inverse operations. Multiplication and division are inverse operations.	***Understand the place-value system.*** **5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.3** Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form (e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NBT.4** Use place-value understanding to round decimals to any place. ***Perform operations with multi-digit whole numbers and with decimals to hundredths.*** **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

KEY TERMS FOR THIS PROGRESSION:

Decimal, Hundredths, Place value, Powers of ten, Rounding, Tenths, Thousandths

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Understand the place-value system.

In Grade 5, the concept of place value is extended to include decimal values to thousandths. The strategies for Grades 3 and 4 should be drawn upon and extended for whole numbers and decimal numbers. For example, students need to continue to represent, write and state the value of numbers including decimal numbers. For students who are not able to read, write and represent multi-digit numbers, working with decimals will be challenging.

Money is a good medium to compare decimals. Present contextual situations that require the comparison of the cost of two items to determine the lower or higher priced item. Students should also be able to identify how many pennies, dimes, dollars and ten dollars, etc., are in a given value. Help students make connections between the number of each type of coin and the value of each coin, and the expanded form of the number. Build on the understanding that it always takes ten of the number to the right to make the number to the left.

Number cards, number cubes, spinners and other manipulatives can be used to generate decimal numbers. For example, have students roll three number cubes, then create the largest and smallest number to the thousandths place. Ask students to represent the number with numerals and words.

Instructional Resources/Tools

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Base Block Decimals:* Student use Ten Frames to demonstrate decimal relationships.

Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and their strategies.

Connections between the algorithm for multiplying multi-digit whole numbers and strategies such as partial products or lattice multiplication are necessary for students' understanding.

You can multiply by listing all the partial products. For example:

```
 234
x  8
----
  32   Multiply the ones (8 × 4 ones = 32 ones)
 240   Multiply the tens (8 × 3 tens = 24 tens or 240
1600   Multiply the hundreds (8 × 2 hundreds = 16 hundreds or 1600)
----
1872   Add the partial products
```

The multiplication can also be done without listing the partial products by multiplying the value of each digit from one factor by the value of each digit from the other factor. Understanding of place value is vital in using the standard algorithm.

In using the standard algorithm for multiplication, when multiplying the ones, 32 ones is 3 tens and 2 ones. The 2 is written in the ones place. When multiplying the tens, the 24 tens is 2 hundreds and 4 tens. But, the 3 tens from the 32 ones need to be added to these 4 tens, for 7 tens. Multiplying the hundreds, the 16 hundreds is 1 thousand and 6 hundreds. But, the 2 hundreds from the 24 tens need to be added to these 6 hundreds, for 8 hundreds.

```
 234
x  8
----
1872
```

As students developed efficient strategies to do whole-number operations, they should also develop efficient strategies with decimal operations.

Students should learn to estimate decimal computations before they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations, not on how many decimal places are involved. For example, to estimate the product of 32.84 × 4.6, the estimate would be more than 120, closer to 150. Students should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84 × 4.6 should be close to 150.

Have students use estimation to find the product by using exactly the same digits in one of the factors with the decimal point in a different position each time. For example, have students estimate the product of 275 × 3.8; 27.5 × 3.8 and 2.75 × 3.8. Discuss why the estimates should or should not be the same.

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Instructional Resources/Tools
Decimal place-value chart

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Base Block Decimals:* Student use Ten Frames to demonstrate decimal relationships.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. (5.MD.4, 5.MD.5)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Number & Operations: Fractions – Adding & Subtracting	5.NF.1, 5.NF.2

UNDERSTAND:

Equivalent fractions are a powerful strategy for adding and subtracting fractions.
Multiple strategies and models can be utilized to solve a variety of problems involving fractional concepts.

KNOW:	DO:
Fractions can be added and subtracted using area models, ratio models, number lines, fraction bars, and finding common denominators. Relationship between numbers and their multiples are used to find equivalent fractions. Benchmark fractions and fraction-number sense can be used to estimate fraction sums and differences and assess the reasonableness of solutions. Methods for recording strategies for adding and subtracting fractions using models or equations.	***Use equivalent fractions as a strategy to add and subtract fractions.*** **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual-fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

KEY TERMS FOR THIS PROGRESSION:

Benchmark fractions, Common denominator, Denominator, Equivalent fractions, Estimate, Fraction, Fraction bar, Mixed numbers, Number line, Numerator, Ratio model, Sum

INSTRUCTIONAL STRATEGIES AND RESOURCES:

***Cluster: Use equivalent fractions as a strategy to add and subtract fractions*.**

To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them.

Students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions.

As with solving word problems with whole-number operations, regularly present word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems. Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable.

Estimation is about getting useful answers, it is not about getting the right answer. It is important for students to learn which strategy to use for estimation. Students need to think about what might be a close answer.

Instructional Resources/Tools

Utah State University. National Library of Virtual Manipulatives. 1999-2010.

Fraction Bars: Learn about fractions using fraction bars.
Fractions - Adding: Illustrates what it means to find a common denominator and combine.
Number-line Bars: Use bars to show addition, subtraction, multiplication, and division on a number line.

Montana Curriculum Organizer: Grade 5 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
Represent and interpret data. (5.MD.2)

STANDARDS FOR MATHEMATICAL PRACTICE:	
1. Make sense of problems and persevere in solving them.	4. Model with mathematics.
2. Reason abstractly and quantitatively.	5. Use appropriate tools strategically.
3. Construct viable arguments and critique the reasoning of others.	6. Attend to precision.
	7. Look for and make use of structure.
	8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Number and Operations – Fractions (Multiplying & Dividing)	5.NF.3, 5.NF.4a-b, 5.NF.5a-b, 5.NF.6, 5.NF.7a-c

UNDERSTAND:

Extending previous understandings of multiplication and division can help solve problems involving multiplying and dividing fractions.

KNOW:	DO:
Fractions can be perceived and utilized as division of the numerator by the denominator. Multiplying a whole number by a number greater than 1 results in a product greater than the given number (e.g., $3^1/_2$ x 5 will result in a number more than $3^1/_2$). Multiplying a whole number by a number smaller than 1 results in a product less than the given number (e.g., $3^1/_2$ x $^1/_4$ will result in a number less than $3^1/_2$). Multiplying a whole number by a number/ fraction equal to 1 results in a number that represents the same quantity. When multiplying fractions, either factor can be the multiplier (e.g., $^2/_3$ x 4 can be interpreted as $^2/_3$ of 4 or 4 groups of $^2/_3$). Visual-fraction models can be used to solve problems like $^1/_3$ divided by 4 or 4 divided by $^1/_3$. An array model can justify the formula: $A = L$ x W.	***Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*** **5.NF.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers (e.g., by using visual-fraction models or equations to represent the problem). *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation within cultural contexts, including those of Montana American Indians. Do the same with (2/3) × (4/5) = 8/15.* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit-fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF.5** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers (e.g., by using visual-fraction models or equations to represent the problem within cultural contexts, including those of Montana American Indians).

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5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for (1/3) ÷ 4, and use a visual-fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context within cultural contexts, including those of Montana American Indians, for 4 ÷ (1/5), and use a visual-fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (e.g., by using visual-fraction models and equations to represent the problem). *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

KEY TERMS FOR THIS PROGRESSION:

Denominators, Factor, Fraction, Groups of, Mixed number, Multiplier, Numerators, Product, Quotient, Scaling

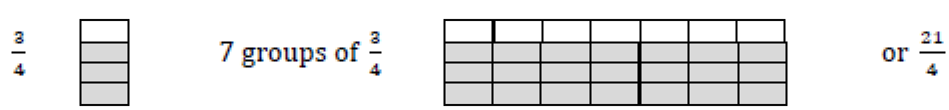
INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division.

Ask questions such as, “What does 2 × 3 mean?” and “What does 12 ÷ 3 mean?” Then, follow with questions for multiplication with fractions, such as, “What does 3/4 × 1/3 mean?”, “What does 3/4 × 7 mean?” (7 sets of 3/4) and “What does 7 × 3/4 mean?” (34 of a set of 7)

The meaning of 4 ÷ 1/2 (“How many 1/2 are in 4?”) and 1/2 ÷ 4 (“How many groups of 4 are in ½?”) also should be illustrated with models or drawings like:



Encourage students to use models or drawings to multiply or divide with fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

Models to consider when multiplying or dividing fractions include, but are not limited to, area models using rectangles or squares, fraction-strips/bars and sets of counters.

Use calculators or models to explain what happens to the result of multiplying a whole number by a fraction (e.g., 3 × 1/2, 4 × 1/2, 5 × 1/2, … and 4 × 1/2, 4 × 1/3, 4 × 1/4, …) and when multiplying a fraction by a number greater than 1.

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. Division of a fraction by a fraction is not a requirement at this grade.

Use calculators or models to explain what happens to the result when dividing a unit fraction by a non-zero whole number (e.g., 1/8 ÷ 4, 1/8 ÷ 8, 1/8 ÷ 16, …) and what happens to the result when dividing a whole number by a unit fraction (e.g., 4 ÷ 1/4 , 8 ÷ 1/4 12 ÷ 1/4, …).

Present problem situations and have students use models and equations to solve the problem. It is important for students to develop understanding of multiplication and division of fractions through contextual situations.

Instructional Resources/Tools

Utah State University. National Library of Virtual Manipulatives. 1999-2010.

Fractions: Rectangle Multiplication: Students can visualize and practice multiplying fractions using an area representation.

Number Line Bars: Fractions: Students can divide fractions using number line bars.

Charles A. Dana Center. University of Texas at Austin. Mathematics TEKS Toolkit. 2012. *Divide and Conquer*: Students can better understand the algorithm for dividing fractions if they analyze division through a sequence of problems starting with division of whole numbers, followed by division of a whole number by a unit fraction, division of a whole number by a non-unit fraction, and finally division of a fraction by a fraction (addressed in Grade 6).

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Graph points on the coordinate plane to solve real-world and mathematical problems. (5.NBT.6, 5.NBT.7)

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
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Montana Curriculum Organizer: Grade 5 Mathematics

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Measurement & Data (Measurement system, line plots, volume)	5.MD.1, 5.MD.2, 5.MD.3a-b, 5.MD.4, 5.MD.5a-c

UNDERSTAND:

There are a multiple ways to organize, recognize, and interpret data for a variety of purposes.
The concepts of volume are related to area, multiplication and division.

KNOW:	DO:
Standard Measurement Units can be used interchangeably. Data can be organized, represented, and interpreted in multiple ways. Volume is an attribute of solid figures relating length, width, and height (depth). Volume is "filling" the inside space if it is a 3-D shape. Volume is additive: The volumes of two non-overlapping rectangular prisms can be added to find a total volume. The formula *V* = *B* x *h* relates the total volume as multiple layers of the Base (area). The area of a rectangular base can be utilized when calculating the volume.	***Convert like-measurement units within a given measurement system.*** **5.MD.1** Convert among different-sized standard-measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems. ***Represent and interpret data.*** **5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (e.g., 1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* ***Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.*** **5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.** a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units. **5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

KEY TERMS FOR THIS PROGRESSION:

Area, Array, Conversion, Cubic units, Line plot, Right rectangular prism, Solid figures, Unit cubes, Volume

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INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Convert like-measurement units within a given measurement system.

Students should gain ease in converting units of measures in equivalent forms within the same system. To convert from one unit to another unit, the relationship between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit. For example, students have discovered that there are 12 inches in 1 foot and 3 feet in 1 yard. This understanding is needed to convert inches to yards. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers are equivalent to one yardstick (3 × 12 inches = 36 inches; 36 inches = 1 yard). Using this knowledge, students can decide whether to multiply or divide when making conversions.

Once students have an understanding of the relationships between units and how to do conversions, they are ready to solve multi-step problems that require conversions within the same system. Allow students to discuss methods used in solving the problems. Begin with problems that allow for renaming the units to represent the solution before using problems that require renaming to find the solution.

Instructional Resources/Tools

Graduated measuring cups (marked with customary and metric units)
Teaspoons and tablespoons
Yardsticks (meter sticks) and rulers (marked with customary and metric units)

National Council of Teachers of Mathematics. 2000-2012.

> *Measuring Up: Discovering Gallon Man:* Students experiment with units of liquid measure used in the customary system of measurement. They practice making volume conversions in the customary system.
> *Measuring Up: Do You Measure Up?* Students learn the basics of the metric system. They identify which units of measurement are used to measure specific objects, and they learn to convert between units within the same system.

Cluster: Represent and interpret data.

Using a line plot to solve problems involving operations with unit fractions now includes multiplication and division. Revisit using a number line to solve multiplication and division problems with whole numbers. In addition to knowing how to use a number line to solve problems, students also need to know which operation to use to solve problems.

Use the tables for common addition and subtraction, and multiplication and division situations (Table 1 and Table 2 in the Common Core State Standards for Mathematics) as a guide to the types of problems students need to solve without specifying the type of problem. Allow students to share methods used to solve the problems. Also have students create problems to show their understanding of the meaning of each operation.

Instructional Resources/Tools

Montana Office of Public Instruction. 2011. *Montana Common Core Standards for School Mathematics Grade-Band*: Table 1 and Table 2.

National Council of Teachers of Mathematics. 2000-2012. *Fractions in Every Day Life:* This activity enables students to apply their knowledge about fractions to a real-life situation. It also provides a good way for teachers to assess students' working knowledge of fraction multiplication and division. Students should have prior knowledge of adding, subtracting, multiplying, and dividing fractions before participating in this activity. This will help students to think about how they use fractions in their lives, sometimes without even realizing it. The basic idea behind this activity is to use a recipe and alter it to serve larger or smaller portions.

Cluster: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

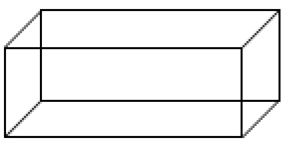
Volume refers to the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters.

Students need to experience finding the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is presented. Provide multiple opportunities for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a 1- or 2-cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box.

Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the same size.

By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how addition plays a part in finding volume. This will lead to an understanding of the formula for the volume of a right rectangular prism, $b \times h$, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Ask what strategies can be used to determine the volume of the prism based on the number of cubes in the bottom layer. Expect responses such as “adding the same number of cubes in each layer as were on the bottom layer” or “multiply the number of cubes in one layer times the number of layers”.

Instructional Resources/Tools
Cubes
Grid paper
Rulers (marked in standard or metric units)

National Council of Teachers of Mathematics. 2000-2012. *Cubes*: Determine the volume of a box by filling it with cubes, rows of cubes, or layers of cubes.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

None

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometry – Graphing & Properties of 2-D Figures	5.G.1, 5.G.2, 5.G.3, 5.G.4

UNDERSTAND:

Coordinate systems can be used to describe locations precisely.
2-D shapes can be identified, classified and analyzed by their properties.

KNOW:	DO:
A pair of perpendicular lines form a coordinated system, with the intersection of the lines (the origin) is coordinated to form the point (0,0). The first number in an ordered pair tells how far to travel from the origin on the *x*-axis, and second number says how far to travel on the *y*-axis. Points that lie on a graphed (linear) line express equivalent ratios. Points graphed on a coordinate plan can be interpreted to solve problems. Two-dimensional shapes are classified by their attributes (i.e., number of sides, number of angles, types of angles, regular vs. irregular polygons, etc.).	***Graph points on the coordinate plane to solve real-world and mathematical problems.*** **5.G.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation including those found in Montana American Indian designs. ***Classify two-dimensional figures into categories based on their properties.*** **5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* **5.G.4** Classify two-dimensional figures in a hierarchy based on properties.

KEY TERMS FOR THIS PROGRESSION:

2-dimensional, Attributes, Axis, Coordinate plane, Coordinates, Intersection, Perpendicular, Points, Properties, Quadrant

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Graph points on the coordinate plane to solve real-world and mathematical problems.
Students need to understand the underlying structure of the coordinate system and see how axes make it possible to locate points anywhere on a coordinate plane. This is the first time students are working with coordinate planes, and only in the first quadrant. It is important that students create the coordinate grid themselves. This can be related to two number lines and reliance on previous experiences with moving along a number line.

Multiple experiences with plotting points are needed. Provide points plotted on a grid and have students name and write the ordered pair. Have students describe how to get to the location. Encourage students to articulate directions as they plot points.

Present real-world and mathematical problems and have students graph points in the first quadrant of the coordinate plane. Gathering and graphing data is a valuable experience for students. It helps them to develop an understanding of coordinates and what the overall graph represents. Students also need to analyze the graph by interpreting the coordinate values in the context of the situation.

Instructional Resources/Tools
Grid/graph paper

National Council of Teachers of Mathematics. 2000-2012.

Finding Your Way Around: Students explore two-dimensional space via an activity in which they navigate the coordinate plane.
Describe the Graph: In this lesson, students will review plotting points and labeling axes. Students generate a set of random points all located in the first quadrant.

Cluster: Classify two-dimensional figures into categories based on their properties.
This cluster builds from Grade 3 when students described, analyzed and compared properties of two-dimensional shapes. They compared and classified shapes by their sides and angles, and connected these with definitions of shapes. In Grade 4 students built, drew and analyzed two-dimensional shapes to deepen their understanding of the properties of two-dimensional shapes. They looked at the presence or absence of parallel and perpendicular lines or the presence or absence of angles of a specified size to classify two-dimensional shapes. Now, students classify two-dimensional shapes in a hierarchy based on properties. Details learned in earlier grades need to be used in the descriptions of the attributes of shapes. The more ways that students can classify and discriminate shapes, the better they can understand them. The shapes are not limited to quadrilaterals.

Students can use graphic organizers such as flow charts or T-charts to compare and contrast the attributes of geometric figures. Have students create a T-chart with a shape on each side. Have them list attributes of the shapes, such as number of side, number of angles, types of lines, etc. they need to determine what's alike or different about the two shapes to get a larger classification for the shapes. Pose questions such as, “Why is a square always a rectangle?” and “Why is a rectangle not always a square?”

Instructional Resources/Tools
National Council of Teachers of Mathematics. 2000-2012.

Rectangles and Parallelograms: Students use dynamic software to examine the properties of rectangles and parallelograms, and identify what distinguishes a rectangle from a more general parallelogram. Using spatial relationships, they will examine the properties of two- and three-dimensional shapes.
Ploygon Capture: In this lesson, students classify polygons according to more than one property at a time. In the context of a game, students move from a simple description of shapes to an analysis of how properties are related.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

Analyze patterns and relationships. (5.OA.3)

STANDARDS FOR MATHEMATICAL PRACTICE:

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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

5th Grade Mathematics Year-at-a-Glance (SAMPLE)

COURSE: 5th				
Unit (Organized by Quarter	**Unit 1: Understand the Place Value System**	**Unit 2: Operations with Whole Numbers and Decimals**	**Unit 3: Convert within a Given System**	**Flex Time**
(Time)	(15 Days)	(20 days)	(5 days)	(5 Days)
STANDARDS	5.NBT.1	5.NBT.7	5.MD.1	
	5.NBT.2			
	5.NBT.3			
	5.NBT.4			
	(Values of numbers, patterns with zeros, comparing, rounding)	(Add, subtract, multiply, divide decimals to hundredths)	(Metric and standard)	

Key:	
	Major Standards (green)
	Supporting Standards (blue)
	Additional Standards (yellow)

5th Grade Mathematics Year-at-a-Glance (SAMPLE)

Unit 4: Classify 2-D Figures	Unit 5: Understand Concepts of Volume and relate to Multiplication and Division	Unit 6: Add, Subtract, Multiply and Divide Decimals to the Hundredths (Review)	Flex Time
(5 days)	(20 days)	(15 days)	(5 Days)
5.G.3	5.MD.3	5.NBT.5	
5.G.4	5.MD.4	5.NBT.6	
	5.MD.5	5.NBT.7	
(attributes of 2-D figures, hierarchy based on properties)	(Cubic units, measuring & counting, relate to multiplication & division, apply formulas V = l x w x h or V = b x h, recognize volume as additive)	(Multiply multi-digit whole numbers, find whole number quotients; 4-digit dividents & 2-digit divisors)	

Key:	
	Major Standards
	Supporting Standards
	Additional Standards

5th Grade Mathematics Year-at-a-Glance (SAMPLE)

Unit 7: Use Equivalent Fractions to Add/Subtract	**Unit 8: Extend Understanding of Multiplication and Divison to Fractions**	**Unit 9: Represent and Interpret Data (Line Plots)**	**Flex Time**
(20 days)	(17 days)	(3 days)	(5 Days)
5.NF.1	5.NF.3	5.MD.2	
5.NF.2	5.NF.4		
	5.NF.5		
	5.NF.6		
	5.NF.7		
(Add, subtract with unlike denominators, use visual-fraction models & equations, evaluate reasonableness)	(Interpret fraction by denominator, whole numbers, mixed numbers, equivalent fractions, area with fractional sides, scaling)	(Create line plot using 1/2, 1/4, 1/8, solve problems using info on line plot)	

Key:	
	Major Standards
	Supporting Standards
	Additional Standards

5th Grade Mathematics Year-at-a-Glance (SAMPLE)

Unit 10: Write and Interpret Numerical Expressions; Analyze Patterns and Relationships	Unit 11: Graph Points on the Coordinate Plane and Analyze Patterns	Unit 12: Comprehensive Review
(10 days)	(10 Days)	(25 Days)
5.OA.1	5.G.1	
5.OA.2	5.G.2	
5.OA.3		
(Parenthesis, brackets/braces, write simple expressions)	(Coordinate system, 2 numerical patterns with 2 rules, graph ordered pairs from 2 patterns)	

Key:	
	Major Standards
	Supporting Standards
	Additional Standards

TEACHER: ____________________ SCHOOL DISTRICT/BUILDING: ____________________

COURSE: MATH GRADE LEVEL(S): 5th Grade

LAST UNIT	CURRENT UNIT	NEXT UNIT
	CURRENT UNIT **1: Understand the Place Value System**	NEXT UNIT 2:Operations with Whole Numbers and Decimals

UNIT SCHEDULE (15 Days)

	-Explore place value
	-Place value is a multiple of ten
	-Recognize patterns of zero when multiplying and dividing
	-Use whole number exponents
	-Read and write decimals to thousandths
	-Compare decimals to thousandths
	-Round decimals

is about… UNIT MAP

Understanding the Place Value System

- *Understand place values is based on multiples of ten*
- *Use place value understanding to round decimals*
- *Use powers of ten in multiplication of whole numbers and decimals*
- *Compare 2 decimals to the thousandths*
- *Read and write decimals to the thousandths*

UNIT SELF TEST QUESTIONS

1. Can I explain the value of each digit in a multi-digit number?
2. Can I use the powers of ten to multiply whole numbers and decimals?
3. Can I read and write decimals to the thousandths?
4. Can I round decimals to any place value?

MATH STANDARDS	
5.NBT.1	
5.NBT.2	
5.NBT.3	
5.NBT.4	

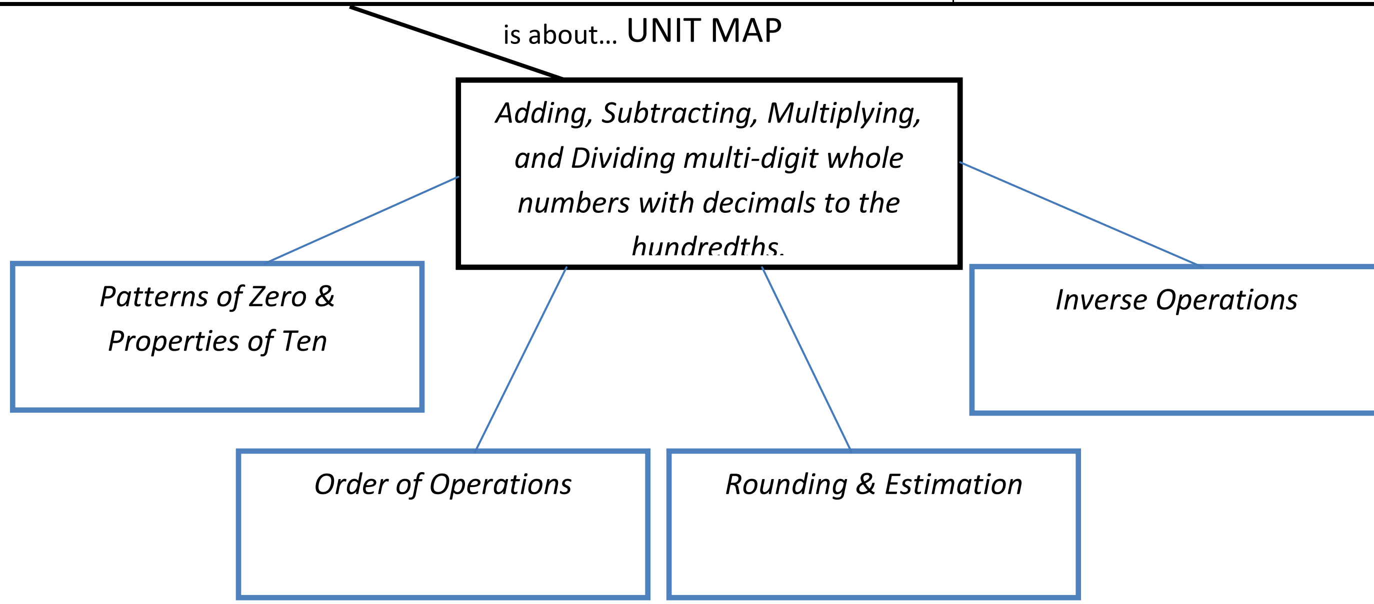
TEACHER: ______________________ SCHOOL DISTRICT/BUILDING: ______________________

COURSE: MATH GRADE LEVEL(S): 5^{th} Grade

LAST UNIT	CURRENT UNIT	NEXT UNIT
1: Understanding the place value system	**2: Operations with Whole Numbers and Decimals**	3: Convert within a given system

UNIT SCHEDULE (20 Days)

is about... UNIT MAP



UNIT SELF TEST QUESTIONS

1. Can I represent, write and state the value of numbers and decimals numbers to the thousandths?
2. Can I multiply and divide whole numbers by ten and see the results in a pattern of zeros?
3. Can I use inverse operations, strategies based on place value, order of operations, and written methods to solve problems when working with whole numbers and decimals?

MATH STANDARDS	
5.NBT.7	

TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: MATH GRADE LEVEL(S): 5th GRADE

LAST UNIT	CURRENT UNIT	NEXT UNIT
2: Operations with whole numbers and decimals	**3: Convert Within a Given Measurement System**	4:Classify 2 dimensional figures

UNIT SCHEDULE (5 Days)

is about… UNIT MAP

Converting units of measurement

- *Convert units of measurement within the metric system*
- *Convert units of measurement within the standard system*
- *Use measurement conversions to solve multi-step real-world problems*

UNIT SELF TEST QUESTIONS

1. Can I convert units of measurement within a given system?
2. Can I apply conversions to solve real-world situations?

MATH STANDARDS	
5.MD.1	

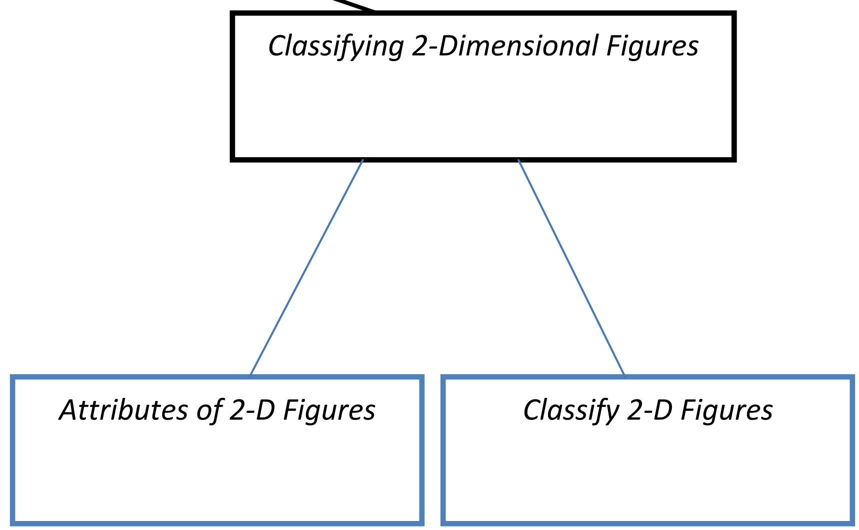
TEACHER: ______________________ SCHOOL DISTRICT/BUILDING: ______________________

COURSE: MATH GRADE LEVEL(S): 5th Grade

LAST UNIT	CURRENT UNIT	NEXT UNIT
3. Convert within a given system	**4. Classify 2-Dimensional Figures**	5. Understand concepts of volume and relate to multiplication & division

UNIT SCHEDULE (5 Days)

is about... UNIT MAP



UNIT SELF TEST QUESTIONS

1. Can I classify 2-dimensional shapes in a hierarchy based on properties?

2. Can I compare and contrast the attributes of geometric figures using graphic organizers?

3. Can I list the attributes and pose questions to determine the classification of shapes?

MATH STANDARDS	
5.G.3	
5.G.4	

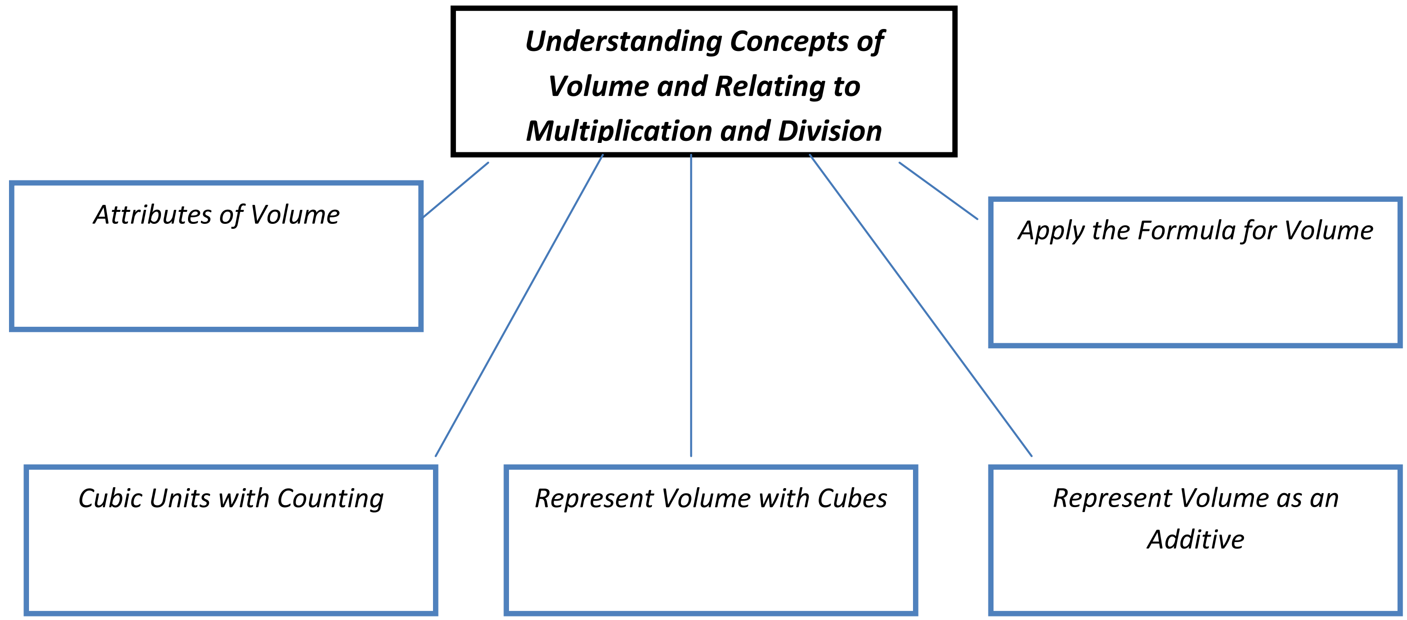
TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: ________MATH______________________________ GRADE LEVEL(S): ___5th GRADE______________________________

LAST UNIT	CURRENT UNIT	NEXT UNIT
4: Classify 2-dimensional figures	**5: Understand Concepts of Volume and Relate to Multiplication and Division**	6:Add, Subtract, Multiply and Divide Decimals

UNIT SCHEDULE (20 Days)

is about… UNIT MAP



UNIT SELF TEST QUESTIONS

1. Can I explain the attributes of volume?
2. Can I count cubic units to measure volume?
3. Can I represent volume with cubes?
4. Can I use a formula to determine volume?
5. Can I apply my understanding of volume to real-world situations?

MATH STANDARDS	
5.MD.3	
5.MD.4	
5.MD.5	

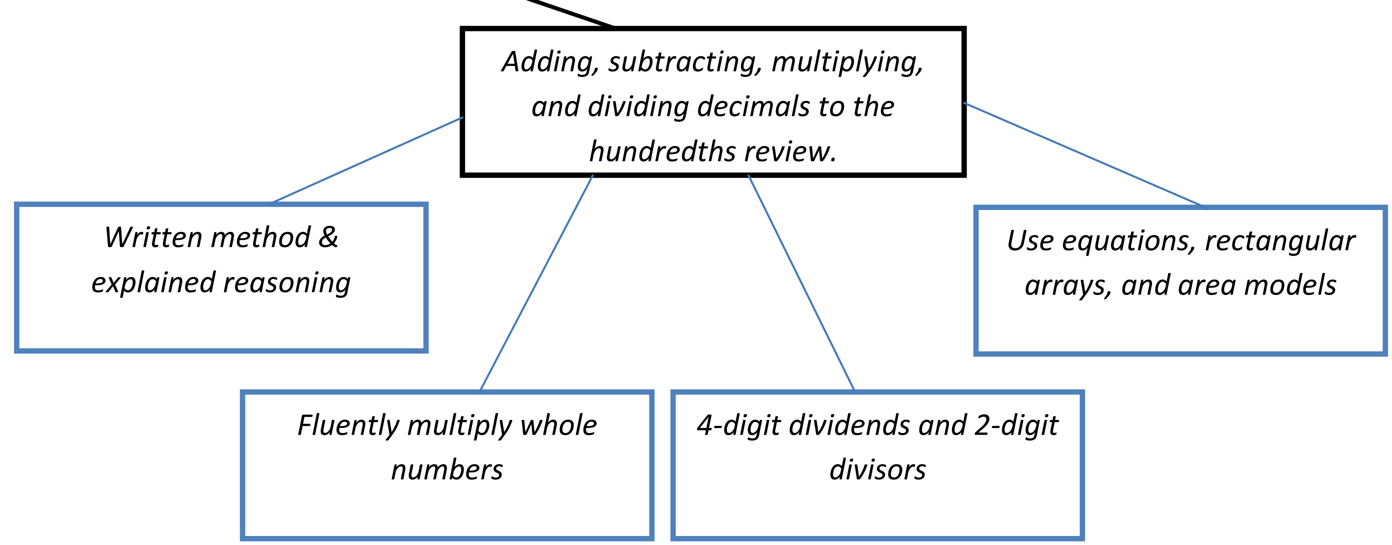
TEACHER: ____ SCHOOL DISTRICT/BUILDING: ____

COURSE: ______MATH____ GRADE LEVEL(S): ___5th Grade____

LAST UNIT	CURRENT UNIT	NEXT UNIT
5: Understand concepts of volume	**6: Add, Subtract, Multiply, & Divide Decimals to the Hundredths (Review)**	7: Use equivalent fractions to add/subtract

UNIT SCHEDULE (15 Days)

is about... UNIT MAP



UNIT SELF TEST QUESTIONS

1. Can I fluently multiply multi-digit whole numbers using standard algorithms?

2. Can I find the quotients of whole numbers with up to 4-digit dividends and 2-digit divisors using various strategies?

3. Can I use all four operations with decimals to the hundredths in a written method and explain my reasoning?

MATH STANDARDS	
5.NBT.5	
5.NBT.6	
5.NBT.7	

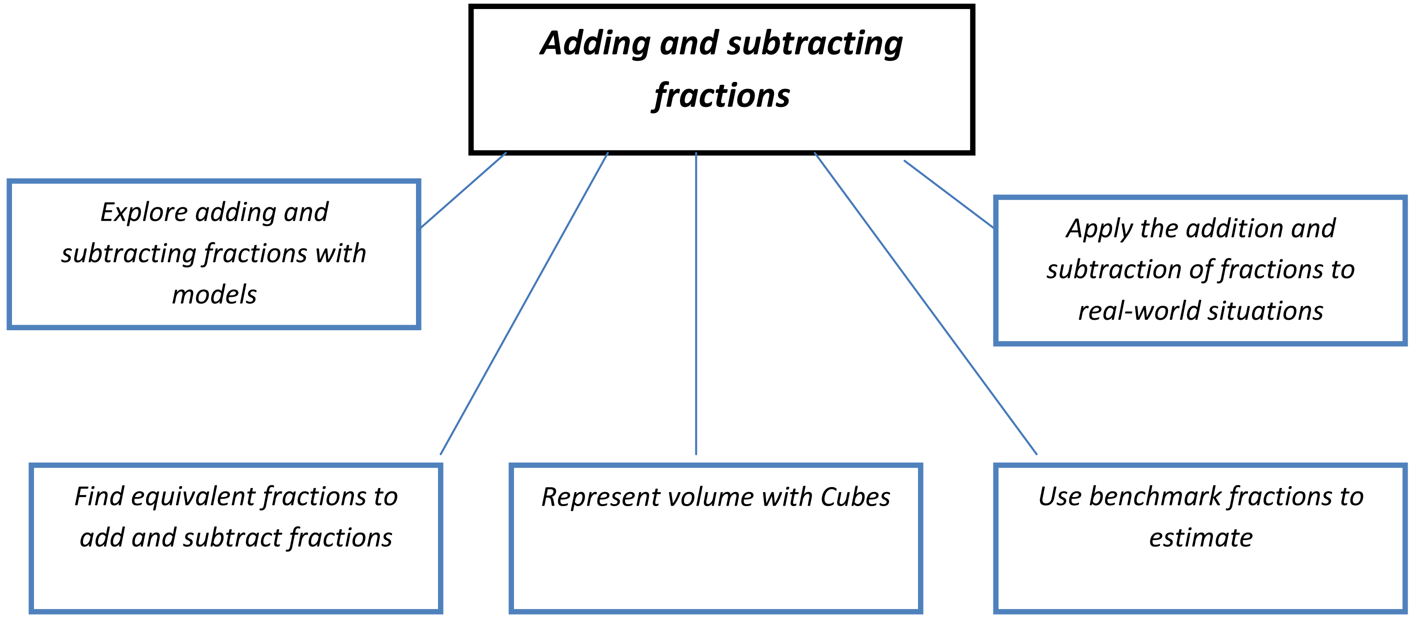
TEACHER: ____ SCHOOL DISTRICT/BUILDING: ____

COURSE: _____MATH____ GRADE LEVEL(S): ___5^{th} Grade____________________________________

LAST UNIT	CURRENT UNIT	NEXT UNIT
6:Add, Subtract, Multiply and Divide Decimals	**7:Use Equivalent Fractions to Add and Subtract**	8: Extend Understanding of Multiplication and Division to Fractions

UNIT SCHEDULE (20 Days)

is about... UNIT MAP



UNIT SELF TEST QUESTIONS

1. Can I use models to add and subtract fractions?
2. Can I find equivalent fractions?
3. Can I use benchmark fractions to estimate?
4. Can I use a formula to determine volume?
5. Can I apply addition and subtraction of fractions to real-world situations?

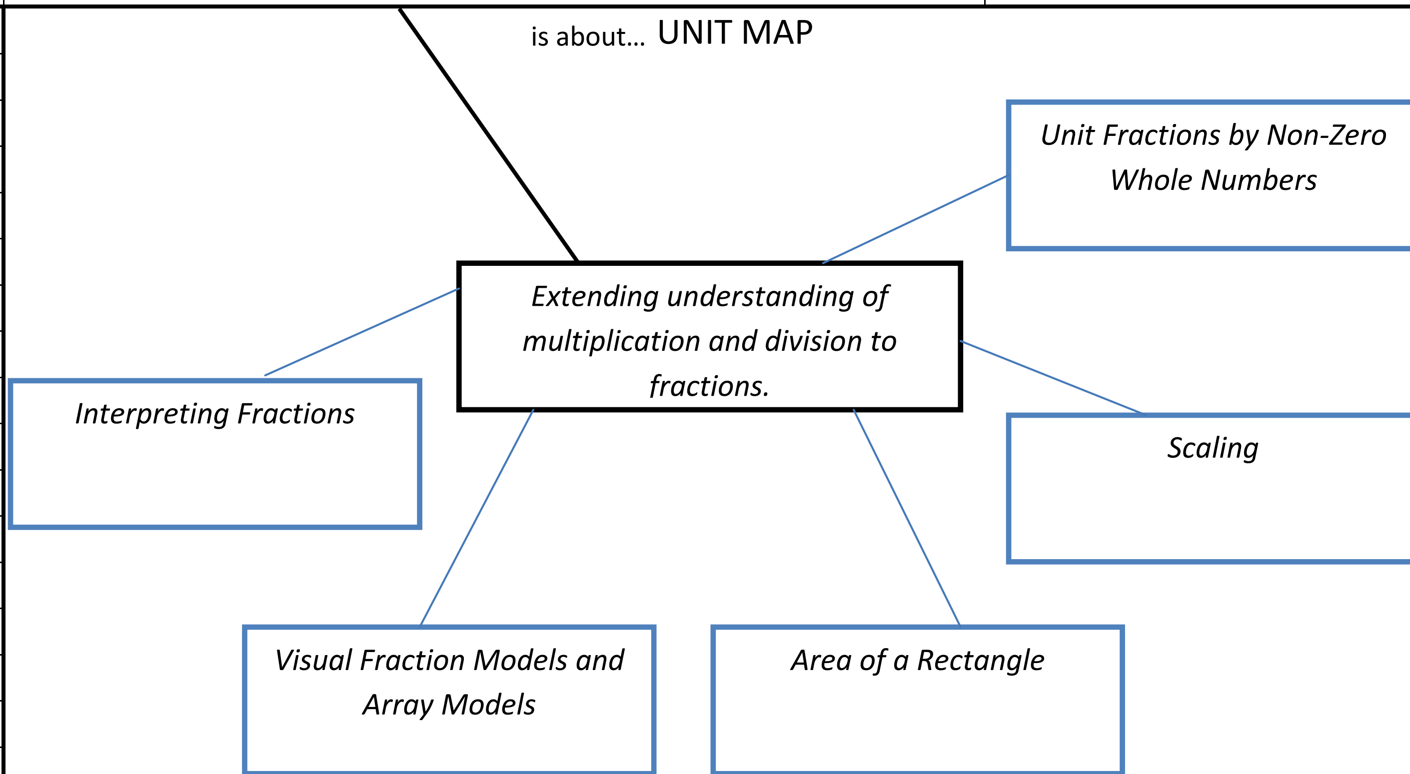
MATH STANDARDS	
5.NF.1	
5.NF.2	

TEACHER: ____ SCHOOL DISTRICT/BUILDING: ____

COURSE: ________MATH________________________________ GRADE LEVEL(S): ___5th GRADE____

LAST UNIT	CURRENT UNIT	NEXT UNIT
7: Use equivalent fractions to add/subtract	**8: Extend Understanding of Multiplication & Division to Fractions**	9: Represent and interpret data (line plots)

UNIT SCHEDULE (17 Days)



UNIT SELF TEST QUESTIONS

1. Can I interpret a fraction?
2. Can I multiply a whole number by a fraction?
3. Can I use multiplication to scale and compare sizes?
4. Can I apply multiplication and division to help solve real-world problems?

MATH STANDARDS	
5.NF.3	
5.NF.4	
5.NF.5	
5.NF.6	
5.NF.7	

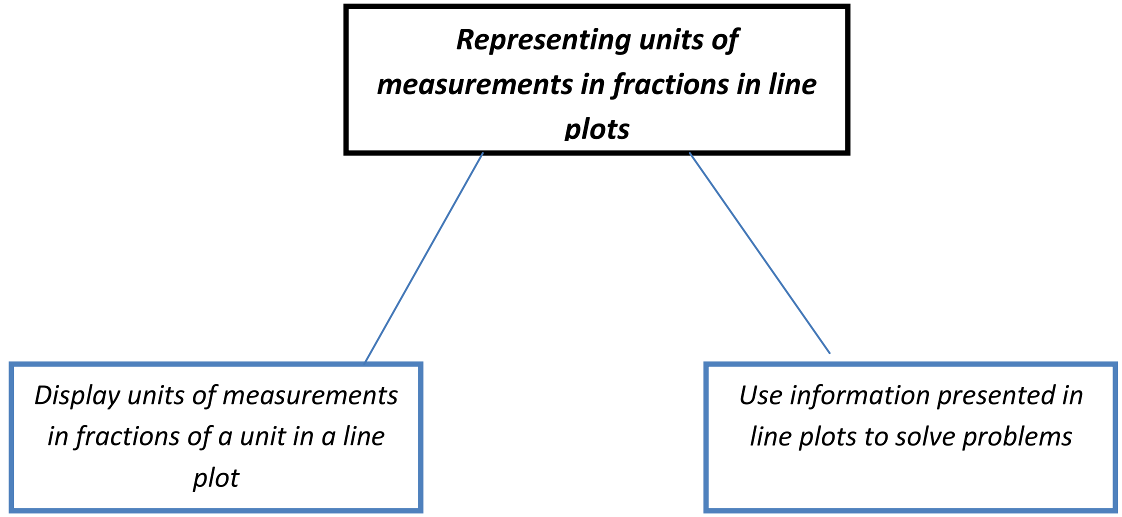
TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: ____________MATH______________________ GRADE LEVEL(S): _____5th GRADE______________________________

LAST UNIT	CURRENT UNIT	NEXT UNIT
8: Extend Understanding of Multiplication and Division to Fractions	**9:Represent and Interpret Data**	10: Write and Interpret Numerical Expressions, Analyze Patterns

UNIT SCHEDULE (3 Days)

is about…UNIT MAP



UNIT SELF TEST QUESTIONS

1. Can I make a line plot to display a data set measured in fractions of a unit?
2. Can I use information presented in a line plot to solve a real-world problem?

MATH STANDARDS	
5.MD.2	

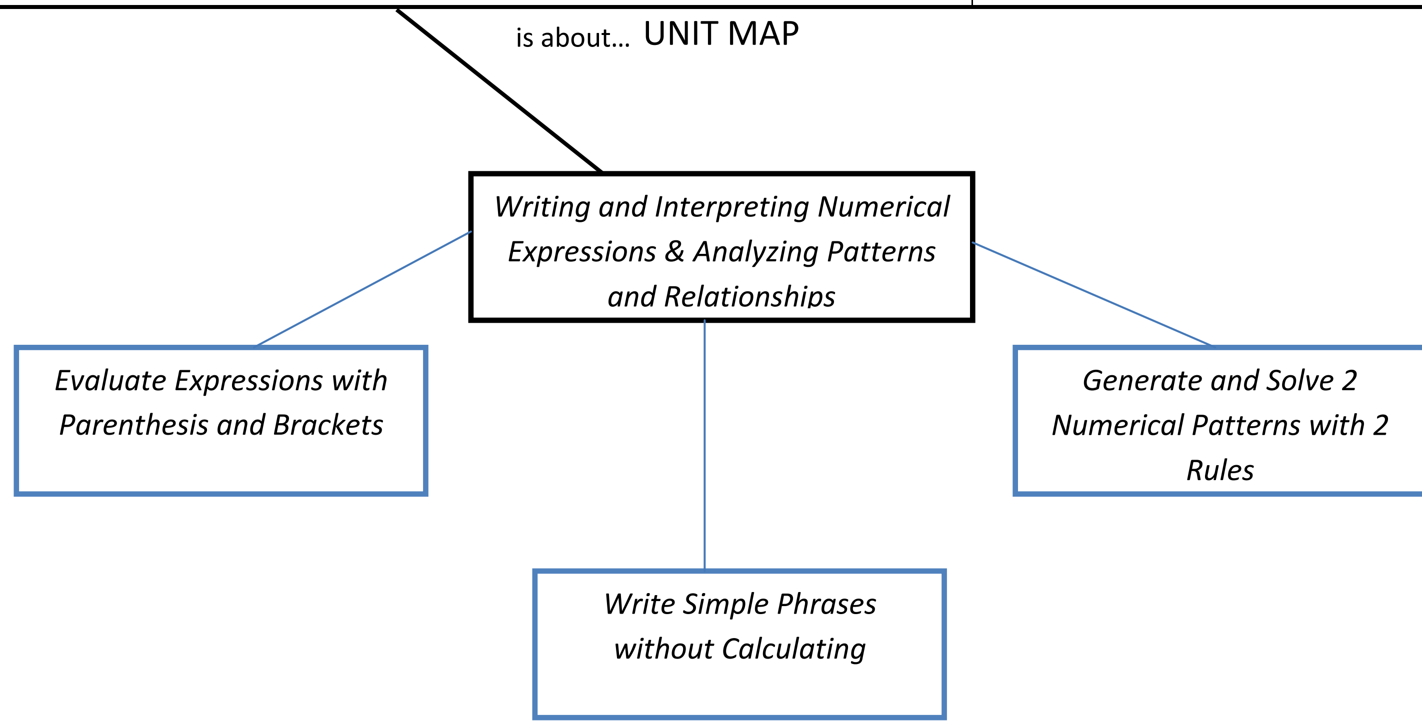
TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: ___________MATH______________________ GRADE LEVEL(S): ____5th GRADE______________________

LAST UNIT	CURRENT UNIT	NEXT UNIT
9: Represent and interpret data (line plots)	**10: Write and Interpret Numerical Expressions & Analyze Patterns and Relationships**	11: Graph points on the coordinate plane and analyze patterns

UNIT SCHEDULE (10 Days)

is about… UNIT MAP



UNIT SELF TEST QUESTIONS

1. Can I evaluate expressions using various symbols?
2. Can I evaluate expressions using the order of operations?
3. Can I generate a pattern that can be represented in multiple ways?

MATH STANDARDS	
5.OA.1	
5.OA.2	
5.OA.3	

TEACHER: ______________________________________ SCHOOL DISTRICT/BUILDING: ______________________________________

COURSE: __________MATH______________________________ GRADE LEVEL(S): ______5th GRADE____

LAST UNIT	CURRENT UNIT	NEXT UNIT
10: Write and Interpret Numerical Expressions, Analyze Patterns	**11: Graph points on the Coordinate Plane and Analyze Patterns**	

UNIT SCHEDULE (10 Days)

is about…UNIT MAP

Graphing points on the coordinate plane and analyzing patterns

Define the parts of a coordinate system

Graph points in the first quadrant of a coordinate plane

Graph and interpret values in a coordinate grid in the a real-world context

UNIT SELF TEST QUESTIONS

1. Can I identify the parts of a coordinate grid?
2. Can I graph points on a coordinate plane?
3. Can I represent real-world problems by graphing and interpreting coordinate values?

MATH STANDARDS	
5.G.1	
5.G.2	

5E 5th Grade Lesson Plan - Volume

Teacher: Ravalli Curriculum Consortium

Date: June 12, 2013

Subject / grade level: Unit 5 - Volume / 5th Grade

Materials: 2 unopened cereal boxes of different brands and sizes

NC SCOS Essential Standards and Clarifying Objectives: 5.MD.3, 5.MD.4, 5.MD.5

Lesson objective(s): Students will be able to accurately calculate the volume of a cereal box and design a new box based on the amount of cereal.

Differentiation strategies to meet diverse learner needs:
Intervention - small group work, smaller cereal boxes

Extension - change to standard/metric system so product can be sold worldwide

ENGAGEMENT

- **Pass out two bags of snacks with varying quantities in each bag. (Don't show the quantity in each bag) Have students pick which bag they would prefer. After they make their selection, show the amount of content in each bag. Discuss their findings.**

EXPLORATION

1. **Measure the height, width, and depth of each box to the nearest centimeter. Record data in a table.**
2. **Calculate the volume of each box. Add this data to your table.**
3. **Open each cereal box. Mark a line on the outside of the box to show the height of the cereal inside the box.**
4. **Measure the height of the cereal in each box and add this data to your table.**
5. **Calculate the volume of the cereal in each box using the height of the cereal in Step 4 and the width and depth of the box in Step 2. Add this data to your table.**

EXPLANATION

Question: *What is the measurement of the empty space in the box? What is the measurement of the used space in the box? How do these measurements compare? What would be a reason for having empty space in the box? How does this apply to production, marketing, and sales of food? Explain your reasoning using complete sentences.*

ELABORATION

- **Design a cereal box that will hold the volume of cereal in one original box with minimal empty space. Write a letter to convince the cereal company why your design is better and more marketable than the original. (Connection to opinion writing standard W.5.1)**
- **Creatively present your project!**

EVALUATION

- **Students precisely measure volume and are able to compare the difference between volumes.**
- **Students are able to accurately articulate their reasoning and process throughout the investigation.**

Students will be presented with a rubric to guide their investigation of volume.

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

5th GRADE SAMPLE PERFORMANCE TASK

GRADE 5 MATH: STUFFED WITH PIZZA

UNIT OVERVIEW

In this unit students will develop and expand the concept of rational numbers by using several interpretations and different types of physical models.

TASK DETAILS

Task Name: Stuffed with Pizza

Grade: 5

Subject: Mathematics

Depth of Knowledge: 2

Task Description: Students use fractional parts of a whole, addition and subtraction of fractions, and comparison, to determine if two boys eat the same amount or a different amount of pizza pieces.

Standards:
5.NF 1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)

5.NF 2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.6 Attend to precision.

http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm

Name___________________________________

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

(5)

Title of Task: Stuffed, with Pizza

Content Strand(s) Addressed: Number operations fractions

State Standard(s) Addressed:

Program Link: Everyday Mathematics, Unit 5

Common Core standard(s):

Underlying Mathematical Concepts

- fractional parts of a whole
- fraction notation
- fraction addition
- comparison of fractions with unlike denominators
- improper/proper fractions

Problem Solving Strategies/Representation

- model (manipulatives/fraction bars)
- diagram/key – area model/circle graph
- chart

Mathematical Language

model, whole, area model, equivalent, circle graph, rectangular, diagram, per, key, greater/less than, more/less than, >, <, fraction $\frac{1}{8}, \frac{2}{8}$ ..., percents %, decimals ., numerator, denominator

Possible Solution(s)

Answer: Luis is correct

pizzas

Key: T Tito, L Luis

Tito $\frac{2}{8} + \frac{3}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$

Luis $\frac{5}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$

Connections

- $\frac{7}{8}$ cheese is eaten
- $\frac{3}{8}$ pepperoni eaten
- 1 whole mushroom eaten
- It appears that Luis likes cheese the most and pepperoni the least.
- It appears that Tito likes mushroom more than cheese
- $\frac{1}{2}$ pizza is 50% or .5
- $\frac{1}{4}$ pizza is 25% or .25
- $\frac{1}{2} = \frac{4}{8}$
- There is $\frac{6}{8}$ slices left but not from 1 pizza

Related Tasks

See Resource Binder

- 100% mushroom eaten
- Relate to a similar problem and state math
- If use rectangular pizzas the amount per boy is the same
- $\frac{5}{8}$ pepperoni left which is greater than 50%, $\frac{1}{2}$

5

Math Grade 5 - Stuffed With Pizza
Common Core Learning Standards/
Universal Design for Learning

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

REPRESENTATION: *The "what" of learning.* How does the task present information and content in different ways? How students gather facts and categorize what they see, hear, and read. How are they identifying letters, words, or an author's style?

In this task, teachers can...

- ✓ **Provide multiple entry points to a lesson and optional pathways through content (e.g., exploring big ideas through dramatic works, arts and literature, film and media)** through the exploration of the understanding of basic fractions, equivalent fractions, and addition of fractions.

ACTION/EXPRESSION: *The "how" of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

In this task, teachers can...

- ✓ **Provide graphic organizers and templates for data collection and organizing information** in order to provide a tool to manage figures and calculations.

ENGAGEMENT: *The "why" of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

In this task, teachers can...

- ✓ **Prompt or require learners to explicitly formulate or restate goal** by having students work in pairs to summarize and define the steps to solving the problem.

Visit http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm to learn more information about UDL.

CCSS Mathematics Content Standards Rubric – Grade 5

Students apply mathematical concepts, reasoning, and procedural skills in problems-solving situations and support their solutions using computations, mathematical language, and appropriate representations/ modeling.

CCSS Math Criteria by Strand	Novice	Apprentice	Practitioner	Expert
Number & Operations in Base Ten & Number & Operations - Fractions	Consistently flawed understanding of: • equivalent fractions and mixed numbers • decimal values • place value Fractional or decimal representations incorrect or not appropriate for task Incorrect computational strategies used or major inaccuracies in computation lead to an incorrect solution A correct answer may be stated, but is not supported by student work or explanations	Some parts of problem correct and those parts supported by student work (e.g., uses visual models to represent fractional or decimal parts of a whole) Mostly consistent and accurate understanding of: • equivalent fractions and mixed numbers • representation of fractional notation • rounding whole numbers and decimals using place value Uses additive reasoning to solve or interpret most problems May include limited Explanations for solutions Displays some inaccuracies in computation (e.g., multiplying multi-digit whole numbers using the standard algorithm)	***Uses clear and consistent fractional and decimal representations (e.g., using visual models- number line, area, sets; symbols, expanded form) when reading, writing, and comparing quantities*** 5.NF-3, 5 5.NBT-3 ***Uses addition and subtraction to solve problems with fractions with unlike denominators*** 5.NF-1, 2 ***May be some minor flaws when performing multi-step computations involving addition, subtraction, multiplication, or division (with fractions, mixed numbers, whole numbers, or decimals), <u>but procedural and conceptual understanding is clearly evident</u>*** 5.NBT-6, 7 5.NF-1, 2, 6, 7	All parts of problem correct, precise, and supported by student work or explanations Demonstrates deeper understanding of fractions, whole numbers, and decimals by relating them to percents or other abstract concepts beyond the scope of a specific task (e.g., verifying the solution or approach using alternative models or equations; making and supporting reasonable estimates in multi-step problems) Uses a variety of strategies and four operations to solve problems with whole numbers, decimals, fractions (including mixed numbers) Consistently applies multiplicative reasoning when appropriate
Operations & Algebraic Thinking	Identifies numeric patterns (e.g., multiplying or diving by powers of 10) or describes differences in numeric patterns (repeating or growing)	Writes a numerical expression for a given situation using one or more of the four operations and some symbols (parentheses, brackets, or braces) Extends a numeric pattern when given a rule (e.g., creates ordered pairs or completes input-output table) Graphs ordered pairs on a coordinate grid	***Writes, interprets, and evaluates numerical expressions*** 5.OA-1, 2 ***Analyzes patterns and numeric relationships using tables, graphs, words, or symbolic rules (e.g., generalizes a pattern to create a rule; forms ordered pairs; explains place value patterns)*** 5.OA-3; 5.NB-1, 2	Generates multiple representations when solving problems (e.g., to show reasonableness of the answer or an alternative approach or solution) Explains or describes functions as linear or nonlinear when represented in graphical or tabular representations

NOTE: Anchor papers will illustrate how descriptors for each performance level are evidenced at each grade.

Exemplars® Standards-Based Math Rubric*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice	No strategy is chosen, or a strategy is chosen that will not lead to a solution. Little or no evidence of engagement in the task present.	Arguments are made with no mathematical basis. No correct reasoning nor justification for reasoning is present.	No awareness of audience or purpose is communicated. or Little or no communication of an approach is evident or Everyday, familiar language is used to communicate ideas.	No connections are made.	No attempt is made to construct mathematical representations.
Apprentice	A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen. Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.	Arguments are made with some mathematical basis. Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.	Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task. or Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols. or Some formal math language is used, and examples are provided to communicate ideas.	Some attempt to relate the task to other subjects or to own interests and experiences is made.	An attempt is made to construct mathematical representations to record and communicate problem solving.

*Based on revised NCTM standards.

Exemplars® Standards-Based Math Rubric (cont.)*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Practitioner	A correct strategy is chosen based on mathematical situation in the task. Planning or monitoring of strategy is evident. Evidence of solidifying prior knowledge and applying it to the problem solving situation is present. Note: The practitioner must achieve a correct answer.	Arguments are constructed with adequate mathematical basis. A systematic approach and/or justification of correct reasoning is present. This may lead to... • clarification of the task. • exploration of mathematical phenomenon. • noting patterns, structures and regularities.	A sense of audience or purpose is communicated. and/or Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response. Formal math language is used throughout the solution to share and clarify ideas.	Mathematical connections or observations are recognized.	Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.
Expert Work at this level is exceeding grade-level expectations	An efficient strategy is chosen and progress towards a solution is evaluated. Adjustments in strategy, if necessary, are made along the way, and / or alternative strategies are considered. Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present. Note: The expert must achieve a correct answer.	Deductive arguments are used to justify decisions and may result in formal proofs. Evidence is used to justify and support decisions made and conclusions reached. This may lead to... • testing and accepting or rejecting of a hypothesis or conjecture. • explanation of phenomenon. • generalizing and extending the solution to other cases.	A sense of audience and purpose is communicated. and/or Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties. Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.	Mathematical connections or observations are used to extend the solution.	Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.

*Based on revised NCTM standards.

SIXTH GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grade 6 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
6.MP.1. Make sense of problems and persevere in solving them.	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
6.MP.2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
6.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
6.MP.4. Model with mathematics.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
6.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6.MP.6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.
6.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; c=6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
6.MP.8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Montana Mathematics Grade 6 Content Standards

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 6 Overview

Ratios and Proportional Relationships
- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations
- Apply and extend previous understandings of arithmetic to algebraic expressions.• Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry
- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability
- Develop understanding of statistical variability.
- Summarize and describe distributions.

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
3. Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours?*
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
7. Understand ordering and absolute value of rational numbers.
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^oC > –7 ^oC to express the fact that –3 ^oC is warmer than –7 ^oC.*
   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
8. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6 s^2$ *to find the volume and surface area of a cube with sides of length s = 1/2.*
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem from a variety of cultural contexts, including those of Montana American Indians, that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians. For example, use Montana American Indian designs to decompose shapes and find the area.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians.

Statistics and Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
2. Understand that a set of data collected (including Montana American Indian demographic data) to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
    a. Reporting the number of observations.
    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

GRADE 6

Domain	Cluster	Code	Common Core State Standard
Ratios and Proportional Relationships	Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
		6.RP.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Note: Expectations for unit rates in this grade are limited to non-complex fractions.)
		6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, , e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours? c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
The Number System	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.). How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?
	Compute fluently with multi-digit numbers and find common factors and multiples.	6.NS.2	Fluently divide multi-digit numbers using the standard algorithm.
		6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
		6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).
	Apply and extend previous understandings of numbers to the system of rational numbers.	6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
		6.NS.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
		6.NS.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3° C > –7° C to express the fact that –3° C is warmer than –7° C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than $30.
		6.NS.8	Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

GRADE 6

Domain	Cluster	Code	Common Core State Standard
Expressions and Equations	Apply and extend previous understandings of arithmetic to algebraic expressions.	6.EE.1	Write and evaluate numerical expressions involving whole-number exponents.
		6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems
		6.EE.3	Apply the properties of operations as strategies to generate equivalent expressions. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply properties of operations to y + y + y to produce the equivalent expression 3y.
		6.EE.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.
	Reason about and solve one-variable equations and inequalities.	6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
		6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
		6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.
		6.EE.8	Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
	Represent and analyze quantitative relationships between dependent and independent variables.	6.EE.9	Use variables to represent two quantities in a real-world problem from a variety of cultural contexts, including those of Montana American Indians, that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

GRADE 6

Domain	Cluster	Code	Common Core State Standard
Geometry	Solve real-world and mathematical problems involving area, surface area, and volume.	6.G.1	Find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians. For example, use Montana American Indian designs to decompose shapes and find the area.
		6.G.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
		6.G.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
		6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians.
Statistics and Probability	Develop understanding of statistical variability.	6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.
		6.SP.2	Understand that a set of data collected including Montana American Indian demographic data) to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
		6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values using a single number, while a measure of variation describes how its values vary using a single number.
	Summarize and describe distributions.	6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
		6.SP.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Getting Every Montana Student College and Career Ready

Montana Common Core
Standards and Assessments

Montana Curriculum Organizer

Grade 6

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

Montana Curriculum Organizer: Grade 6 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

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HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process, is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade-level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 5 through 26 consist of tables organized into learning progressions that can function as units. The table for each learning progression, unit, includes 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

<table>
<tr><th>LEARNING PROGRESSION</th><th colspan="2">STANDARDS IN LEARNING PROGRESSION</th></tr>
<tr><td>Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.</td><td colspan="2">Standards covered in this learning progression.</td></tr>
<tr><th colspan="3">UNDERSTAND:</th></tr>
<tr><td colspan="3">What students need to understand by the end of this learning progression.</td></tr>
<tr><th>KNOW:</th><th colspan="2">DO:</th></tr>
<tr><td>What students need to know by the end of this learning progression.</td><td colspan="2">What students need to be able to do by the end of this learning progression, organized by cluster and standard.</td></tr>
<tr><th colspan="3">KEY TERMS FOR THIS PROGRESSION:</th></tr>
<tr><td colspan="3">Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.</td></tr>
<tr><th colspan="3">INSTRUCTIONAL STRATEGIES AND RESOURCES:</th></tr>
<tr><td colspan="3">Cluster: Title
Strategies for this cluster

Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><td colspan="3">Cluster: Title
Strategies for this cluster

Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><th colspan="3">CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:</th></tr>
<tr><td colspan="3">Standards that connect to this learning progression are listed here, organized by cluster.</td></tr>
<tr><th colspan="3">STANDARDS FOR MATHEMATICAL PRACTICE:</th></tr>
<tr><td colspan="3">A quick reference guide to the 8 standards for mathematical practice is listed here.</td></tr>
</table>

Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and

Montana Curriculum Organizer: Grade 6 Mathematics

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statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Grade 6 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
6.MP.1. Make sense of problems and persevere in solving them.	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
6.MP.2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
6.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
6.MP.4. Model with mathematics.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
6.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6.MP.6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.
6.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e., $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e., $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume.
6.MP.8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

CRITICAL AREAS FOR GRADE 6 MATH

In Grade 6, instructional time should focus on four critical areas:
(1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;
(2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers;
(3) writing, interpreting, and using expressions and equations; and
(4) developing understanding of statistical thinking.

Montana Curriculum Organizer: Grade 6 Mathematics

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Rates & Ratios	6.RP.1, 6.RP.2, 6.RP.3a-d,

UNDERSTAND:

Reasoning about multiplication and division is critical to the understanding of ratio concepts and their application to solving problems.
A rate is a set of infinitely many equivalent ratios.

KNOW:

A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.

Ratios and rates are connected to multiplication and division.

Various ways for representing ratios (e.g., in words, with a colon, in fraction notation).

Ratios are connected to fractions:
- Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
- Ratios are often used to make "part-part" comparisons, but fractions are not.
- Ratios and fractions can be thought of as overlapping sets.
- Ratios can be meaningfully re-interpreted as fractions.

(Source: *Essential Understanding of Ratios, Proportions, & Proportional Reasoning gr 6-8* from NCTM 2010.)

When representing a ratio, quantities must correspond to the context of the situation.

Idea of unit rate is comparing quantity per one. A variety of representations (ratio table, graph, double-number lines, scale drawings, etc.) model proportionality.

Flexibility between representations of ratios and proportions (listed above) empowers problem solving.

Scaling up/down a ratio proportionally maintains equivalency.

Percent of a quantity is per 100.

Flexibility between representations of numbers (fractions, decimals, percent) empowers problem solving.

Equivalent measurements within measurement systems can be used to solve problems.

DO:

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double-number-line diagrams, or equations).

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

KEY TERMS FOR THIS PROGRESSION:

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

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Unit rate, Ratio, Tables of equivalent ratios, Tape diagrams (double-number lines), Equations, Percent (rate per 100)

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Understand ratio concepts and use ratio reasoning to solve problems.

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative thinking. Examples with ratio and proportion must involve measurements, prices and geometric contexts, as well as rates of miles per hour or portions per person within contexts that are relevant to sixth-graders. Experience with proportional and non-proportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. Although algorithms provide efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Students develop the understanding that ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percent's are examples of part-to-whole ratios. Fractions are written as the part being identified compared to the whole amount. A percent is the part identified compared to the whole (100). Provide students with multiple examples of ratios, fractions and percent's of this type. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28.

Percents are often taught in relationship to learning fractions and decimals. This cluster indicates that percent's are to be taught as a special type of rate. Provide students with opportunities to find percent's in the same ways they would solve rates and proportions.

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16. This form of ratios is often used to compare the event that can happen to the event that cannot happen.

Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost $2.48 at Store A and 6 cans of the same pudding costs $4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem:

- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking ½ of $4.50.

Using ratio tables develops the concept of proportion. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

Store A	
3 cans	6 cans
$2.48	#4.96

Store B	
6 cans	3 cans
$4.50	$2.25

Students should also solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with models such as ratio tables, T-charts or double-number-line diagrams will help students internalize the size relationships between same system measurements and relate the process of converting to the solution of a ratio.

Multiplicative reasoning is used when finding the missing element in a proportion. For example, use 2 cups of syrup to 5 cups of water to make fruit punch. If 6 cups of syrup are used to make punch, how many cups of water are needed?

$2/5 = 6x$ Recognize that the relationship between 2 and 6 is 3 times; $2 \cdot 3 = 6$
To find x, the relationship between 5 and x must also be 3 times. $3 \cdot 5 = x$, therefore, $x = 15$
$2/5 = 6/15$ The final proportion.

Other ways to illustrate ratios that will help students see the relationships follow. Begin written representation of ratios with the words "out of" or "to" before using the symbolic notation of the colon and then the fraction bar. For example, 3 out of 7, 3 to 5, 6:7 and then 4/5.

Use skip counting as a technique to determine if ratios are equal.

Labeling units helps students organize the quantities when writing proportions.

$$3\ eggs/2\ cups\ of\ flour = z\ eggs/8\ cups\ of\ flour$$

Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used. For example, have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

Instructional Resources/Tools
100 grids (10 x 10) for modeling percents
Ratio tables – to use for proportional reasoning
Bar Models – for example, 4 red bars to 6 blue bars as a visual representation of a ratio and then expand the number of bars to show other equivalent ratios

Public Broadcasting Service. 1995-2012.
Something Fishy: Students will estimate the size of a large population by applying the concepts of ratio and proportion through the capture/recapture statistical procedure.
How Many Noses Are in Your Arm?: Students will apply the concept of ratio and proportion to determine the length of the Statue of Liberty's torch-bearing arm.

Schwartz, David M. *If You Hopped Like a Frog*. Scholastic Press, 1999.
This book introduces the concepts of ratio.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
6.NS.6a-c, 6.NS.7a-d

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
The Number System – Operations w/ Rational Numbers (excluding integers)	6.NS.1, 6.NS.2, 6.NS.3, 6.NS.4

UNDERSTAND:

Multiple strategies and models can be utilized to solve a variety of problems involving fractions and decimals.
Relative magnitude of a solution (quotient) depends on the size of the divisor.

KNOW:

Computation with rational numbers is an extension of computation with whole numbers, but introduces some new ideas and processes.

- Strategies utilized for multiplying and dividing whole numbers (and whole numbers by fractions) extend to fractions/decimals (e.g., The same reasoning used to solve 6 ÷ 2 (*How many 2's are in 6?*) can be used to solve problems involving fractions such as 8 ÷ ½ (*How many one-half's are in 8?*) and ¾ ÷ ¼ (*How many one-fourths are in ¾?*))
- When dividing by a fraction, the solution may be larger than what you started with (the original value).
- Multiplying by a number greater than 1 results in a product greater than the given number (e.g., $3^1/_2$ x 5 will result in a number more than $3^1/_2$).
- Multiplying by a number smaller than 1 results in a product less than the given number (e.g., $3^1/_2$ x $^1/_4$ will result in a number less than $3^1/_2$)

Number line, ratio table, area model, arrays, bar model, fraction circles, picture/visual.

Estimation as a means for predicting and assessing the reasonableness of a solution.

Computational fluency is built upon understandings of models and decomposing and recomposing numbers.

Fluency with mental math and estimation facilitates efficient problem solving.

Flexibility with the equivalent forms of the distributive property (expanded form and factored form) allows for efficient problem solving.

DO:

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions (e.g., by using visual-fraction models and equations to represent the problem). *For example, create a story context for (2/3) ÷ (3/4) and use a visual-fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

KEY TERMS FOR THIS PROGRESSION:

Multiplication, Division, Fractions, Strategies, Estimation, Factors, Multiples, LCM, GCF, Distributive property, Algorithm

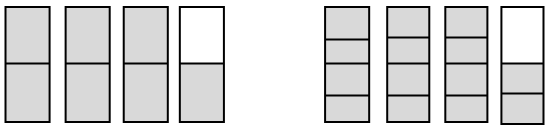
INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought.

For example, 12 ÷ 3 means; How many groups of three would make 12? Or how many in each of 3 groups would make 12? Thus 7/2 ÷ 1/4 can be solved the same way. How many groups of 1/4 make 7/2? Or, how many objects in a group when 7/2 fills one fourth?

Creating the picture that represents this problem makes seeing and proving the solutions easier.



Set the problem in context and represent the problem with a concrete or pictorial model. 5/4 ÷ 1/2. 5/4 cups of nuts fills 1/2 of a container. How many cups of nuts will fill the entire container?

Teaching "invert and multiply" without developing an understanding of why it works leads to confusion as to when to apply the shortcut.

Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed.

Instructional Resources/Tools
Anneberg Foundation, 2012. *Models for Multiplying and Dividing Fractions:* This teacher resource shows how the area model can be used in multiplication and division of fractions. There is also a section on the relationship to decimals.

- Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Fractions - Rectangle Multiplication:* Use this virtual manipulative to graphically demonstrate, explore, and practice multiplying fractions.

Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.
As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of standard algorithms. Fluency with an algorithm denotes an ability that is efficient, accurate, appropriate and flexible. Division was introduced in Grade 3 conceptually, as the inverse of multiplication. In Grade 4, division continues using place-value strategies, properties of operations, the relationship with multiplication, area models, and rectangular arrays to solve problems with one digit divisors. In Grade 6, fluency with the algorithms for division and all operations with decimals is developed.

Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper pencil algorithms, mental math or a computing tool is also a necessary skill and should be provided in problem solving situations.

Greatest common factor and *least common multiple* are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as prime and composite numbers in Grade 4. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples. For example, consider answering the question: "If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?" Being able to see and write the relationships between numbers will be beneficial as further algebraic understandings are developed. Another focus is to be able to see how the GCF is useful in expressing the numbers using the distributive property, (36 + 24) = 12(3+2), where 12 is the GCF of 36 and 24. This concept will be extended in Expressions and Equations as work progresses from understanding the number system and solving equations to simplifying and solving algebraic equations in Grade 7.

Instructional Resources/Tools
Math.com. 2000-2005. *Greatest Common Factor Lesson:* This lesson is a resource for teachers or for students after participating in lessons exploring GCF.

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CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
None

STANDARDS FOR MATHEMATICAL PRACTICE:	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Expanding Rational Numbers	6.NS.5, 6.NS.6a-c, 6.NS.7a-d, 6.NS.8

UNDERSTAND:

Rational numbers are a subset of the number system including and beyond whole numbers.
Although every whole number corresponds to a single numeral, every rational number can be written in many different ways.

KNOW:

Rational numbers are a set of numbers that includes the whole numbers, integers, as well as numbers that can be written as quotient of two integers, $a \div b$, where $b \neq 0$.

Much of the space between adjacent integers on a number line is taken by rational numbers that are not integers.

Between any two rational numbers, there are infinitely many rational numbers.

Rational numbers can be represented as fractions, decimals, and percents in infinitely many equivalent forms.

Rational numbers have multiple interpretations, and making sense of them depends on identifying the unit.
- The concept of *unit* is fundamental to the interpretation of rational numbers.
- Rational numbers can be interpreted as:
  - A part-whole relationship;
  - As a measure;
  - As a quotient;
  - As a ratio; and
  - As an operator.

(Source: *Essential Understanding of Rational Numbers* by NCTM 2010.)

Estimation and mental math are more complex with rational numbers than with whole numbers.

DO:

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge), use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself (e.g., -(-3) = 3), and that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by **reflections** across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3^oC > -7^oC$ to express the fact that -3^oC is warmer than -7^oC.*

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.*

d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*

6.NS.8 Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

KEY TERMS FOR THIS PROGRESSION:

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Positive, Negative, Location on a number line, Reflection, Rational number, Absolute value, Coordinate plane, Inequality

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.

The purpose of this cluster is to begin study of the existence of negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. Starting with examples of having/owing and above/below zero sets the stage for understanding that there is a mathematical way to describe opposites. Students should already be familiar with the counting numbers (positive whole numbers and zero), as well as with fractions and decimals (also positive). They are now ready to understand that all numbers have an opposite. These special numbers can be shown on vertical or horizontal number lines, which then can be used to solve simple problems. Demonstration of understanding of positives and negatives involves translating among words, numbers and models: given the words "7 degrees below zero," showing it on a thermometer and writing -7; given -4 on a number line, writing a real-life example and mathematically -4. Number lines also give the opportunity to model absolute value as the distance from zero.

Simple comparisons can be made and order determined. Order can also be established and written mathematically: -3°C > -5°C or -5°C < -3°C. Finally, absolute values should be used to relate contextual problems to their meanings and solutions.

Using number lines to model negative numbers, prove the distance between opposites, and understand the meaning of absolute value easily transfers to the creation and usage of four-quadrant coordinate grids. Points can now be plotted in all four quadrants of a coordinate grid. Differences between numbers can be found by counting the distance between numbers on the grid. Actual computation with negatives and positives is handled in Grade 7.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

6.G.3, 6.RP.3a-d

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Algebraic Reasoning – Expressions	6.EE.1, 6.EE.2a-c, 6.EE.3, 6.EE.4

UNDERSTAND:

Mathematical expressions can be used to represent and solve real-world and mathematical problems.
Flexibility in manipulating expressions to suit a particular purpose (rewriting an expression to represent a quantity in a different way to make it more compact or to feature different information) helps with solving problems efficiently.

KNOW:	DO:
Variables can be used to represent numbers whose exact values are not yet specified. Expressions can be manipulated to generate equivalent expressions to simplify the problem. Properties of Operations and Order of Operations are used to simplify, evaluate, or find equivalent expressions. The equals sign as a symbol of equivalence.	***Apply and extend previous understandings of arithmetic to algebraic expressions.*** **6.EE.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.* **6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **6.EE.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

KEY TERMS FOR THIS PROGRESSION:

Coefficient, Term, Expressions, Equivalence, Equations, Exponents, Variable, Order of operations, Commutative property, Associative property, Distributive property

INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Apply and extend previous understandings of arithmetic to algebraic expressions. The skills of reading, writing and evaluating expressions are essential for future work with expressions and equations, and are a Critical Area of Focus for Grade 6. In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Students should now focus on what terms are to be solved first rather than invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar (3/5). Less confusion will occur as students write algebraic expressions and equations if *x* represents only variables and not multiplication. The use of a dot (●) or parentheses between number terms is preferred.

Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression *x* – 10 could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees," "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4 \cdot 2$, students could write $2x + 2x + 8$ or some other equivalent expression. Make the connection to the simplest form of this expression as $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses. Include whole-number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise, start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in Grade 6 The Number System; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

Instructional Resources/Tools
BBC. 2013. ***Late Delivery*:** In this game, the student helps the mail carrier deliver five letters to houses with numbers such as $3(a + 2)$.

Utah State University. National Library of Virtual Manipulatives. 1999-2010. ***Algebra tiles*** -can be used to represent expressions and equations.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

6.G.1, 6.G.2, 6.EE.5–9

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 6 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Algebraic Reasoning – Equations & Inequalities	6.EE.5, 6.EE.6, 6.EE.7, 6.EE.8

UNDERSTAND:

Mathematical expressions, equations, and inequalities are used to represent and solve real-world and mathematical problems.

KNOW:	DO:
Variables can be used to represent numbers whose exact values are not yet specified. Inverse operations are used to solve equations and inequalities. Solutions to an equation/inequality are the values of the variables that make the equation/inequality true. There are some inequalities that have infinitely many solutions (those in the form of $x > c$ or $x < c$). Solutions to an inequality are represented symbolically or using a number line. The equals sign as a symbol of equivalence.	***Reason about and solve one-variable equations and inequalities.*** **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all non-negative rational numbers. **6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

KEY TERMS FOR THIS PROGRESSION:

1. **Equation, Inequality, Inverse operations, Substitution, Variables, Expressions, Number line**

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Reason about and solve one-variable equations and inequalities.

The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation $x + 21 = 32$ students know that $21 + 9 = 30$ therefore the solution must be 2 more than 9 or 11, so $x = 11$.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations/inequalities for a situation. This process should go both ways; students should be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story does not come naturally for many students. A strategy for assisting with this is to give students an equation and ask them to come up with the situation/story that the equation could be referencing.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

6.EE.1–4, 6.EE.9, 6.G.1, 6.G.2

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STANDARDS FOR MATHEMATICAL PRACTICE:	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Algebraic Reasoning – Quantitative Relationships	6.EE.9

UNDERSTAND:

Quantitative relationships between dependent and independent variables can be represented in multiple ways including algebraic (equation), graphical, verbal (scenario), and tabular.

KNOW:	DO:
Quantities that change in relationship to one another can be represented using variables. There is a relationship between independent and dependent variables. Different representations of the relationship provide varied opportunities to analyze changes in quantities (e.g., as in linear relationships). Various representations of quantitative relationships including: scenario (context) table, graph, and equation.	***Represent and analyze quantitative relationships between dependent and independent variables.*** **6.EE.9** Use variables to represent two quantities in a real-world problem from a variety of cultural contexts, including those of Montana American Indians, that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

KEY TERMS FOR THIS PROGRESSION:

Equation, Variable, Quantity, Independent variable, Dependent variable, Table, Graph, Equation, Axes, *X*-axis, *Y*-axis, Scale, Coordinate pairs, Relationship

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Represent and analyze quantitative relationships between dependent and independent variables.
The goal is to help students connect the pieces together. This can be done by having students use multiple representations for the mathematical relationship. Students need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs and equations. They also need to be able to start with any of the representations and develop the others.

Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components. For example, how far I travel is dependent on the time and rate that I am traveling.

Throughout the expressions and equations domain in Grade 6, students need to have an understanding of how the expressions or equations relate to situations presented, as well as the process of solving them.

The use of technology, including computer apps, CBL's, and other hand-held technology allows the collection of real-time data or the use of actual data to create tables and charts. It is valuable for students to realize that although real-world data often is not linear, a line sometimes can model the data.

Instructional Resources/Tools
Use graphic organizers as tools for connecting various representations.

National Council of Teachers of Mathematics. 2000-2012. *Pedal Power*: A lesson on translating a graph to a story.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

6.RP.3a-b, 6.EE.3–8

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometry – Area, Surface Area, Volume	6.G.1, 6.G.2, 6.G.3, 6.G.4

UNDERSTAND:

By decomposing two- and three-dimensional shapes into smaller component shapes, the concept of surface area and the formulas for area and volume can be developed and justified.

KNOW:

Areas of polygons can be found by decomposing them into triangles and/or other shapes, rearranging or removing shapes, and relating the shapes to rectangles. (Formulas for the areas of triangles and parallelograms can be developed and justified with these methods.)

Linear, square, and cubic measurements are described with different units (connect the units to what is being measured).

Area is 'covering', and is measured in square units.

Surface area is 'covering' all faces of a 3-D shape with square units.

Volume is 'filling' and is measured in cubic units.

The volume of a rectangular prism can be decomposed into multiple layers.

Volume can be thought of as multiple layers (*h*) of the number of cubes needed for the base (*B*). (V = *B* • *h*)

DO:

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians. For example, use Montana American Indian designs to decompose shapes and find the area.

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians.

KEY TERMS FOR THIS PROGRESSION:

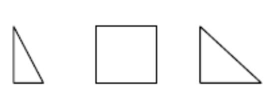
Area, Surface area, Quadrilaterals, Composing, Decomposing, Volume, Edges, Unit cubes, Polygons, Vertices, Nets, Base of rectangular prisms, 2-dimensional, 3-dimensional, Square units, Cubic units, Rectangular prisms

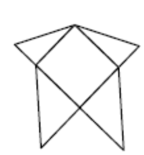
INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.

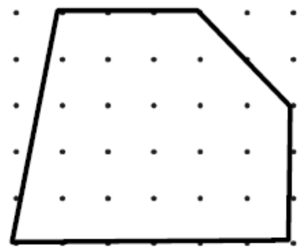
It is very important for students to continue to physically manipulate materials and make connections to the symbolic and more abstract aspects of geometry. Exploring possible nets should be done by taking apart (unfolding) three-dimensional objects. This process is also foundational for the study of surface area of prisms. Building upon the understanding that a net is the two-dimensional representation of the object, students can apply the concept of area to find surface area. The surface area of a prism is the sum of the areas for each face.

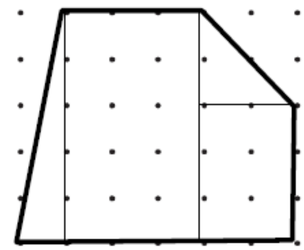
Multiple strategies can be used to aid in the skill of determining the area of simple two-dimensional composite shapes. A beginning strategy should be to use rectangles and triangles, building upon shapes for which they can already determine area to create composite shapes. This process will reinforce the concept that composite shapes are created by joining together other shapes, and that the total area of the two-dimensional composite shape is the sum of the areas of all the parts.





A follow-up strategy is to place a composite shape on grid or dot paper. This aids in the decomposition of a shape into its foundational parts. Once the composite shape is decomposed, the area of each part can be determined and the sum of the area of each part is the total area.





Fill prisms with cubes of different edge lengths (including fractional lengths) to explore the relationship between the length of the repeated measure and the number of units needed. An essential understanding to this strategy is the volume of a rectangular prism does not change when the units used to measure the volume changes. Since focus in Grade 6 is to use fractional lengths to measure, if the same object is measured using one centimeter cubes and then measured using half-centimeter cubes, the volume will appear to be eight times greater with the smaller unit. However, students need to understand that the value or the number of cubes is greater but the volume is the same.

Instructional Resources/Tools
Cubes of fractional edge length
Squares that can be joined together used to develop possible nets for a cube.
Use floor plans as a real world situation for finding the area of composite shapes.

National Council of Teachers of Mathematics. 2000-2012.
Area Formulas: Finding the Area of Trapeziods: Lessons on area.
Online dot paper

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

6.NS.8, 6.EE.6, 6.EE.7

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Statistics – Describing Data (Measures of Center & Variability)	6.SP.1, 6.SP.2, 6.SP.3, 6.SP.4, 6.SP.5a-d

UNDERSTAND:

Descriptive statistics (mean, median, mode, range, inter-quartile range) and the various graphical representations allow you to summarize and compare data sets.

KNOW:	DO:
Numerical data can be summarized with one or more numbers. Descriptive statistics (mean, median, mode, range, inter-quartile range) allow you to summarize and compare data sets. A single measure of center will not thoroughly describe a data set because very different data sets can share the same measure of center. Different measures of center may be selected to describe data for different purposes or contexts. Different representations of data may be appropriate based on the context or purpose for displaying the data.	***Develop understanding of statistical variability.*** **6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.2** Understand that a set of data collected including Montana American Indian demographic data) to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. ***Summarize and describe distributions.*** **6.SP.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **6.SP.5** Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

KEY TERMS FOR THIS PROGRESSION:

Distribution, Measures of center, Mean, Median, Mode, Range, Spread, Inter-quartile range, Shape, Statistics, Variability, Number line, Dot plots, Histograms, Box plots, Mean absolute deviation, Outliers

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Develop understanding of statistical variability.

Grade 6 is the introduction to the formal study of statistics for students. Students need multiple opportunities to look at data to determine and word statistical questions. Data should be analyzed from many sources, such as organized lists, box-plots, bar graphs and stem-and-leaf plots. This will help students begin to understand that responses to a statistical question will vary, and that this variability is described in terms of spread and overall shape. At the same time, students should begin to relate their informal knowledge of mean, mode and median to understand that data can also be described by single numbers. The single value for each of the measures of center (mean, median or mode) and measures of spread (range, interquartile range, mean absolute deviation) is used to summarize the data. Given measures of center for a set of data, students should use the value to describe the data in words. The important purpose of the number is not the value itself, but the interpretation it provides for the variation of the data. Interpreting different measures of center for the same data develops the understanding of how each measure sheds a different light on the data. The use of a similarity and difference matrix to compare mean, median, mode and range may facilitate understanding the distinctions of purpose between and among the measures of center and spread.

Include activities that require students to match graphs and explanations, or measures of center and explanations prior to interpreting graphs based upon the computation measures of center or spread. The determination of the measures of center and the process for developing graphical representation is the focus of the cluster "Summarize and describe distributions" in the Statistics and Probability domain for Grade 6. Classroom instruction should integrate the two clusters.

Instructional Resources/Tools
Newspaper and magazine graphs for analysis of the spread, shape and variation of data

National Council of Teachers of Mathematics, Illuminations: *Numerical and Categorical Data:*
In this unit of three lessons, students formulate and refine questions, and collect, display and analyze data.

Data Analysis and Probability Virtual Manipulatives Grades 6-8: Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without requiring students to spend time hand-drawing the display. Classroom time can then be spent discussing the patterns and variability of the data.

Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, American Statistics Association

Cluster:- Summarize and describe distributions.
This cluster builds on the understandings developed in the Grade 6 cluster "Develop understanding of statistical variability." Students have analyzed data displayed in various ways to see how data can be described in terms of variability. Additionally, in Grades 3-5 students have created scaled picture and bar graphs, as well as line plots. Now students learn to organize data in appropriate representations such as box plots (box-and-whisker plots), dot plots, and stem-and-leaf plots. Students need to display the same data using different representations. By comparing the different graphs of the same data, students develop understanding of the benefits of each type of representation.

Further interpretation of the variability comes from the range and center-of-measure numbers. Prior to learning the computation procedures for finding mean and median, students will benefit from concrete experiences.

To find the median visually and kinesthetically, students should reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch. This number is the median.

The concept of mean (concept of fair shares) can be demonstrated visually and kinesthetically by using stacks of linking cubes. The blocks are redistributed among the towers so that all towers have the same number of blocks. Students should not only determine the range and centers of measure, but also use these numbers to describe the variation of the data collected from the statistical question asked. The data should be described in terms of its shape, center, spread (range) and interquartile range or mean absolute deviation (the absolute value of each data point from the mean of the data set). Providing activities that require students to sketch a representation based upon given measures of center and spread and a context will help create connections between the measures and real-life situations.

Continue to have students connect contextual situations to data to describe the data set in words prior to computation. Therefore, determining the measures of spread and measures of center mathematically need to follow the development of the conceptual understanding. Students should experience data which reveals both different and identical values for each of the measures. Students need opportunities to explore how changing a part of the data may change the measures of center and measure of spread. Also, by discussing their findings, students will solidify understanding of the meanings of the measures of center and measures of variability, what each of the measures do and do not tell about a set of data, all leading to a better understanding of their usage.

Using graphing calculators to explore box plots (box-and-whisker plots) removes the time intensity from their creation and permits more time to be spent on the meaning. It is important to use the interquartile range in box plots when describing the variation of the data. The mean absolute deviation describes the distance each point is from the mean of that data set. Patterns in the graphical displays should be observed, as should any outliers in the data set. Students should identify the attributes of the data and know the appropriate use of the attributes when describing the data. Pairing contextual situations with data and its box-and-whisker plot is essential.

Instructional Resources/Tools

Graphing calculators may also be used for creating lists and displaying the data.
Guidelines for Assessment and Instruction in Statistics Education (GAISE) report. American Statistical Association

Ohio Resource Center

Hollywood Box Office: This rich problem focuses on measures of center and graphical displays.
Stella's Stumpers Basketball Team Weight: This problem situation uses the mean to determine a missing data element.
Learning Conductor Lessons: Use the interactive applets in these standards-based lessons to improve understanding of mathematical concepts. Scroll down to the **statistics** section for your specific need.

Public Broadcasting Service. 1995-2013.*Wet Heads:* In this lesson, students create stem-and-leaf plots and back-to-back stem-and-leaf plots to display data collected from an investigative activity.

National Council of Teachers of Mathematics. 2000-2010. *Height of Students in our Class:* This lesson has students creating box-and-whisker plots with an extension of finding measures of center and creating a stem-and-leaf plot.

National Library of Virtual Manipulatives. Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without the tediousness of the student hand drawing the display.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

6.NS

STANDARDS FOR MATHEMATICAL PRACTICE:

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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

SAMPLE YEAR LONG PLAN

COURSE: SIXTH GRADE MATH

Unit	Unit 1 Rates and Ratios	Unit 2 The Number System-Operations w/Rational Numbers (excluding integers)	Unit 3 Expanding Rational Number	Unit 4 Algebraic Reasoning-Expressions
(Time)	(35 days)	(25 days)	(25 days)	(15 days)
STANDARDS	6.RP.1 6.Rp.2 6.Rp.3a-d	6.NS.1 6.NS.2 6.NS.3 6.NS.4	6.NS.5 6.NS.6a-c 6.Ns.7 a-d 6.NS.8	6.EE.1 6.EE.2a-c 6.EE.3 6.EE.4
Connections To Other Domains:	6.NS. 6a-c 6.NS.7a-d		6.G.3 6.RP.3a-d	6.G.1 6.G.2 6.EE.5-9

SIXTH GRADE MATH p. 2

Unit 5 Algebraic Reasoning- Equations and Inequalities	Unit 6 Algebraic Reasoning- Quantitative Relationships	Unit 7 Geometry- Area, SurfaceArea, and Volume	Unit 8 Statistics _ Describing Data (Measures of Center and Variability)
(15 days)	(15 days)	(25 days)	(25 days)
6.EE.5	6.EE.9	6.G.1	6.SP.1
6.EE.6		6.G.2	6.SP.2
6.EE.7		6.G.3	6.SP.3
6.EE.8		6.G.4	6. SP.4
			6.SP.5a-d
6.EE.1-4	6.RP.3a-b	6.NS.8	6.NS
6.EE.9	6.EE.3-8	6.EE.6	
6.G.1		6.EE.7	
6.G.2			

TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

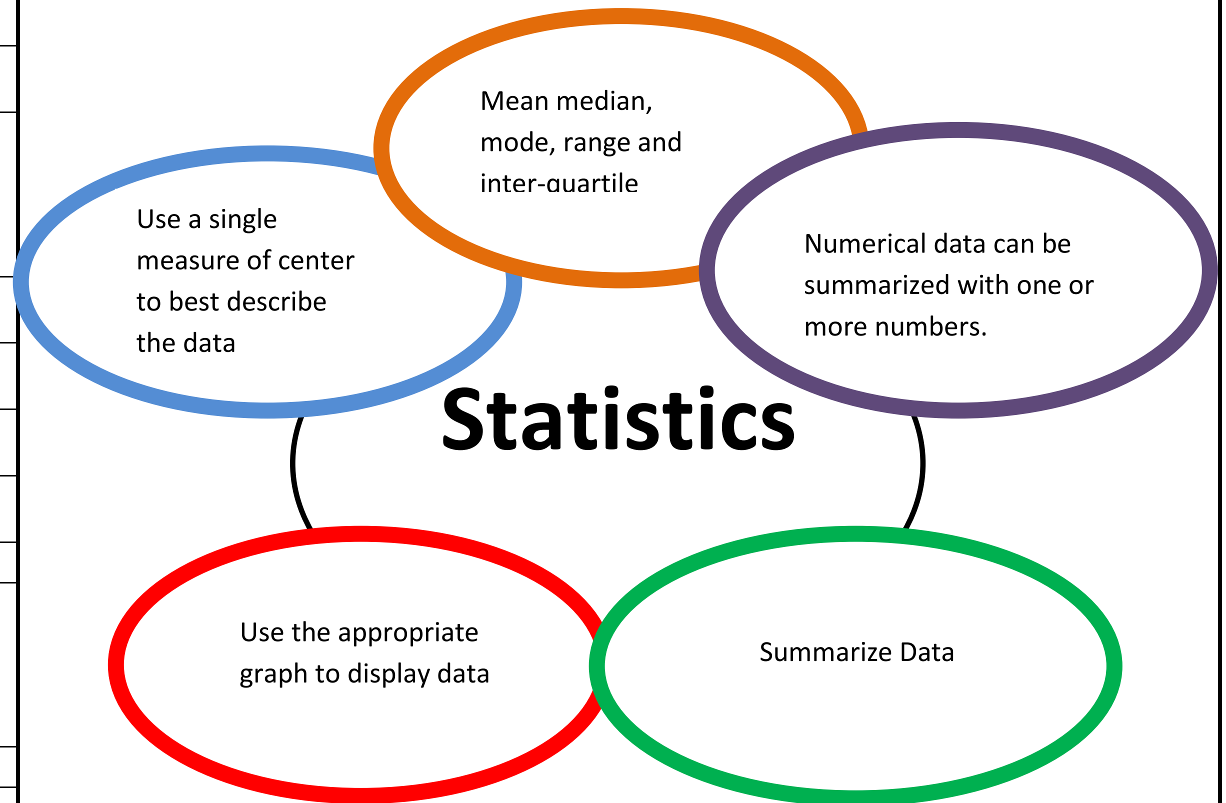
COURSE: ___MATH______________________ GRADE LEVEL(S): _6th______________________

LAST UNIT	**CURRENT UNIT**	NEXT UNIT

UNIT SCHEDULE

Day	
1	6.SP.1 -Develop Statistical questions and variability
2	-Examples of statistical questions and variability
3	6.SP.2 6.SP.4 -Collect/Organize Data (Frequency Tables)
4	-Histogram (interval of data)
5	-Bar Graph (individual pieces of data)
6	-Line Graph (show change in data over time)
7	-Line Plots/Box Plots (Display Data)
8	-Circle Graph
9-11	6.SP.3 Calculate and Measure - Mean - Median - Mode
12	-Range of Data and Inter-Quartile
13	Outliers of Data/Mean absolute Deviation
14-16	6.SP.5 Summarize Set of Data in a Report Include: Measure of center, Range, Outliers, and best measure of center to describe data.
17	REVIEW! -Statistics-Describing Data (Measures of Center and Variability)
18	Assessment

UNIT MAP ON STATISTICS



UNIT SELF TEST QUESTIONS	1.) What are the measures of center that can numerically summarize data? 2.) What are statistics used to measure the spread of the data? 3.) How do you determine the best descriptor of the data using the centers of measures? 4.) How can you determine which graph is used to display data?	**MATH STANDARDS**	
			Statistics Describing Data (Measures of Center and Variability)
			6.SP.1
			6.SP.2
			6.SP.3
			6.SP.4
			6.SP.5 a-d

6th GRADE LESSON PLAN SAMPLE

Teacher:

Subject: Statistics

Lesson Topic: Median, and Mode

Grade and Learner Profile:

- 6th Grade
- 29 Students

Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attended to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Materials: - Tape Measure

Time: 5-10 min

Engage: The students will pair up and measure each others height. Once they have their measurements their will recorded the recommended data on the front board. The students will not include their names. The students must record the height to the closest inch.

- **Mean Median and Mode**

http://youtu.be/k3aKKasOmIw

Time: 5-10 min

Explore: How do you find the median of a set of data?

How will you find the median height in the class?

(Without using the collected data points)

How do you find the mode of a set of data?

How will you find the mode height in the class?

(Without using the collected data points)

Find the mean of the actual set of data

The students must record their results in finding the median and the mode of the classroom students.

Have a group discussion on how to find the median of a set of data and how to find the mode of a set of data.

Now with the correct data points the students must calculate the median and find mode.

Time: 10-15 min

Explain: The students will get into their groups of two and brainstorm how they came up with finding the median and mode. Once they have agreed on a decision they will explain their reasoning to the class.

Use a foldable or graphic organizer to define the meaning of median and mode in your own words. Students must share their definitions with the whole group.

Time: 10-15 min

Elaborate: Show a YouTube video on how to find mean, median and mode.

- **It's not hard (Averages song)**

  http://youtu.be/QH2obAPwfqk

Now the students will be given sets of data to calculate the mean, median and mode.

Time: 5-10 min

Evaluate: Check for understanding

- Use white boarded to check class as a whole
- Provided evidence students understand new skills (Formative Assessment)

Time: 5 min

Extension: Mean, Median and Mode are important statistics of the class, however which one of the three can be used to best describe the class as a whole? Your answer must be justified.

SAMPLE 6th GRADE CONSTRUCTED RESPONSE

Name:________________________

Constructive Response

Standard: 6. SP. 3

The data below shows David's test scores in history class.

65, 85, 85, 85, 90, 90, 95, 95, 100, 100

David's teacher allows him to use the mean, median, or mode of his test scores to represent his final grade. Which of the following gives David the highest final grade?

Part A: Calculate the mean, median and mode of David's Test Scores

Part B: Which one of the central measures would David use for his final test score and why?

Part C: David's teacher would like to receive a clear understanding of David's overall test scores. What central measure would David's teacher use?

Depth of Knowledge Rubri

DOK Level	
Level 4	Student has clearly stated their understanding towards mean, median and mode. Within Parts B, and C the student has taken the time to fully explain and extend his/her understanding of the questions being asked.
Level 3	Student has calculated their understanding towards mean, median and mode. Within Parts B, and C the student has thought deeply towards the questions given.
Level 2	Student has calculated the mean, median and mode. Within Parts B, and C the students had constructed understanding towards the questions.
Level 1	Student has remembered how to calculate the mean, median and mode. Within Parts B, and C the student has not transferred the knowledge of mean, median and mode to answer each question professionally.

Performance Task

Choose one of the following performance tasks. The analysis of the performance task must be backed by the statistical information.

1.) Pick a sports team and analyze their win loss percentage over 20 years.
2.) Find the cost of personal computers over the last 20 years and analyze the change of price.
3.) Choose a stock and analyze the price fluctuation over the past 2o years.
4.) Analyze the population of an animal breed over the past 20 years and justify why or why not is should be on the endanger species list.

Performance Task Rubric

DOK Level	
Level 4	Student used a highly advanced level of thinking skills to synthesize and well processed conclusion.
Level 3	Student clarified supporting evidence to process a conclusion toward a performance task.
Level 2	Student used evidence to rectal and obtain information
Level 1	Student has remembered how to calculate the mean, median and mode. Within Parts B, and C the student has not transferred the knowledge of mean, median and mode to answer each question professionally.

SEVENTH GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grade 7 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
7.MP.1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
7.MP.2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
7.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?". They explain their thinking to others and respond to others' thinking.
7.MP.4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
7.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
7.MP.6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
7.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
7.MP.8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Explanations and Examples Grade 7
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Grade 7 Content Standards

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 7 Overview

Ratios and Proportional Relationships
- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations
- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry
- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability
- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*
2. Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. A contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gatherings, community celebrations, etc.*
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems within cultural contexts, including those of Montana American Indians (e.g., percent of increase and decrease of tribal land). *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.
   c. Apply properties of operations as strategies to multiply and divide rational numbers.
   d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
3. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers.[1]

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Expressions and Equations 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
4. Use variables to represent quantities in a real-world or mathematical problem, including those represented in Montana American Indian cultural contexts, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry 7.G

Draw construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems from a variety of cultural contexts, including those of Montana American Indians; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data, including Montana American Indian demographic data, from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data, predict how many text messages your classmates receive in a day. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. For example, when playing Montana American Indian Hand/Stick games, you can predict the approximate number of accurate guesses.*
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
   a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
   b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
   c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions

GRADE 7

Domain	Cluster	Code	Common Core State Standard
Ratios and Proportional Relationships	Analyze proportional relationships and use them to solve real-world and mathematical problems.	7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, If a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.
		7.RP.2	Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. A contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gatherings, community celebrations, etc. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
		7.RP.3	Use proportional relationships to solve multistep ratio and percent problems within cultural contexts, including those of Montana American Indians (e.g., percent of increase and decrease of tribal land). Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
The Number System	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	7.NS.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.
		7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats
		7.NS.3	Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

GRADE 7

Domain	Cluster	Code	Common Core State Standard
Expressions and Equations	Use properties of operations to generate equivalent expressions.	7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
		7.EE.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."
	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
		7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, including those represented in Montana American Indian cultural contexts, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.
Geometry	Draw, construct, and describe geometrical figures and describe the relationships between them.	7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
		7.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
		7.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	7.G.4	Know the formulas for the area and circumference of a circle and use them to solve problems from a variety of cultural contexts, including those of Montana American Indians; give an informal derivation of the relationship between the circumference and area of a circle.
		7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
		7.G.6	Solve real-world and mathematical problems a variety of cultural contexts, including those of Montana American Indians, involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

GRADE 7

Domain	Cluster	Code	Common Core State Standard
Statistics and Probability	Use random sampling to draw inferences about a population.	7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
		7.SP.2	Use data , including Montana American Indian demographics data, from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data, predict how many text messages your classmates receive in a day. Gauge how far off the estimate or prediction might be.
	Draw informal comparative inferences about two populations.	7.SP.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
		7.SP.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
	Investigate chance processes and develop, use, and evaluate probability models.	7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
		7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. . For example, when playing Montana American Indian Hand/Stick games, you can predict the approximate number of accurate guesses.
		7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
		7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Getting Ready
Every Montana Student College and Career

Montana Common Core Standards and Assessments

Montana Curriculum Organizer

Grade 7

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

November 2012 Materials adapted from Arizona, Delaware and Ohio Departments of Education

Montana Curriculum Organizer: Grade 7 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

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HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 7 through 32 consist of tables organized into learning progressions that can function as units. The table for each learning progression, unit, includes 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

<table>
<tr><th>LEARNING PROGRESSION</th><th>STANDARDS IN LEARNING PROGRESSION</th></tr>
<tr><td>Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.</td><td>Standards covered in this learning progression.</td></tr>
<tr><th colspan="2">UNDERSTAND:</th></tr>
<tr><td colspan="2">What students need to understand by the end of this learning progression.</td></tr>
<tr><th>KNOW:</th><th>DO:</th></tr>
<tr><td>What students need to know by the end of this learning progression.</td><td>What students need to be able to do by the end of this learning progression, organized by cluster and standard.</td></tr>
<tr><th colspan="2">KEY TERMS FOR THIS PROGRESSION:</th></tr>
<tr><td colspan="2">Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.</td></tr>
<tr><th colspan="2">INSTRUCTIONAL STRATEGIES AND RESOURCES:</th></tr>
<tr><td colspan="2">Cluster: Title
Strategies for this cluster
Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><td colspan="2">Cluster: Title
Strategies for this cluster
Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
<tr><th colspan="2">CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:</th></tr>
<tr><td colspan="2">Standards that connect to this learning progression are listed here, organized by cluster.</td></tr>
<tr><th colspan="2">STANDARDS FOR MATHEMATICAL PRACTICE:</th></tr>
<tr><td colspan="2">A quick reference guide to the 8 standards for mathematical practice is listed here.</td></tr>
</table>

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Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Grade 7 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
7.MP.1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
7.MP.2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
7.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?", "Does that always work?" They explain their thinking to others and respond to others' thinking.
7.MP.4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
7.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
7.MP.6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
7.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e., $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e., $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
7.MP.8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

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CRITICAL AREAS FOR GRADE 7 MATH
In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Ratios and Proportional Relationships	7.RP.1, 7.RP.2a-d, 7.RP.3

UNDERSTAND:

Extending an understanding of ratios develops a deeper understanding of proportionality builds the knowledge and skill levels needed to solve single- and multi-step problems.

<table>
<tr><th>KNOW:</th><th>DO:</th></tr>
<tr><td>
Multiple strategies for finding unit rates.

There are various strategies for deciding if a relationship is proportional (e.g., equivalent ratios in a table, observing points graphed on a coordinate plane, analyzing ratios for equivalence, etc.).

Ratio (rate) tables are used to build equivalent ratios/ rates.

Proportional relationships can be represented symbolically (equation), graphically (coordinate plane), in a table, in diagrams, and verbal descriptions.

The coordinates representing a proportional linear context can be interpreted in terms of the context.

Special attention should be spent on analyzing the points (0,0) and 1, r where r is the unit rate.

Ratio and proportional-reasoning strategies can be extended and applied to multi-step ratio and percent problems.
</td><td>
Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.
7.RP.2 Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.
a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. A contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gatherings, community celebrations, etc.
d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1,r) where r is the unit rate.
7.RP.3 Use proportional relationships to solve multi-step ratio and percent problems within cultural contexts, including those of Montana American Indians (e.g., percent of increase and decrease of tribal land). For example, simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
</td></tr>
</table>

KEY TERMS FOR THIS PROGRESSION:

Proportional Relationship, Equivalent, Coordinate plane, Origin, Unit rate, Ratio, Rate, Simple interest, Tax, Tip, Percent increase/decrease, Commission, Percent error

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.
Building from the development of rate and unit concepts in Grade 6, applications now need to focus on solving unit-rate problems with more sophisticated numbers: fractions per fractions.

Proportional relationships are further developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. This is the time to push for a deep understanding of what a representation of a proportional relationship looks like and what the characteristics are: a straight line through the origin on a graph, a "rule" that applies for all ordered pairs, an equivalent ratio or an expression that describes the situation, etc. This is not the time for students to learn to cross multiply to solve problems.

Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on “out of 100”; now percents above 100 are encountered.

Providing opportunities to solve problems based within contexts that are relevant to seventh-graders will connect meaning to rates, ratios and proportions. Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-ended problem scenarios with and without numbers to give students the opportunity to demonstrate conceptual understanding, inviting students to create a similar problem to a given problem and explain their reasoning.

Instructional Resources/Tools

Play money - act out a problem with play money
Advertisements in newspapers
Unlimited manipulatives or tools (don’t restrict the tools to one or two, give students many options)

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

7.G.1, 7.EE, 7.NS

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
The Number System – Operations w/ Rational Numbers	7.NS.1a-d, 7.NS.2a-d, 7.NS.3

UNDERSTAND:

Rational numbers are a subset of the number system including and beyond whole numbers.
Properties of whole-number operations can be applied to solving real-world and mathematical problems involving rational numbers, including integers.

KNOW:	DO:
Properties of whole-number operations can be applied when solving problems involving operations with rational numbers (Distributive Property, Commutative, Associative, and Identity Properties of Addition and Multiplication, Additive Inverse Property). Strategies to represent and solve problems involving operations with rational numbers (including decimals, fractions, integers). Strategies for converting a rational number into a decimal. The decimal form of a rational number terminates in zeros or eventually repeats. Opposites and absolute value of rational numbers. A negative number can also be interpreted as the opposite of the positive number (e.g., -5 can be interpreted as the opposite of 5). Computation with integers is an extension of computation with fractions and decimals. Strategies utilized for multiplying and dividing fractions and decimals numbers extend to integers. *For example, The same reasoning used to solve* ***6 x 2*** *(What is 6 groups of 2?) can be used to solve problems involving integers such as* ***6 x -2*** *(What is 6 groups of -2?)* The result of multiplication/division by negative numbers. **Models**: Number line, chip model, area model, arrays, bar model, fraction circles, picture/visual Estimation as a means for predicting and assessing the reasonableness of a solution. Computational fluency is built upon understandings of models and decomposing and recomposing numbers. Fluency with mental math and estimation	***Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*** **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats. **7.NS.3** Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)[1]

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

facilitates efficient problem solving. Flexibility with the equivalent forms of the distributive property (expanded form and factored form) allows for efficient problem solving. Rational numbers are a set of numbers that includes the whole numbers, integers, as well as numbers that can be written as quotient of two integers, $a \div b$, where $b \neq .0$. Between any two rational numbers, there are infinitely many rational numbers. Rational numbers can be represented as fractions, decimals, and percents in infinitely many equivalent forms. Rational numbers have multiple interpretations, and making sense of them depends on identifying the unit. • The concept of *unit* is fundamental to the interpretation of rational numbers • Rational numbers can be interpreted as: o A part-whole relationship; o As a measure; o As a quotient; o As a ratio; or o As an operator. (Source: *Essential Understanding of Rational Numbers* by NCTM 2010.) Estimation and mental math are more complex with rational numbers than with whole numbers.	

KEY TERMS FOR THIS PROGRESSION:

***Properties of Operations*: Commutative, Associative, and Identity Properties for Addition and Multiplication, Distributive Property, Additive Inverse Property**

Absolute value, Opposites, Integer, Rational number, Positive, Negative, Equivalent

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

This cluster builds upon the understandings of rational numbers in Grade 6:

- quantities can be shown using + or – as having opposite directions or values;
- points on a number line show distance and direction;
- opposite signs of numbers indicate locations on opposite sides of 0 on the number line;
- the opposite of an opposite is the number itself;
- the absolute value of a rational number is its distance from 0 on the number line;
- the absolute value is the magnitude for a positive or negative quantity; and
- locating and comparing locations on a coordinate grid by using negative and positive numbers.

Learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers.

Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Number lines present a visual image for students to explore and record addition and subtraction results. Two-color counters or colored chips can be used as a physical and kinesthetic model for adding and subtracting integers. With one color designated to represent positives and a second color for negatives, addition/subtraction can be represented by placing the appropriate numbers of chips for the addends and their signs on a board. Using the notion of opposites, the board is simplified by removing pairs of opposite colored chips. The answer is the total of the remaining chips with the sign representing the appropriate color. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. Fractional-rational numbers and whole numbers should be used in computations and explorations. Students should be able to give contextual examples of integer operations, write and solve equations for real-world problems and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers.

For example, beginning with known facts, students predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2 and 3 below).

Table 1	Table 2	Table 3
4 x 4 = 16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x - 4 =
4 x - 2 =	-4 x 2 =	-2 x - 4 =
4 x - 3 =	-4 x 3 =	-3 x - 4 =
4 x - 4 =	-4 x 4 =	-4 x - 4 =

Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers (-4 x -4 = 16, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products and commutativity of multiplication. Then students should be asked to answer these questions and prove their responses:

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?
- How is the numerical value of the product of any two numbers found?

Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language ($-(p/q) = (-p)/q = p/(-q)$) is written for the teacher's information, not as an expectation for students.)

Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals.

In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

Instructional Resources/Tools

Two-color counters

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Calculators

Utah State University. National Library of Virtual Manipulatives. 1999-2010.

Circle 3: A puzzle involving adding positive real numbers to sum to three.

Circle 21: A puzzle involving adding positive and negative integers to sum to 21.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4a-b, 7.RP.1, 7.RP.3

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Problem Solving with Expressions & Equations	7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4a-b

UNDERSTAND:

Expressions can be manipulated to suit a particular purpose and solving problems efficiently.
Mathematical expressions, equations, and inequalities are used to represent and solve real-world and mathematical problems.

KNOW:

Variables can be used to represent numbers whose exact values are not yet specified.

Expressions can be manipulated to generate equivalent expressions to simplify the problem.

Expressions can be decomposed and recomposed in different ways to generate equivalent forms.

Flexibility with the equivalent forms of an expression (expanded form, factored form, etc.) allows for efficient problem solving.

Properties of Operations and Order of Operations are used to simplify, evaluate, or find equivalent expressions.

The equals sign demonstrates equivalence (e.g.; $2x + x = 3x$ (equivalent expressions) $2x + x = 3x + 4 = 3x + 4$ (not equivalent expressions).

Rational numbers can be represented in equivalent forms to solve problems efficiently (25% can be represented as ¼ or 0.25).

Estimation as a means for predicting and assessing the reasonableness of a solution.

Fluency with mental math and estimation facilitates efficient problem solving.

Inverse operations are used to solve equations and inequalities.

Solutions to an equation/inequality are the values of the variables that make the equation/inequality true.

There are some inequalities that have infinitely many solutions (those in the form of $x > c$ or $x < c$).

Solutions to an inequality are represented symbolically or using a number line.

DO:

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example, If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, including those represented in Montana American Indian cultural contexts, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Montana Curriculum Organizer: Grade 7 Mathematics

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KEY TERMS FOR THIS PROGRESSION:
Linear, Expression, Equivalent, Coefficient, Rational number, Commutative property, Associative property, Distributive property, Identity properties, Expanded form, Factored form, Equation, Inequality

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Use properties to generate equivalent expressions.
Have students build on their understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions that were developed in Grade 6. Students continue to use properties that were initially used with whole numbers and now develop the understanding that properties hold for integers, rational and real numbers.

Provide opportunities to build upon this experience of writing expressions using variables to represent situations and use the properties of operations to generate equivalent expressions. These expressions may look different and use different numbers, but the values of the expressions are the same.

Provide opportunities for students to experience expressions for amounts of increase and decrease. In Standard 2, the expression is rewritten and the variable has a different coefficient. In context, the coefficient aids in the understanding of the situation. Another example is this situation which represents a 10% decrease: $b - 0.10b = 1.00b - 0.10b$ which equals $0.90b$ or 90% of the amount.

One method that students can use to become convinced that expressions are equivalent is by substituting a numerical value for the variable and evaluating the expression. For example, $5(3 + 2x)$ is equal to
$5•3 + 5•2x$. Let $x = 6$ and substitute 6 for x in both equations.

5(3 + 2•6)	5•3 + 5•2•6
5(3 + 12)	15 + 60
5(15)	75
75	

Provide opportunities for students to use and understand the properties of operations. These include: the commutative, associative, identity, and inverse properties of addition and of multiplication, and the zero property of multiplication. Another method students can use to become convinced that expressions are equivalent is to justify each step of simplification of an expression with an operation property.

Instructional Resources/Tools
Utah State University. National Library of Virtual Manipulatives. 1999-2010. *Algebra Tiles:* Visualize multiplying and factoring algebraic expressions using tiles.

Cluster: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
To assist students' assessment of the reasonableness of answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation. Connections between performing the inverse operation and undoing the operations are appropriate here. It is appropriate to expect students to show the steps in their work. Students should be able to explain their thinking using the correct terminology for the properties and operations.

Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also the context for students to practice using rational numbers including: integers, positive and negative fractions and decimals. As students analyze a situation, they need to identify what operation should be completed first, then the values for that computation. Each set of the needed operation and values is determined in order. Finally an equation matching the order of operations is written. *For example, Bonnie goes out to eat and buys a meal that costs $12.50 that includes a tax of $.75. She only wants to leave a tip based on the cost of the food*. In this situation, students need to realize that the tax must be subtracted from the total cost before being multiplied by the percent of tip and then added back to obtain the final cost. $C = (12.50 - .75)(1 + T) + .75 = 11.75(1 + T) + .75$ where C = cost and T = tip.

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Instructional Resources/Tools

Solving for a Variable This activity for students uses a pan balance to model solving equations for a variable.
Solving an Inequality This activity for students illustrates the solution to inequalities modeled on a number line.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

7.NS.1a-d, 7.NS.2a-d, 7.NS.3

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometry – Relationships in 2-D Geometric Figures	7.G.1, 7.G.2, 7.G.3

UNDERSTAND:

Two-dimensional geometric figures are representations of our three-dimensional world.
Experimenting with and investigating the relationships between 2-D and 3-D geometric figures connects and integrates these concepts for problem solving.

KNOW:

Scaling up/down is an application of proportional reasoning.

The relationship between dimensions of a scale drawing and the original figure is proportional.

Attributes of triangles and angles.

Depending on the attributes given, a unique triangle, more than one triangle, or no triangle can be the result.

There are certain given conditions that will produce only one, unique triangle. Some given conditions may produce more than one triangle or no triangle at all.

Slicing/ cross-sectioning 3-D shapes (including but not limited to right rectangular prisms and right rectangular pyramids) will result in 2-D shapes.

DO:

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

KEY TERMS FOR THIS PROGRESSION:

Scale drawing, Scalene, Isosceles, Equilateral, Acute, Obtuse, Right, Cross-section

INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Draw, construct, and describe geometrical figures and describe the relationships between them.

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems.

Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of time you multiple the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks (not the hexagon) provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. Questions about the relationship of the original block to the created shape should be asked and recorded. A sample of a recording sheet is shown.

Shape	Original Side Length	Created Side Length	Scale Relationship of Created to Original
Square	1 unit		
Triangle	1 unit		
Rhombus	1 unit		

This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. In other words, if the original side can be stated to represent 2.5 inches, what would be the new lengths and what would be the scale?

Shape	Original Side Length	Created Side Length	Scale
Square	2.5 inches		
Parallelogram	3.25 cms		
Trapezoid	(Actual measurements)	Length 1 Length 2	

Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

After students have explored multiple iterations with a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Starting with simple shapes and whole-number side lengths allows all students access to discover and understand the relationships. An interesting discovery is the relationship of the scale of the side lengths to the scale of the respective perimeters (same scale) and areas (scale squared). A sample recording sheet is shown:

Shape	Side Length	Scale	Original Perimeter	Scaled Perimeter	Perimeter Scale	Original Area	Scaled Area	Area Scale
Rectangle	2 x 3 in.	2	10 inches	20 inches	2	6 sq. in.	24 sq in.	4
Triangle	1.5 inches	2	4.5 inches	9 inches	2	2.25 sq. in.	9 sq in.	4

Students should move on to drawing scaled figures on grid paper with proper figure labels, scale and dimensions. Provide word problems that require finding missing side lengths, perimeters or areas. For example, if a 4 by 4.5 cm rectangle is enlarged by a scale of 3, what will be the new perimeter? What is the new area? If the scale is 6, what will the new side length look like? Suppose the area of one triangle is 16 sq units and the scale factor between this triangle and a new triangle is 2.5. What is the area of the new triangle?

Reading scales on maps and determining the actual distance (length) is an appropriate contextual situation.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles. Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.

This cluster is related to Grade 7 cluster: "Solve real-life and mathematical problems involving angle measure, area, surface area, and volume." Further construction work can be replicated with quadrilaterals, determining the angle sum, noticing the variety of polygons that can be created with the same side lengths but different angle measures, and ultimately generalizing a method for finding the angle sums for regular polygons and the measures of individual angles. For example, subdividing a polygon into triangles using a vertex $(N - 2)180°$ or subdividing a polygons into triangles using an interior point $180°N - 360°$ where N = the number of sides in the polygon. An extension would be to realize that the two equations are equal.

Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found. Challenges can also be given: "See how many different two-dimensional figures can be found by slicing a cube." or "What three-dimensional figure can produce a hexagon slice?" This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices and resulting two-dimensional figures.

Instructional Resources/Tools
Straws, clay, angle rulers, protractors, rulers, grid paper
Road Maps - convert to actual miles
Dynamic computer software - Geometer's SketchPad. This cluster lends itself to using dynamic software. Students sometimes can manipulate the software more quickly than do the work manually. However, being able to use a protractor and a straight edge are desirable skills.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

7.RP.1, 7.RP.2a

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Geometry – Problem Solving with Angles, Area, SA, Volume	7.G.4, 7.G.5, 7.G.6

UNDERSTAND:

Relationships between geometric figures are useful for building new knowledge and solving real-world and mathematical problems accurately.

KNOW:	DO:
There is a relationship between the circumference and the diameter of a circle. The ratio of the circumference to the diameter of a circle is pi (π). There is a proportional relationship between the circumference and area of a circle. (This is informal.) [The area of a circle can be found by multiplying half the circumference by the radius (A = 1/2 * C * r) or multiplying one-fourth the circumference by the diameter (A = 1/4 * C * d). Relate this to the formula for finding the area of a rectangle ($A = l * w$).] Circumference of a circle: $C = 2\pi r$ or πd Area of a circle: $A = \pi r^2$ Supplementary angles sum to 180º. Complementary angles sum to 90º. Vertical angles are created by intersecting lines and are congruent. Adjacent angles in parallelogram are supplementary (sum to 180º). Previous knowledge of area and volume to solve problems involving area, volume, and SA of additional 2-D and 3-D figures.	***Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.*** **7.G.4** Know the formulas for the area and circumference of a circle and use them to solve problems from a variety of cultural contexts, including those of Montana American Indians; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.6** Solve real-world and mathematical problems a variety of cultural contexts, including those of Montana American Indians, involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

KEY TERMS FOR THIS PROGRESSION:

Supplementary angles, Complementary angles, Vertical angles, Adjacent angles, Sum of interior angles, Circumference, Diameter, Radius, Area

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
This is the students' initial work with circles. Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, pi and area. Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius. Given multiple-size circles, students should then explore the relationship between the radius and the length measure of the circle (circumference) finding an approximation of pi and ultimately deriving a formula for circumference. Laying string or yarn over the circle and comparing to a ruler is an adequate estimate of the circumference. This same process can be followed in finding the relationship between the diameter and the area of a circle by using grid paper to estimate the area.

Another visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. In figuring area of a circle, the squaring of the radius can also be explained by showing a circle inside a square. Again, the formula is derived and then learned. After explorations, students should then solve problems, set in relevant contexts, using the formulas for area and circumference.

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh-graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Instructional Resources/Tools
Circular objects of several different sizes
String or yarn
Tape measures, rulers
Grid paper
Paper plates

National Council of Teachers of Mathematics. 2000-2012.NCTM Illuminations:

Apple Pi: Using estimation and measurement skills, students will determine the ratio of circumference to diameter and explore the meaning of π. Students will discover the circumference and area formulas based on their investigations.

Circle Tool: With this three-part online applet, students can explore with graphic and numeric displays how the circumference and area of a circle compare to its radius and diameter. Students can collect data points by dragging the radius to various lengths and clicking the "Add to Table" button to record the data in the table.

Geometry of Circles: Using a MIRA™ geometry tool, students determine the relationships between radius, diameter, circumference and area of a circle.

Square Circles: This lesson features two creative twists on the standard lesson of having students measure several circles to discover that the ratio of the circumference to the diameter seems always to be a little more than 3. This lesson starts with squares, so students can first identify a simpler constant ratio (4) of perimeter to length of a side before moving to the more difficult case of the circle. The second idea is to measure with a variety of units, so students can more readily see that the ratio of the measurements remains constant, not only across different sizes of figures, but even for the same figure with different measurements. From these measurements, students will discover the constant ratio of 1:4 for all squares and the ratio of approximately 1:3.14 for all circles.

Ohio Resource Center. 2013. *Circles and Their Areas:* Given that units of area are squares, how can we find the area of a circle or other curved region? Imagine a waffle-like grid inside a circle and a larger grid containing the circle. The area of the circle lies between the area of the inside grid and the area of the outside grid..

Charles A. Dana Center. University of Texas at Austin. Mathematics TEKS Toolkit. 2012. *Exploring c/d = π:* Students measure circular objects to collect data to investigate the relationship between the circumference of a circle and its diameter. They find that, regardless of the size of the object or the size of the measuring unit, it always takes a little more than three times the length of the diameter to measure the circumference.

National Security Agency. 2009. *Parallel Lines*: Students use Geometer's Sketchpad® to explore relationships among the angles formed when parallel lines are cut by a transversal. The software is integral to the lesson, and step-by-step instructions are provided.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

7.RP.

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STANDARDS FOR MATHEMATICAL PRACTICE:	
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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Statistics	7.SP.1, 7.SP.2, 7.SP.3, 7.SP.4

UNDERSTAND:

Formulating questions, designing studies, and collecting data about a population through random sampling allow us to make inferences and compare data.

KNOW:	DO:
Random sampling tends to produce representative samples. Finding a valid, representative sample will enable valid inferences to be made about a population. What it means to have a valid, random sample representative of a population(s). Inferences about a population are only valid if the sample is random and representative. Proportional reasoning is used to make estimates or predictions about a population. Having multiple samples for the same population allows for gauging the variation of estimates or predictions. Measures of center can be used to compare data and measure variability between data sets. Data displays are used to visually compare data sets and draw informal comparative inferences. Box plots are way to show measures of variability such as the range (other data displays may highlight other measures of variability). Mean absolute deviation is an element of a data set that is the absolute difference between that element and a given point. Typically the point from which the deviation is measured is a measure of central tendency, most often the median or sometimes the mean of the data set.	***Use random sampling to draw inferences about a population.*** **7.SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.2** Use data, including Montana American Indian demographics data, from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data, predict how many text messages your classmates receive in a day. Gauge how far off the estimate or prediction might be. ***Draw informal comparative inferences about two populations*** **7.SP.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variability's, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team. On a dot plot, the separation between the two distributions of heights is noticeable.* **7.SP.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh grade science book are generally longer than the words in a chapter of a fourth grade science book.*

KEY TERMS FOR THIS PROGRESSION:

Population, Sample, Random sample, Simulated sample, Absolute deviation, Data distributions, Variability, Inference, Conjecture, Measures of center

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Use random sampling to draw inferences about a population.

In Grade 6, students used measures of center and variability to describe data. Students continue to use this knowledge in Grade 7 as they use random samples to make predictions about an entire population and judge the possible discrepancies of the predictions. Providing opportunities for students to use real-life situations from science and social studies shows the purpose for using random sampling to make inferences about a population.

Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is to understand that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample. Have students compare the random sample to population, asking questions like “Are all the elements of the entire population represented in the sample?” and “Are the elements represented proportionally?” Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis.

Provide students with random samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.

Instructional Resources/Tools

Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, American Statistical Association

Ohio Resource Center:
Public Broadcasting Service. 1995-2012. *Mathline: Something Fishy*: Students estimate the size of a large population by applying the concepts of ratio and proportion through the capture/recapture statistical procedure.

Anneberg Foundation. 2012.

Bias in Sampling: This content resource addresses statistics topics that teachers may be uncomfortable teaching due to limited exposure to statistical content and vocabulary. This resource focuses a four-component statistical problem-solving process and the meaning of variation and bias in statistics and to investigate how data vary

Random Sampling and Estimation: In this session, students estimate population quantities from a random sample.

National Council of Teachers of Mathematics. 2000-2012., Illuminations - *Capture/Recapture:* In this lesson, students experience an application of proportion that scientists use to solve real-life problems. Students estimate the size of a total population by taking samples and using proportions.

Cluster: Draw informal comparative inferences about two populations.

In Grade 6, students used measures of center and variability to describe sets of data. In the cluster “Use random sampling to draw inferences about a population” of Statistics and Probability in Grade 7, students learn to draw inferences about one population from a random sampling of that population. Students continue using these skills to draw informal comparative inferences about two populations.

Provide opportunities for students to deal with small populations, determining measures of center and variability for each population. Then have students compare those measures and make inferences. The use of graphical representations of the same data (Grade 6) provides another method for making comparisons. Students begin to develop understanding of the benefits of each method by analyzing data with both methods.

When students study large populations, random sampling is used as a basis for the population inference. This builds on the skill developed in the Grade 7 cluster “Use random sampling to draw inferences about a population” of Statistics and Probability. Measures of center and variability are used to make inferences on each of the general populations. Then the students have make comparisons for the two populations based on those inferences.

This is a great opportunity to have students examine how different inferences can be made based on the same two sets of data. Have students investigate how advertising agencies uses data to persuade customers to use their products. Additionally, provide students with two populations and have them use the data to persuade both sides of an argument.

Instructional Resources/Tools

Advancing Science, Serving Society. 2013. *Baseball Stats:* In this lesson students explore and compare data sets and statistics in baseball.

Ohio Resource Center. *Representation of Data—Cholera and War*: The object of this activity is to study excellent examples of the presentation of data. Students analyze a map of cholera cases plotted against the location of water wells in London in 1854 and a map of Napoleon's march on Moscow in 1812-1813 to see what inferences they can draw from the data displays.

Representation_of_Data-The_US_Census.pdf: The object of this activity is to study an excellent example of the presentation of data. Students analyze an illustration of the 1930 U.S. census compared to the 1960 census to see what inferences they can draw from the data displays.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

7.RP, 7.SP.5-8

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Probability	7.SP.5, 7.SP.6, 7.SP.7a-b, 7.SP.8a-c

UNDERSTAND:

Probabilities are fractions derived from modeling real-world experiments and simulations of chance.

KNOW:	DO:
Probability is the likelihood of an event occurring. The likelihood of a chance event is a number between 0 and 1. Larger numbers (closer to 1) indicate greater likelihood of an event occurring. The benchmark of ½ can be used to determine if an event is more likely or less likely to occur. Theoretical probability is the likelihood of a happening based on all possible outcomes. Experimental probability of an event occurring after an experiment was conducted. Theoretical and experimental probabilities and proportional reasoning are used to make predictions. Equivalent fractions (and prior fraction knowledge) are used for making predictions. Probability in a uniform probability model is the number of favorable outcomes as 1 (numerator) out of the number of all possible outcomes (denominator). For example, probability of rolling a 5 on a regular die is 1/6. Probabilities of compound events are found by first finding the probabilities of each independent event, and then multiply the probabilities of the independent events. Multiplication of fractions can be used to find probabilities of compound events. Sample space represents all possible outcomes. Models can be used to represent the sample space of a compound event in order to connect and build understanding of the probability calculations. Models of probability – area model, tree diagrams, organized lists, table.	***Investigate chance processes and develop, use, and evaluate probability models.*** **7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. For example, when playing Montana American Indian Hand/Stick games, you can predict the approximate number of accurate guesses. **7.SP.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

KEY TERMS FOR THIS PROGRESSION:

Likely event, Unlikely event, Probability of an event, Experimental probability (observed frequency), Theoretical probability, Expected value (frequency), Compound events, Probability model, Uniform probability, Model, Organized lists, Tables, Tree diagrams, Simulations, Sample space

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Investigate chance processes and develop, use, and evaluate probability models.

Grade 7 is the introduction to the formal study of probability. Through multiple experiences, students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions.

Help students understand the probability of chance is using the benchmarks of probability: 0, 1 and ½. Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as 1/2. Then advance to situations in which the probability is somewhere between any two of these benchmark values. This builds to the concept of expressing the probability as a number between 0 and 1. Use this understanding to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen. Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes that are developed to be used in determining the probabilities of events.

Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data (i.e., a sample space of the compound event). From each sample space, students determine the probability or fraction of each possible outcome. Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability.

Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities.

Instructional Resources/Tools

National Council of Teachers of Mathematics. 2000-2012., Illuminations:

Boxing Up: In this lesson, students explore the relationship between theoretical and experimental probabilities.

Capture-Recapture: In this lesson, students estimate the size of a total population by taking samples and using proportions to estimate the entire population.

Ohio Resource Center. 2013.

Probability Basics: This is a 7+ minute video that explores theoretical and experimental probability with tree diagrams and the fundamental counting principle.
Probability Using Dice: This activity explores the probabilities of rolling various sums with two dice. Extensions of the problem and a complete discussion of the underlying mathematical ideas are included.
How to Fix an Unfair Game: This activity explores a fair game and "How to Fix an Unfair Game."
Dart Throwing: The object of this activity is to study an excellent example of the presentation of data. Students analyze an illustration of the 1930 U.S. census compared to the 1960 census to see what inferences they can draw from the data displays.

Public Broadcasting Service. 1995-2013. *Remove One:* A game is analyzed and the concepts of probability and sample space are discussed. In addition to the lesson plan, the site includes ideas for teacher discussion, extensions of the lesson, additional resources (including a video of the lesson procedures) and a discussion of the mathematical content.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

7.NS.2

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 7 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

SAMPLE YEAR LONG PLAN

COURSE: 7th Grade MATH

Unit	Unit 1: Rational Numbers	Unit 2: Expressions and Equations	Unit 3: Ratios and Proportion	Unit 4: Proportional Relationships
(Time)	(25 days)	(25 days)	(20 days)	(20 days)
STANDARDS	7.NS.1	7.EE.1	7.RP.1	7.RP.2
	7.NS.2	7.EE.2	7.RP.3	
	7.NS.3	7.EE.3		7.EE.3
		7.EE.4	7.G.1	7.EE.4

7th Grade Math p. 2

Unit 5: Statistics and Data Analysis	Unit 6: Probability	Unit 7: Geometry	Unit 8: Front-Load for 8th Grade (i.e. exponents, radicals, pythagorean Theorem)
(25 days)	(25 days)	(25 days)	(15 days)
7.SP.1	7.SP.5	7.G.1	
7.SP.2	7.SP.6	7.G.2	
7.SP.3	7.SP.7	7.G.3	
7.SP.4	7.SP.8	7.G.4	
		7.G.5	
7.NS.3		7.G.6	

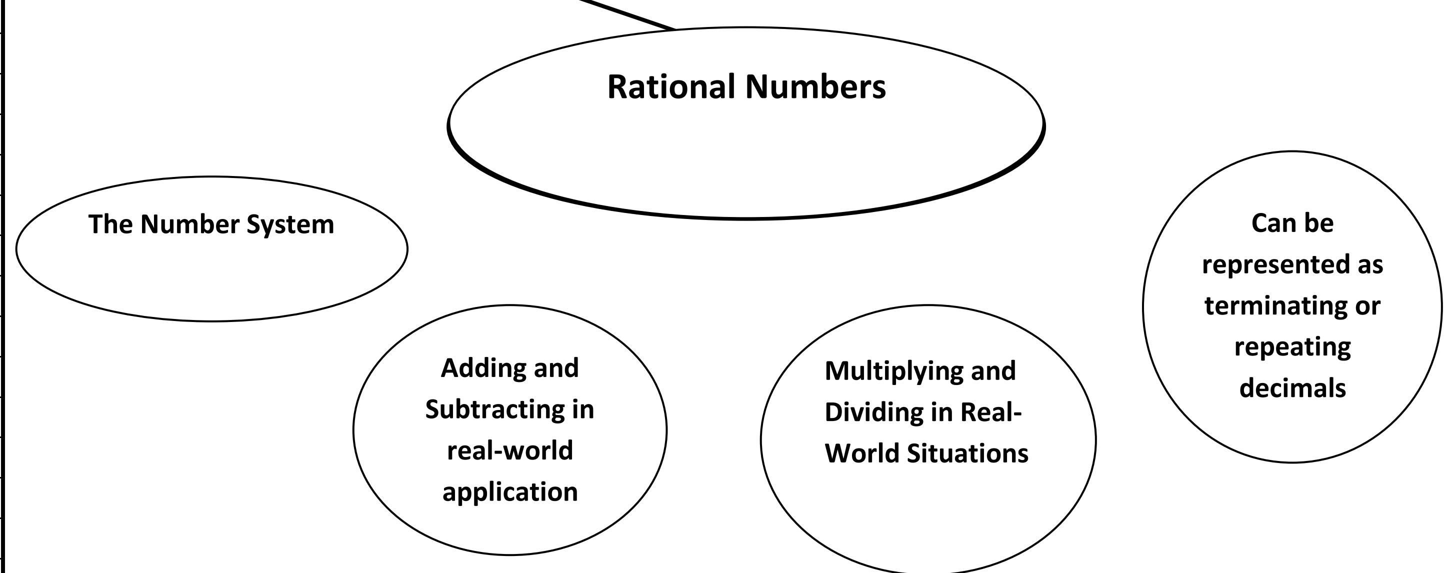
TEACHER: ____________________ SCHOOL DISTRICT/BUILDING: ____________________

COURSE: ________7th Grade Math____________________ GRADE LEVEL(S): ________7____________________

LAST UNIT	CURRENT UNIT	NEXT UNIT
N/A (6th Grade)	**Rational Numbers**	Expressions and Equations

UNIT SCHEDULE	
	Review the Number System
	(real-world examples include
	gaining/losing yards in football,
	positive/negative temperature,
	and balancing a checkbook. Review
	the four basic operations with
	integers.
	Addition and Subtraction
	of Rational Numbers
	Review fractions visually with
	manipulatives.
	Finding LCM/LCD via prime
	factorization/ the "cake" method
	Multiplication and Division
	of Rational Numbers
	Review reciprocals.

is about… UNIT MAP



UNIT SELF TEST QUESTIONS

1.) How do rational numbers relate to the rest of the number system?

2.) How do whole number operations (addition, subtraction, multiplication, and division) apply to rational numbers?

3.) How can we model and solve real-world and mathematical problems involving rational numbers?

MATH STANDARDS	
7.NS.1	
7.NS.2	
7.NS.3	

7th GRADE SAMPLE LESSON PLAN

1.) Grade Level: 7

2.) Title: Exploring Proportions and Scale Through Photographs

Photographs Lesson Website PDF

3.) Standards: 7.RP.1, 7.RP.2, 7.RP.3

4.) Engage
Explore
Explain
Elaborate
Evaluate
Extend

Engage and Explore: The activity at the outstart of the lesson plunges students right in to a real-life situation where they need to make similar photographs fit into a certain size page. Students are paired up and given the activity with little to no direction. (See PDF for details on the activity). Students need to ensure their reduced photographs are proportional to the original, otherwise the image will be distorted (tall and skinny or short and fat). Students will thus learn the importance of proportionality and similarity between preimage and image through a hands-on, real-life application from the beginning of the lesson. This activity helps students discover the **WHY** through their own engagement and exploration!

Explain: Students will now need to defend and explain their reasoning to their peers (and teacher). One student from each pair will act as the "explorer" and will move to another group. The other student from the pair will act as the "defender". The explorer goes to another group's defender and questions the defender about the group's solution to the photograph activity. The defender must articulate her group's solution using mathematical vocabulary. Once this interaction is finished, the explorer and defender from each group switch roles. The new explorer then moves to a new group and the process repeats.

Elaborate: This is where the teacher would present the specific algorithm for solving proportions involving scale drawings and models. The teacher will use book materials and solidify the concept as students take notes for about 10 minutes. (See book materials for details). Setting up equivalent fractions (proportions), defining where your variable goes and why, and cross-multiplying to solve for the unknown are all taught here.

Example from the activity: "6 is to 4 as 3 is to x."

Evaluate: This is an introductory activity, so the teacher would informally evaluate students as he walks around the classroom during the explain (and explore and defend) phase of the activity. The teacher would then give an assignment consisting of similar problems involving photographs in magazines. Students may work together in partners for the rest of the class period, and remaining problems will be homework (see below).

Extension: as homework, students will be given the assignment to cut pictures out of magazines they have at home, measure the pictures, and either reduce or enlarge the picture to fit on specified dimensions. If students do not have magazines at home they can cut, the teacher will have some in the room to provide. This homework activity requires students to apply what they have learned to real-world problems that may not be familiar.

7^{th} GRADE SAMPLE CONSTRUCTED RESPONSE

Strategic Thinking: DOK Level 3

- Requires deep understanding exhibited through planning, using evidence, and more demanding cognitive reasoning
- The cognitive demands are complex and abstract
- An assessment item that has more than one possible answer and requires students to justify the response would most likely be a Level 3

Constructed Response Question #1. 7.RP.3

While on vacation, your group can rent bicycles and scooters by the week. From Company A you get a reduced rental rate if you rent 5 bicycles for every 2 scooters rented. Company A's reduced rate per bicycle is $15.50 per week and the reduced rate per scooter is $160 per week. The sales tax on each rental is 12%.

Company B also gives a reduced rental rate if you rent 7 bicycles for every 3 scooters rented. Company B charges a reduced rate of $16.00 per bike per week and their reduced rate per scooter is $150 per week. The sales tax is fixed in your state, so it is also 12% for Company B.

Your group has $2000 available to spend on bicycle and scooter rentals. There are 25 people in your group. You are frugal, thus you must get the discounted rate from either Company A or Company B. Any extra money left over goes back into the group's vacation fund for next year. Which company would you rent from, and how many bicycles and scooters would you rent from that company and **why**? Justify your answer mathematically by showing all your work!

Our Rubric for scoring this Constructed Response Question is on the next page.

This lesson was adapted from the following web url:

http://www.engageny.org/sites/default/files/resource/attachments/math-grade-7.pdf

(page 18 of the PDF contains question 13)

MATH STUDENT RUBRIC[3]

	Understanding	Reasoning	Accuracy	Communication
Expert 4	• I can show a deep understanding of the problem. • I completely address all parts of the task. • I got it! I can use big math ideas to solve the problem.	• I can use powerful and thorough strategies to get to effective solutions. • I can explore, analyze, and justify all my claims. • I can observe and make connections beyond the problem to real-life situations.	• My procedures are organized so others can follow it. • All of my work is correct. • I can label every item.	• I clearly explain how I solved the problem. • I use visual designs to show how my ideas match the solution. • I can use math language to explain my thinking.
Practitioner 3	• I have a thorough understanding of the problem. • I address the important parts the task. • I logically use big math ideas to solve the problem.	• I use effective strategies for the solutions. • I give evidence for my claims. • I can observe and make connections.	• My procedures are organized and can be followed by others. • If I made mistakes, they are not important ones. • I can label most of the items.	• I explain how I solved the problem. • I use visual designs to show my ideas. • I can use some math language.
Apprentice 2	• I show a limited understanding of the problem. • I address some of the important parts of the task. • My big math ideas did not work very well to solve the problem.	• My strategies worked for part of the problem. • I did not give clear evidence for my claims. • I tried to observe and make connections.	• My procedures are difficult for others to follow. • I have many mistakes in my work. • Some of my items are labeled.	• I did not explain how the problem was solved very well. • My visual designs do not match the solution. • I can use a little math language.
Novice 1	• I did not show that I understand the problem. • I did not address the important parts of the task. • My solution does not use big math ideas.	• I did not use a strategy that helps solve the problem. • The evidence for my claims does not make sense. • I did not make connections to the problem.	• My procedures are not organized for others to follow. • There are too many big mistakes in my work. • None of my items are labeled.	• I did not explain how my solution works to solve the problem. • I did not create designs to help explain the solution. • I did not use math language.

EIGHTH GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

CURRICULUM ORGANIZERS

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Montana Mathematics Grade 8 Content Standards

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

   Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8 Overview

The Number System
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Statistics and Probability
- Investigate patterns of association in bivariate data.

Geometry
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Functions
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

The Number System — 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations — 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrati onal.
3. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
    c. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions — 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations from a variety of cultural contexts, including those of Montana American Indians:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures from a variety of cultural contexts, including those of Montana American Indians: using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. *For example, determine the unknown height of a Plains Indian tipi when given the side length and radius.*
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data including data from Montana American Indian sources on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

[1]Function notation is not required in Grade 8.

Getting Ready
Every Montana Student
College and Career

Montana Common Core
Standards and Assessments

Montana Curriculum Organizer

Grade 8

Mathematics

Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

November 2012 Materials adapted from Arizona, Delaware and Ohio Departments of Education

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HOW TO USE THE MONTANA CURRICULUM ORGANIZER

The Montana Curriculum Organizer supports curriculum development and instructional planning. The Montana Guide to Curriculum Development, which outlines the curriculum development process is another resource to assemble a complete curriculum including scope and sequence, units, pacing guides, outline for use of appropriate materials and resources and assessments.

Page 4 of this document is important for planning curriculum, instruction and assessment. It contains the Standards for Mathematical Practice grade level explanations and examples that describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise. The Critical Areas indicate two to four content areas of focus for instructional time. Focus, coherence and rigor are critical shifts that require considerable effort for implementation of the Montana Common Core Standards. Therefore, a copy of this page for easy access may help increase rigor by integrating the Mathematical Practices into all planning and instruction and help increase focus of instructional time on the big ideas for that grade level.

Pages 7 through 34 consist of tables organized into learning progressions that can function as units. The table for each learning progression, unit, includes 1) domains, clusters and standards organized to describe what students will Know, Understand, and Do (KUD), 2) key terms or academic vocabulary, 3) instructional strategies and resources by cluster to address instruction for all students, 4) connections to provide coherence, and 5) the specific standards for mathematical practice as a reminder of the importance to include them in daily instruction.

Description of each table:

<table>
<tr><th>LEARNING PROGRESSION</th><th>STANDARDS IN LEARNING PROGRESSION</th></tr>
<tr><td>Name of this learning progression, often this correlates with a domain, however in some cases domains are split or combined.</td><td>Standards covered in this learning progression.</td></tr>
<tr><th colspan="2">UNDERSTAND:</th></tr>
<tr><td colspan="2">What students need to understand by the end of this learning progression.</td></tr>
<tr><th>KNOW:</th><th>DO:</th></tr>
<tr><td>What students need to know by the end of this learning progression.</td><td>What students need to be able to do by the end of this learning progression, organized by cluster and standard.</td></tr>
<tr><th colspan="2">KEY TERMS FOR THIS PROGRESSION:</th></tr>
<tr><td colspan="2">Mathematically proficient students acquire precision in the use of mathematical language by engaging in discussion with others and by giving voice to their own reasoning. By the time they reach high school they have learned to examine claims, formulate definitions, and make explicit use of those definitions. The terms students should learn to use with increasing precision in this unit are listed here.</td></tr>
<tr><th colspan="2">INSTRUCTIONAL STRATEGIES AND RESOURCES:</th></tr>
<tr><td colspan="2">Cluster: Title
Strategies for this cluster
Instructional Resources/Tools
Resources and tools for this cluster</td></tr>
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<tr><th colspan="2">CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:</th></tr>
<tr><td colspan="2">Standards that connect to this learning progression are listed here, organized by cluster.</td></tr>
<tr><th colspan="2">STANDARDS FOR MATHEMATICAL PRACTICE:</th></tr>
<tr><td colspan="2">A quick reference guide to the 8 standards for mathematical practice is listed here.</td></tr>
</table>

Mathematics is a human endeavor with scientific, social, and cultural relevance. Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and

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statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians. Cultural context and the Standards for Mathematical Practices together provide opportunities to engage students in culturally relevant learning of mathematics and create criteria to increase accuracy and authenticity of resources. Both mathematics and culture are found everywhere, therefore, the incorporation of contextually relevant mathematics allows for the application of mathematical skills and understandings that makes sense for all students.

Standards for Mathematical Practice: Grade 8 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
8.MP.1. Make sense of problems and persevere in solving them.	In grade 8, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?” and “Can I solve the problem in a different way?”
8.MP.2. Reason abstractly and quantitatively.	In grade 8, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
8.MP.3. Construct viable arguments and critique the reasoning of others.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?”, “Does that always work?” They explain their thinking to others and respond to others’ thinking.
8.MP.4. Model with mathematics.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
8.MP.5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
8.MP.6. Attend to precision.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
8.MP.7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8.MP.8. Look for and express regularity in repeated reasoning.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

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CRITICAL AREAS FOR GRADE 8 MATH
In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean theorem.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
The Number System – Rational and Irrational Numbers (This is not a stand-alone unit, but topics that are embedded within other units.)	8.NS.1, 8.NS.2

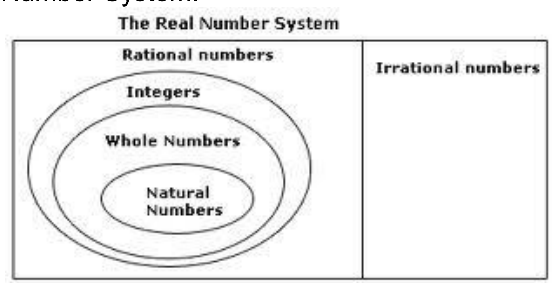
UNDERSTAND:

In the real-number system, numbers can be defined by their decimal representations.

KNOW:

There are numbers that are not rational called “irrational”.

Irrational numbers are a subset of the Real Number System.



Every number has a decimal representation:

- Irrational decimals are non-repeating and non-terminating; and
- Rational number decimals eventually terminate or repeat.

Irrational numbers can be approximated for comparing and ordering them.

DO:

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

KEY TERMS FOR THIS PROGRESSION:

Real-number system, Rational, Irrational, Square root, Repeating decimal, Terminating decimal, Radical, Non-repeating, Non-terminating, Integers

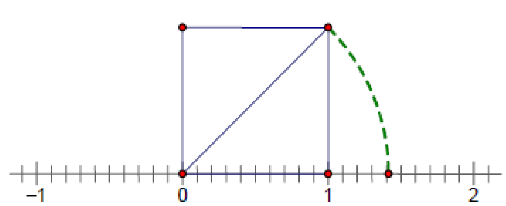
INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Know that there are numbers that are not rational, and approximate them by rational numbers.

The distinction between rational and irrational numbers is an abstract distinction, originally based on ideal assumptions of perfect construction and measurement. In the real world, however, all measurements and constructions are approximate. Nonetheless, it is possible to see the distinction between rational and irrational numbers in their decimal representations.

A rational number is of the form *a/b*, where *a* and *b* are both integers, and *b* is not 0. In the elementary grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into *b* equal parts; then, beginning at 0, count out *a* of those parts. The surprising fact, now, is that there are numbers on the number line that cannot be expressed as *a/b*, with *a* and *b* both integers, and these are called irrational numbers.

Students construct a right isosceles triangle with legs of 1 unit. Using the Pythagorean theorem, they determine that the length of the hypotenuse is √2. In the figure below, they can rotate the hypotenuse back to the original number line to show that indeed √2 is a number on the number line.



In the elementary grades, students become familiar with decimal fractions, most often with decimal representations that terminate a few digits to the right of the decimal point. For example, to find the exact decimal representation of 2/7, students might use their calculator to find 2/7 = 0.2857142857…, and they might guess that the digits 285714 repeat. To show that the digits do repeat, students in Grade 7 actually carry out the long division and recognize that the remainders repeat in a predictable pattern — a pattern that creates the repetition in the decimal representation (see 7.NS.2d).

Thinking about long division generally, ask students what will happen if the remainder is 0 at some step. They can reason that the long division is complete, and the decimal representation terminates. If the reminder is never 0, in contrast, then the remainders will repeat in a cyclical pattern because at each step with a given remainder, the process for finding the next remainder is always the same. Thus, the digits in the decimal representation also repeat. When dividing by 7, there are 6 possible nonzero remainders, and students can see that the decimal repeats with a pattern of at most 6 digits. In general, when finding the decimal representation of *m/n*, students can reason that the repeating portion of decimal will have at most *n* - 1 digits. The important point here is that students can see that the pattern will repeat, so they can imagine the process continuing without actually carrying it out.

Conversely, given a repeating decimal, students learn strategies for converting the decimal to a fraction. One approach is to notice that rational numbers with denominators of 9 repeat a single digit. With a denominator of 99, two digits repeat; with a denominator of 999, three digits repeat, and so on. For example,

13/99 = 0.13131313…
74/99 = 0.74747474…
237/999 = 0.237237237…
485/999 = 0.485485485…

From this pattern, students can go in the other direction, conjecturing, for example, that the repeating decimal 0.285714285714… = 285714/999999. And then they can verify that this fraction is equivalent to 2/7.

Once students understand that (1) every rational number has a decimal representation that either terminates or repeats, and (2) every terminating or repeating decimal is a rational number, they can reason that on the number line, irrational numbers (i.e., those that are not rational) must have decimal representations that neither terminate nor repeat. And although students at this grade do not need to be able to prove that √2 is irrational, they need to know that √2 is irrational (see 8.EE.2), which means that its decimal representation neither terminates nor repeats. Nonetheless, they can approximate √2 without using the square root key on the calculator. Students can create tables like those below to approximate √2 to one, two, and then three places to the right of the decimal point:

x	x^2
1.0	1.00
1.1	1.21
1.2	1.44
1.3	1.69
1.4	1.96
1.5	2.25
1.6	2.56
1.7	2.89
1.8	3.24
1.9	3.61
2.0	4.00

x	x^2
1.40	1.9600
1.41	1.9881
1.42	2.0164
1.43	2.0449
1.44	2.0736
1.45	2.1025
1.46	2.1316
1.47	2.1609
1.48	2.1904
1.49	2.2201
1.50	2.2500

x	x^2
1.410	1.988100
1.411	1.990921
1.412	1.993744
1.413	1.996569
1.414	1.999396
1.415	2.002225
1.416	2.005056
1.417	2.007889
1.418	2.010724
1.419	2.013561
1.420	2.016400

From knowing that $1^2 = 1$ and $2^2 = 4$, or from the picture above, students can reason that there is a number between 1 and 2 whose square is 2. In the first table above, students can see that between 1.4 and 1.5, there is a number whose square is 2. Then in the second table, they locate that number between 1.41 and 1.42. And in the third table they can locate √2 between 1.414 and 1.415. Students can develop more efficient methods for this work. For example, from the picture above, they might have begun the first table with 1.4. And once they see that $1.42^2 > 2$, they do not need generate the rest of the data in the second table.

Use set diagrams to show the relationships among real, rational, irrational numbers, integers, and counting numbers. The diagram should show that the all real numbers (numbers on the number line) are either rational or irrational.

Given two distinct numbers, it is possible to find both a rational and an irrational number between them.

Instructional Resources/Tools
Graphing calculators
Dynamic geometry software

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.EE.1, 8.EE.2, 8.EE.3, 8.EE.4, 8.G.6, 8.G.7, 8.G.8

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Expressions & Equations – Radicals & Integer Exponents	8.EE.1, 8.EE.2, 8.EE.3, 8.EE.4

UNDERSTAND:

The properties of number systems and their relationships remain consistent when applied to integer exponents.

KNOW:	DO:
Properties of integer exponents. A perfect square is a number in which the square root is an integer. A perfect cube is a number in which the cube root is a whole number. Perfect squares and perfect cube numbers up to 100. The √2 is irrational. The base-ten number system can be applied to represent very large and very small numbers using powers of 10. Flexibility with the equivalent forms of an expression allows for efficient problem solving. Strategies for computing with numbers expressed in scientific notation. Properties of Operations and Order of Operations are used to simplify, evaluate, or find equivalent expressions. Estimation as a means for predicting and assessing the reasonableness of a solution.	***Work with radicals and integer exponents.*** **8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. **8.EE.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* **8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

KEY TERMS FOR THIS PROGRESSION:

Radical, Square root, Cube root, Scientific notation, Integer exponents, Perfect squares, Perfect cubes

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Work with radicals and integer exponents.

Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to know and use the properties of exponents and to extend the meaning beyond counting-number exponents. It is no accident that these expectations are simultaneous, because it is the properties of counting-number exponents that provide the rationale for the properties of integer exponents. In other words, s*tudents should not be told these properties, but rather should derive them through experience and reason.*

For counting-number exponents (and for nonzero bases), the following properties follow directly from the meaning of exponents.

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$

Students should have experience simplifying numerical expressions with exponents so that these properties become natural and obvious. For example,

$2^3 \cdot 2^5 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2\cdot2) = 2^8$
$(5^3)^4 = (5\cdot5\cdot5)\cdot(5\cdot5\cdot5)\cdot(5\cdot5\cdot5)\cdot(5\cdot5\cdot5) = 5^{12}$
$(3\cdot7)^4 = (3\cdot7)\cdot(3\cdot7)\cdot(3\cdot7)\cdot(3\cdot7) = (3\cdot3\cdot3\cdot3)\cdot(7\cdot7\cdot7\cdot7) = 3^4\cdot7^4$

If students reason about these examples with a sense of generality about the numbers, they begin to articulate the properties. For example, "I see that 3 twos is being multiplied by 5 twos, and the results is 8 twos being multiplied together, where the 8 is the sum of 3 and 5, the number of twos in each of the original factors. That would work for a base other than two (as long as the bases are the same)."

Note: When talking about the meaning of an exponential expression, it is easy to say (incorrectly) that "35 means 3 multiplied by itself 5 times." But by writing out the meaning, 35 = 3·3·3·3·3, students can see that there are only 4 multiplications. So a better description is "35 means 5 3's multiplied together."

Students also need to realize that these simple descriptions work only for counting-number exponents. When extending the meaning of exponents to include 0 and negative exponents, these descriptions are limiting. Is it sensible to say "30 means 0 3's multiplied together" or that "3-2 means -2 3's multiplied together?"

The motivation for the meanings of 0 and negative exponents is the following principle: *The properties of counting-number exponents should continue to work for integer exponents.*

For example, Property 1 can be used to reason what 3^0 should be. Consider the following expression and simplification: $3^0\cdot3^5 = 3^{0+5} = 3^5$. This computation shows that the when 3^0 is multiplied by 3^5, the result (following Property 1) should be 3^5, which implies that 3^0 must be 1. Because this reasoning holds for any base other than 0, we can reason that $a^0 = 1$ for any nonzero number *a*.

To make a judgment about the meaning of 3^{-4}, the approach is similar: $3^{-4}\cdot3^4 = 3^{-4+4} = 3^0 = 1$. This computation shows that 3^{-4} should be the reciprocal of 3^4, because their product is 1. And again, this reasoning holds for any nonzero base. Thus, we can reason that $a^{-n} = 1/a^n$.

Properties of Integer Exponents
For any nonzero real numbers *a* and *b* and integers *n* and *m*:
1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

Putting all of these results together, we now have the properties of integer exponents, shown in the above chart. For mathematical completeness, one might prove that properties 1-3 continue to hold for integer exponents, but that is not necessary at this point.

A supplemental strategy for developing meaning for integer exponents is to make use of patterns, as shown in the chart below:

Patterns in Exponents

⋮	⋮
5^4	625
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	1/5
5^{-2}	1/25
5^{-3}	1/125
⋮	⋮

As the exponent decreases by 1, the value of the expression is divided by 5, which is the base. Continue that pattern to 0 and negative exponents.

The meanings of 0 and negative-integer exponents can be further explored in a place-value chart:

thousands	*hundreds*	*tens*	*ones*		*tenths*	*hundredths*	*thousandths*
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
3	2	4	7	.	5	6	8

Thus, integer exponents support writing any decimal in expanded form like the following:

$3247.568 = 3\cdot10^3 + 2\cdot10^2 + 4\cdot10^1 + 7\cdot10^0 + 5\cdot10^{-1} + 6\cdot10^{-2} + 8\cdot10^{-3}$.

Expanded form and the connection to place value is important for helping students make sense of scientific notation, which allows very large and very small numbers to be written concisely, enabling easy comparison. To develop familiarity, go back and forth between standard notation and scientific notation for numbers near, for example, 1012 or 10-9. Compare numbers, where one is given in scientific notation and the other is given in standard notation. Real-world problems can help students compare quantities and make sense about their relationship.

Provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally.

Opportunities for conceptually understanding irrational numbers should be developed. One way is for students to draw a square that is one unit by one unit and find the diagonal using the Pythagorean Ttheorem. The diagonal drawn has an irrational length of $\sqrt{2}$. Other irrational lengths can be found using the same strategy by finding diagonal lengths of rectangles with various side lengths.

Instructional Resources/Tools

Square tiles and cubes to develop understanding of squared and cubed numbers
Calculators to verify and explore patterns
Webquests using data mined from sites like the U.S. Census Bureau, scientific data (planetary distances)
Place value charts to connect the digit value to the exponent (negative and positive)

Michael W. Davidson and the Florida State University. 1995-2012. *Powers of 10*: Online video.

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CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:
8.NS.1, 8.NS.2, 8.G.6, 8.G.7, 8.G.8

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Expressions & Equations – Proportional Linear Relationships	8.EE.5, 8.EE.6

UNDERSTAND:

The connection between linearity and proportionality (as a special case of linearity) is based on an understanding of slope as the constant rate of change and the *y*-intercept.

KNOW:	DO:
Proportional relationships can be represented symbolically (equation), graphically (coordinate plane), in a table, in diagrams, and verbal descriptions. The coordinates representing a proportional linear situation can be interpreted in terms of the context. In a proportional linear relationship, the point (0,0) is the *y*-intercept and (1,*r*) is the slope, where *r* is the unit rate. Slope of a line is a constant rate of change. The *y*-intercept is the point at which a line intersects the vertical axis (*y*-axis). One form of an equation for a line is $y = mx + b$, where *m* is the slope and *b* is the *y*-intercept. A special case of linear equations (proportional relationships) are in the form of $y/x = m$ and $y = mx$.	***Understand the connections between proportional relationships, lines, and linear equations.*** **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

KEY TERMS FOR THIS PROGRESSION:

Equation, Slope, Constant rate of change, Unit rate, Origin, *Y*-intercept, Proportional relationship, Linear, Similar triangles, Coordinate

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

This cluster focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade, unit rates were expanded to complex fractions and percents through solving multistep problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance/time problems are notorious in mathematics. In this cluster, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed and described in different ways: graphically and algebraically. Emphasize the creation of representative graphs and the meaning of various points. Then compare the same information when represented in an equation.

By using coordinate grids and various sets of three similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students can be led to generalize the slope to $y = mx$ for a line through the origin and $y = mx + b$ for a line through the vertical axis at *b*.

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Instructional Resources/Tools
Carnegie Math™
Graphing calculators
SMART™ technology with software emulator

National Library of Virtual Manipulatives. 2000-2012. (NLVM)©
The National Council of Teachers of Mathematics, Illuminations Capture-Recapture
Annenberg™ video tutorials, http://www.nsdl.org/to access applets
Texas Instruments® website (http://www.ticares.com/)

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.EE.7, 8.EE.8

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 8 Mathematics

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Expressions & Equations – Systems of Equations	8.EE.7a-b, 8.EE.8a-c

UNDERSTAND:

Linear equations, systems of equations, linear functions, and their understanding of slope of a line can be used to analyze situations and solve problems.

KNOW:	DO:
Inverse operations are used to solve equations. Linear equations in one variable can have one solution, infinitely many solutions, or no solution. Solutions to a system of equations are the values of the variables that make both equations true (one point of intersection). Systems of equations that have infinitely many solutions are equivalent forms of the same equation, and represent the same line in a plane (all points intersect because they are the same line). Systems of equations that have no solution are parallel lines having equivalent slopes (no point of intersection).	***Analyze and solve linear equations and pairs of simultaneous linear equations.*** **8.EE.7** Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.EE.8** Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

KEY TERMS FOR THIS PROGRESSION:

Equation, System of Equations, Slope, *Y*-intercept, Solution, Linear, Coordinate, Inverse Operations, Distributive Property

INSTRUCTIONAL STRATEGIES AND RESOURCES

Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.

In Grade 6, students applied the properties of operations to generate equivalent expressions, and identified when two expressions are equivalent. This cluster extends understanding to the process of solving equations and to their solutions, building on the fact that solutions maintain equality, and that equations may have only one solution, many solutions, or no solution at all. Equations with many solutions may be as simple as $3x = 3x$, $3x + 5 = x + 2 + x + x + 3$, or $6x + 4x = x(6 + 4)$, where both sides of the equation are equivalent once each side is simplified.

Table 4 on page 74 in the [Montana Common Core Standards for School Mathematics Grade-Band](), generalizes the properties of operations and serves as a reminder for teachers of what these properties are. Eighth-graders should be able to describe these relationships with real numbers and justify their reasoning using words and not necessarily with the algebraic language of Table 3. In other words, students should be able to state that 3(-5) = (-5)3 because multiplication is commutative and it can be performed in any order (it is commutative), or that 9(8) + 9(2) = 9(8 + 2) because the distributive property allows us to distribute multiplication over addition, or determine products and add them. Grade 8 is the beginning of using the generalized properties of operations, but this is not something on which students should be assessed.

Pairing contextual situations with equation solving allows students to connect mathematical analysis with real-life events. Students should experience analyzing and representing contextual situations with equations, identify whether there is one, none, or many solutions, and then solve to prove conjectures about the solutions. Through multiple opportunities to analyze and solve equations, students should be able to estimate the number of solutions and possible values(s) of solutions prior to solving. Rich problems, such as computing the number of tiles needed to put a border around a rectangular space, or solving proportional problems, as in doubling recipes, help ground the abstract symbolism to life.

Experiences should move through the stages of concrete, conceptual and algebraic/abstract. Utilize experiences with the pan-balance model as a visual tool for maintaining equality (balance) first with simple numbers, then with pictures symbolizing relationships, and finally with rational numbers allows understanding to develop as the complexity of the problems increases. Equation-solving in Grade 8 should involve multistep problems that require the use of the distributive property, collecting like terms, and variables on both sides of the equation.

This cluster builds on the informal understanding of slope from graphing unit rates in Grade 6 and graphing proportional relationships in Grade 7 with a stronger, more formal understanding of slope. It extends solving equations to understanding solving systems of equations, or a set of two or more linear equations that contain one or both of the same two variables. Once again the focus is on a solution to the system. Most student experiences should be with numerical and graphical representations of solutions. Beginning work should involve systems of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection, simplify the computation and hone in on finding a solution. More complex systems can be investigated and solved by using graphing technology.

Contextual situations relevant to eighth-graders will add meaning to the solution to a system of equations. Students should explore many problems for which they must write and graph pairs of equations leading to the generalization that finding one point of intersection is the single solution to the system of equations. Provide opportunities for students to connect the solutions to an equation of a line, or solution to a system of equations, by graphing, using a table and writing an equation. Students should receive opportunities to compare equations and systems of equations, investigate using graphing calculators or graphing utilities, explain differences verbally and in writing, and use models such as equation balances.

Problems such as, “Determine the number of movies downloaded in a month that would make the costs for two sites the same, when Site A charges \$6 per month and \$1.25 for each movie and Site B charges \$2 for each movie and no monthly fee.” Students write the equations letting y = the total charge and x = the number of movies: Site A: $y = 1.25x + 6$; Site B: $y = 2x$.

Students graph the solutions for each of the equations by finding ordered pairs that are solutions and representing them in a T-chart. Discussion should encompass the realization that the intersection is an ordered pair that satisfies both equations. And finally, students should relate the solution to the context of the problem, commenting on the practicality of their solution.

Problems should be structured so that students also experience equations that represent parallel lines and equations that are equivalent. This will help them to begin to understand the relationships between different pairs of equations: When the slope of the two lines is the same, the equations are either different equations representing the same line (thus resulting in many solutions), or the equations are different equations representing two not intersecting, parallel, lines that do not have common solutions.

System-solving in Grade 8 should include estimating solutions graphically, solving using substitution, and solving using elimination. Students again should gain experience by developing conceptual skills using models that develop into abstract skills of formal solving of equations. Provide opportunities for students to change forms of equations (from a given form to slope-intercept form) in order to compare equations.

Instructional Resources/Tools
SMART Board’s new tools for solving equations
Graphing calculators
Index cards with equations/graphs for matching and sorting

National Council of Teachers of Mathematics. 2000-2012. *Supply and Demand*: This activity focuses on having students create and solve a system of linear equations in a real-world setting. By solving the system, students will find the equilibrium point for supply and demand. Students should be familiar with finding linear equations from two points or slope and *y*-intercept. This lesson was adapted from the October 1991 edition of *Mathematics Teacher*.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.EE.5, 8.EE.6

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Functions – Linear & Nonlinear	8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.F.5

UNDERSTAND:

Functions can be classified into different families of functions (linear and nonlinear) that can be used to model different real-world phenomena.

KNOW:	DO:
Input/output tables can be used as a tool to generate a function rule. Functions can be represented algebraically, graphically, numerically in tables (ordered pairs), or by verbal descriptions. Changing the way a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations may be more useful than others and may highlight different characteristics. Some representations of functions may show only part of the function. Functions are used to model real-world phenomena.	***Define, evaluate, and compare functions.*** **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1] **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **8.F.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* ***Use functions to model relationships between quantities.*** **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

KEY TERMS FOR THIS PROGRESSION:

Functions, Linear function, Nonlinear

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Define, evaluate, and compare functions.

In Grade 6, students plotted points in all four quadrants of the coordinate plane. They also represented and analyzed quantitative relationships between dependent and independent variables. In Grade 7, students decided whether two quantities are in a proportional relationship. In Grade 8, students begin to call relationships functions when each input is assigned to exactly one output. Also, in Grade 8, students learn that proportional relationships are part of a broader group of linear functions, and they are able to identify whether a relationship is linear. Nonlinear functions are included for comparison. Later, in high school, students use function notation and are able to identify types of nonlinear functions.

To determine whether a relationship is a function, students should be expected to reason from a context, a graph, or a table, after first being clear which quantity is considered the input and which is the output. When a relationship is not a function, students should produce a counterexample: an "input value" with at least two "output values." If the relationship is a function, the students should explain how they verified that for each input there was exactly one output. The "vertical line test" should be avoided because (1) it is too easy to apply without thinking, (2) students do not need an efficient strategy at this point, and (3) it creates misconceptions for later mathematics, when it is useful to think of functions more broadly, such as whether x might be a function of y.

[1] Function notation is not required in Grade 8.

"Function machine" pictures are useful for helping students imagine input and output values, with a rule inside the machine by which the output value is determined from the input.

Notice that the standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically (symbolically, with letters). This is sometimes called the "rule of four." For fluency and flexibility in thinking, students need experiences translating among these. In Grade 8, the focus, of course, is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$. Students also need experiences with nonlinear functions, including functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule, such as a girl's height as a function of her age.

In the elementary grades, students explore number and shape patterns (sequences), and they use rules for finding the next term in the sequence. At this point, students describe sequences both by rules relating one term to the next and also by rules for finding the *n*th term directly. (In high school, students will call these recursive and explicit formulas.) Students express rules in both words and in symbols. Instruction should focus on additive and multiplicative sequences as well as sequences of square and cubic numbers, considered as areas and volumes of cubes, respectively.

When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots can be misleading. For example, if a function is used to model the height of a stack of n paper cups, it does not make sense to have 2.3 cups, and thus there will be no ordered pairs between $n = 2$ and $n = 3$.

Provide multiple opportunities to examine the graphs of linear functions and use graphing calculators or computer software to analyze or compare at least two functions at the same time. Illustrate with a slope triangle where the run is "1" that slope is the "unit rate of change." Compare this in order to compare two different situations and identify which is increasing/decreasing as a faster rate.

Students can compute the area and perimeter of different-size squares and identify that one relationship is linear while the other is not by either looking at a table of value or a graph in which the side length is the independent variable (input) and the area or perimeter is the dependent variable (output).

Instructional Resources/Tools
Graphing calculators
Graphing software (including dynamic geometry software)

Cluster: Use functions to model relationships between quantities.
In Grade 8, students focus on linear equations and functions. Nonlinear functions are used for comparison.

Students will need many opportunities and examples to figure out the meaning of $y = mx + b$. What does m mean? What does b mean? They should be able to "see" m and b in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts. For example, if a function is used to model the height of a stack of n paper cups, then the rate of change, m, which is the slope of the graph, is the height of the "lip" of the cup: the amount each cup sticks above the lower cup in the stack. The "initial value" in this case is not valid in the context because 0 cups would not have a height, and yet a height of 0 would not fit the equation. Nonetheless, the value of b can be interpreted in the context as the height of the "base" of the cup: the height of the whole cup minus its lip.

Use graphing calculators and web resources to explore linear and nonlinear functions. Provide context as much as possible to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.

Give students opportunities to gather their own data or graphs in contexts they understand. Students need to measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They also need opportunities to draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar. Probe student thinking by asking them to determine which input values make sense in the problem situations.

Provide students with a function in symbolic form and ask them to create a problem situation in words to match the function. Given a graph, have students create a scenario that would fit the graph. Ask students to sort a set of "cards" to match a graphs, tables, equations, and problem situations. Have students explain their reasoning to each other.

From a variety of representations of functions, students should be able to classify and describe the function as linear or nonlinear, increasing or decreasing. Provide opportunities for students to share their ideas with other students and create their own examples for their classmates.

Use the slope of the graph and similar triangle arguments to call attention to not just the change in *x* or *y*, but also to the rate of change, which is a ratio of the two.

Emphasize key vocabulary. Students should be able to explain what key words mean (e.g., model, interpret, initial value, functional relationship, qualitative, linear, non-linear. Use a "word wall" to help reinforce vocabulary.

Instructional Resources/Tools
Graphing calculators
Graphing software for computers, including dynamic geometry software
Data-collecting technology, such as motion sensors, thermometers, CBL's, etc.
Graphing applets online

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.EE.5-8

STANDARDS FOR MATHEMATICAL PRACTICE:

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LEARNING PROGRESSION:	STANDARDS IN LEARNING PROGRESSION:
Transformations & Angle Relationships	8.G.1a-c, 8.G.2, 8.G.3, 8.G.4, 8.G.5

UNDERSTAND:

Use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarities can describe and analyze two-dimensional figures and solve problems.

KNOW:	DO:
Reflections, rotations, and translations are rigid transformations and maintain congruence (do not change the size and shape of the object being transformed). Congruent figures have the same size and shape. Dilations may change the size of the object being transformed, but not the shape. Similar figures have the same shape; corresponding angle measures remain congruent, with a scale factor relating corresponding sides (corresponding sides are proportional). Scale factors larger than 1 enlarge a figure. Scale factors less than 1 shrink a figure. A scale factor of exactly 1 maintains congruence. Two triangles are similar if at least two pairs corresponding angles are congruent (AA postulate for similarity) Symbols for congruency and similarity (≅ and ~). The sum of the angles of a triangle is 180̊. The sum of interior angles of a polygon is related to the number of triangles that the polygon is composed of, and that relates to a generalization (formula).	***Understand congruence and similarity using physical models, transparencies, or geometry software.*** **8.G.1** Verify experimentally the properties of rotations, reflections, and translations from a variety of cultural contexts, including those of Montana American Indians: a. lines are taken to lines, and line segments to line segments of the same length; b. angles are taken to angles of the same measure; and c. parallel lines are taken to parallel lines. **8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **8.G.3** Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures from a variety of cultural contexts, including those of Montana American Indians, using coordinates. **8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

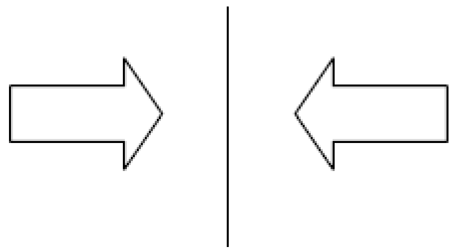
KEY TERMS FOR THIS PROGRESSION:

Transformations, Rotations, Reflections, Translations, Symmetry, Similarity, Similar triangles, Congruent, Pre-image, Image, Interior angles, Exterior angles, Parallel lines, Transversal, Vertical angles, Supplementary angles, Complementary angles, Adjacent angles

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.
A major focus in Grade 8 is to use knowledge of angles and distance to analyze two- and three-dimensional figures and space in order to solve problems. This cluster interweaves the relationships of symmetry, transformations, and angle relationships to form understandings of similarity and congruence. Inductive and deductive reasoning are utilized as students forge into the world of proofs. Informal arguments are justifications based on known facts and logical reasoning. Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). Students are expected to use logical thinking, expressed in words using correct terminology. They are not expected to use theorems, axioms, postulates or a formal format of proof as in two-column proofs.

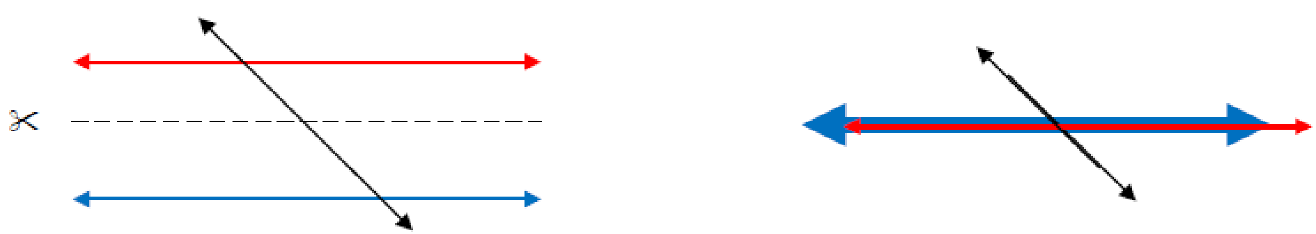
Transformational geometry is about the effects of rigid motions, rotations, reflections and translations on figures. Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. For example, when reflecting over a line, each vertex is the same distance from the line as its corresponding vertex. This is easier to visualize when not using regular figures. Time should be allowed for students to cut out and trace the figures for each step in a series of transformations. Discussion should include the description of the relationship between the original figure and its image(s) in regards to their corresponding parts (length of sides and measure of angles) and the description of the movement, including the attributes of transformations (line of symmetry, distance to be moved, center of rotation, angle of rotation and the amount of dilation).The case of distance – preserving transformation leads to the idea of congruence. It is these distance-preserving transformations that lead to the idea of congruence.



Work in the coordinate plane should involve the movement of various polygons by addition, subtraction and multiplied changes of the coordinates. For example, add 3 to *x*, subtract 4 from *y*, combinations of changes to *x* and *y*, multiply coordinates by 2 then by 12. Students should observe and discuss such questions as: ”What happens to the polygon?” and ”What does making the change to all vertices do?” Understandings should include generalizations about the changes that maintain size or maintain shape, as well as the changes that create distortions of the polygon (dilations). Example dilations should be analyzed by students to discover the movement from the origin and the subsequent change of edge lengths of the figures. Students should be asked to describe the transformations required to go from an original figure to a transformed figure (image). Provide opportunities for students to discuss the procedure used, whether different procedures can obtain the same results, and if there is a more efficient procedure to obtain the same results. Students need to learn to describe transformations with both words and numbers.

Through understanding symmetry and congruence, conclusions can be made about the relationships of line segments and angles with figures. Students should relate rigid motions to the concept of symmetry and to use them to prove congruence or similarity of two figures. Problem situations should require students to use this knowledge to solve for missing measures or to prove relationships. It is an expectation to be able to describe rigid motions with coordinates.

Provide opportunities for students to physically manipulate figures to discover properties of similar and congruent figures. For example, the corresponding angles of similar figures are equal. Additionally use drawings of parallel lines cut by a transversal to investigate the relationship among the angles. For example, what information can be obtained by cutting between the two intersections and sliding one onto the other?

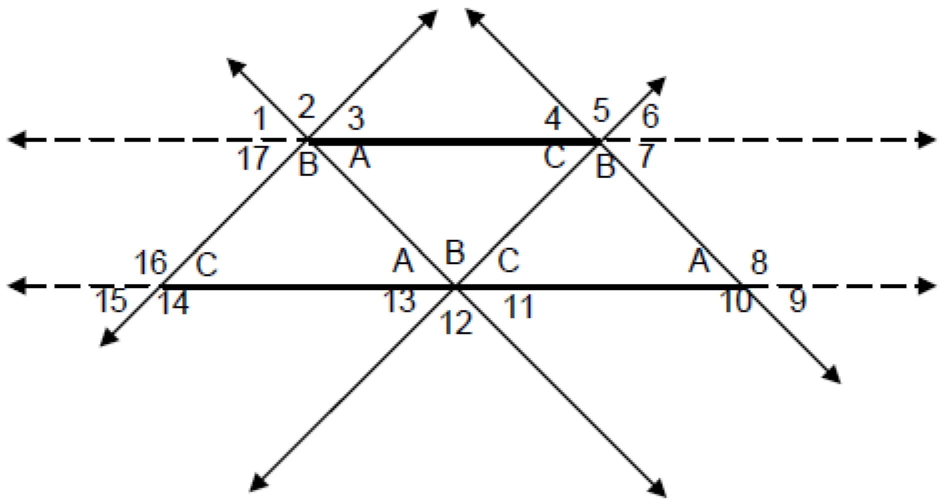


In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, vertical). Now, the focus is on learning the about the sum of the angles of a triangle and using it to, find the measures of angles formed by transversals (especially with parallel lines), or to find the measures of exterior angles of triangles and to informally prove congruence.

By using three copies of the same triangle labeled and placed so that the three different angles form a straight line, students can:

- explore the relationships of the angles;
- learn the types of angles (interior, exterior, alternate interior, alternate exterior, corresponding, same -side interior, same -side exterior),; and
- explore the parallel lines, triangles and parallelograms formed.

Further examples can be explored to verify these relationships and demonstrate their relevance in real-life.



Investigations should also lead to the Angle-Angle criterion for similar triangles. For instance, pairs of students create two different triangles with one given angle measurement, then repeat with two given angle measurements and finally with three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.

Students should solve mathematical and real-life problems involving understandings from this cluster. Investigation, discussion, justification of their thinking, and application of their learning will assist in the more formal learning of geometry in high school.

Instructional Resources/Tools

Pattern blocks or shape sets
Mirrors - Miras
Geometry software like Geometer's Sketchpad, Cabri Jr. or GeoGebra
Graphing calculators grid paper
Patty paper

Utah State University. National Library of Virtual Manipulatives. 1999-2010.

Congruent Triangles: Build similar triangles by combining sides and angles.
Geoboard - Coordinate: Rectangular geoboard with *x* and *y* coordinates.
Transformations – Composition: Explore the effect of applying a composition of translation, rotation, and reflection transformations to objects.
Transformations - Dilation: Dynamically interact with and see the result of a dilation transformation.
Transformations - Reflection: Dynamically interact with and see the result of a reflection transformation.
Transformations - Rotation: Dynamically interact with and see the result of a rotation transformation.
Transformations - Translation: Dynamically interact with and see the result of a translation transformation.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.G.6–8, 8.EE.6

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1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Geometry – Pythagorean Theorem	8.G.6, 8.G.7, 8.G.8

UNDERSTAND:

The Pythagorean theorem, the special relationship between side lengths of a right triangle, can be used to find and describe unknown length.

KNOW:	DO:
The square root of the area of a square represents the side length of the square (e.g., a square with an area of 9 cm^2 has a side length of √9 = 3cm). There is a special relationship between the side lengths of a right triangle that states that the sums of the squares of the legs equal the square of the hypotenuse. This relationship is called the Pythagorean theorem. Numbers that have two identical factors are called perfect squares (e.g., 16 is a square number because 4 * 4 = 16), so the square root (as a side length) of these numbers are whole numbers (√16 = 4). Estimating can help to assess the reasonableness of a square root calculation (e.g., √15 is between 3 and 4, but closer to 4 because √9 = 3 and √16 = 4 and 15 is between 9 and 16, but closer to 16).	***Understand and apply the Pythagorean theorem.*** **8.G.6** Explain a proof of the Pythagorean theorem and its converse. **8.G.7** Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. For example, determine the unknown height of an American Plains Indian tipi when given the side length and radius. **8.G.8** Apply the Pythagorean theorem to find the distance between two points in a coordinate system.

KEY TERMS FOR THIS PROGRESSION:

Pythagorean theorem, Right triangle, Legs, Hypotenuse, Square, Square root, Area

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Understand and apply the Pythagorean theorem.

Previous understanding of triangles, such as the sum of two side measures is greater than the third side measure, angles sum, and area of squares, is furthered by the introduction of unique qualities of right triangles. Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs and the measure of the hypotenuse. Experiences should involve using grid paper to draw right triangles from given measures and representing and computing the areas of the squares on each side. Data should be recorded in a chart such as the one below, allowing for students to conjecture about the relationship among the areas within each triangle.

Triangle	Measure of Leg 1	Measure of Leg 2	Area of Square on Leg 1	Area of Square on Leg 2	Area of Square on Hypotenuse
1					

Students should then test out their conjectures, then explain and discuss their findings. Finally, the Pythagorean theorem should be introduced and explained as the pattern they have explored. Time should be spent analyzing several proofs of the Pythagorean theorem to develop a beginning sense of the process of deductive reasoning, the significance of a theorem, and the purpose of a proof. Students should be able to justify a simple proof of the Pythagorean theorem or its converse.

Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need situations that explore using the Pythagorean theorem to test whether or not side lengths represent right triangles.

(Recording could include Side length *a*, Side length *b*, Sum of $a^2 + b^2$, c^2, $a^2 + b^2 = c^2$. Through these opportunities, students should realize that there are Pythagorean (triangular) triples such as (3, 4, 5), (5, 12, 13), (7, 24, 25), (9, 40, 41) that always create right triangles, and that their multiples also form right triangles. Students should see how similar triangles can be used to find additional triples. Students should be able to explain why a triangle is or is not a right triangle using the Pythagorean theorem.

The Pythagorean theorem should be applied to finding the lengths of segments on a coordinate grid, especially those segments that do not follow the vertical or horizontal lines, as a means of discussing the determination of distances between points. Contextual situations, created by both the students and the teacher, that apply the Pythagorean theorem and its converse should be provided. For example, apply the concept of similarity to determine the height of a tree using the ratio between the student's height and the length of the student's shadow. From that, determine the distance from the tip of the tree to the end of its shadow and verify by comparing to the computed distance from the top of the student's head to the end of the student's shadow, using the ratio calculated previously. Challenge students to identify additional ways that the Pythagorean theorem is or can be used in real-world situations or mathematical problems, such as finding the height of something that is difficult to physically measure, or the diagonal of a prism.

Instructional Resources/Tools

Utah State University. National Library of Virtual Manipulatives. 1999-2010.

Pythagorean Theorem: Solve two puzzles that illustrate the proof of the Pythagorean theorem.

Right Triangle Solver: Practice using the Pythagorean theorem and the definitions of the trigonometric functions to solve for unknown sides and angles of a right triangle.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.G.1–5, 8.EE.2, 8.NS.1-2

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Montana Curriculum Organizer: Grade 8 Mathematics

This document is a curriculum organizer adapted from other states to be used for planning scope and sequence, units, pacing and other materials that support a focused, coherent, and rigorous study of mathematics K-12.

LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Geometry – Volume	8.G.9

UNDERSTAND:

The volume of any three dimensional shape is dependent on the area of its base, height of the shape and the number of parallel bases (layers) that the shape has.

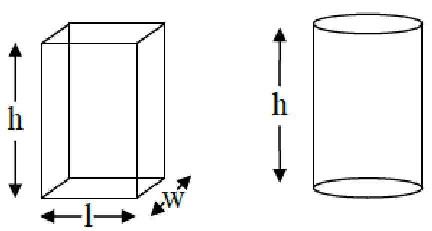
KNOW:	DO:
Volume is measured in cubic units. There is a relationship between the volumes of cylinders, cones, and spheres. The volume of a cone is 1/3 of the volume of a cylinder (with same radius and height). The volume of a sphere is 2/3 the volume of a cylinder (with same radius and height).	***Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.*** **8.G.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

KEY TERMS FOR THIS PROGRESSION:

Volume, Area, Circle, Cone, Cylinder, Sphere, Cubic units

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
Begin by recalling the formula, and its meaning, for the volume of a right rectangular prism: $V = l \times w \times h$. Then ask students to consider how this might be used to make a conjecture about the volume formula for a cylinder:



Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a "base" times the height, and so because the area of the base of a cylinder is π r2 the volume of a cylinder is $V_c = \pi r^2 h$.

To motivate the formula for the volume of a cone, use cylinders and cones with the same base and height. Fill the cone with rice or water and pour into the cylinder. Students will discover/experience that 3 cones full are needed to fill the cylinder. This non-mathematical derivation of the formula for the volume of a cone, V = 1/3 π r2h, will help most students remember the formula.

In a drawing of a cone inside a cylinder, students might see that that the triangular cross-section of a cone is 1/2 the rectangular cross-section of the cylinder. Ask them to reason why the volume (three dimensions) turns out to be *less* than 1/2 the volume of the cylinder. It turns out to be 1/3.

For the volume of a sphere, it may help to have students visualize a hemisphere "inside" a cylinder with the same height and "base." The radius of the circular base of the cylinder is also the radius of the sphere and the hemisphere. The area of the "base" of the cylinder and the area of the section created by the division of the sphere into a hemisphere is πr^2. The height of the cylinder is also r so the volume of the cylinder is πr^3. Students can see that the volume of the hemisphere is less than the volume of the cylinder and more than half the volume of the cylinder. Illustrating this with concrete materials and rice or water will help students see the relative difference in the volumes. At this point, students can reasonably accept that the volume of the hemisphere of radius r is $2/3 \pi r^3$ and therefore volume of a sphere with radius r is twice that or $4/3 \pi r^3$. There are several websites with explanations for students who wish to pursue the reasons in more detail. (Note that in the pictures above, the hemisphere and the cone together fill the cylinder.)

Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions.

Instructional Resources/Tools
Ohio Resource Center. 2013.

The Cylinder Problem: Students build a family of cylinders, all from the same-sized paper, and discover the relationship between the dimensions of the paper and the resulting cylinders. They order the cylinders by their volumes and draw a conclusion about the relationship between a cylinder's dimensions and its volume.
Finding Surface Area and Volume:
Blue Cube, 27 Little Cubes (Stella Stunner):
Volume of a Spheres and Cones (Rich Problem):

Utah State University. National Library of Virtual Manipulatives. 1999-2010. *How High:* an applet that can be used to take an inquiry approach to the formula for volume of a cylinder or cone.

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.G.3-6

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
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LEARNING PROGRESSION	STANDARDS IN LEARNING PROGRESSION
Statistics & Probability – Scatter Plots, Linear vs. Nonlinear Data Associations & Linear Regression	8.SP1, 8.SP.2, 8.SP.3, 8.SP.4

UNDERSTAND:

The same characteristics used to describe linear relationships allow us to describe, classify, and analyze the association of bivariate measurement data.

KNOW:	DO:
The characteristics of a linear relationship, such as: • Can be written in the form *y* = *mx* + *b* in which *x* is the independent variable, *y* is the dependent variable, *m* is the slope, and *b* is the *y*-intercept. • Appears to be a straight line in a *xy* coordinate graph. • When the constant rate is positive, the line will extend northeast and southwest. • When the constant rate is negative, the line will extend northwest and southeast. • Strategies for modeling a line of best fit given a set of data. • Line drawn with the least total deviation from the actual data points. • Strategies include using a graphing calculator, strand of spaghetti, ruler, etc. Linear trends can be identified as positive or negative, while some trends have no correlation. Identifying outliers and other data characteristics allows for meaningful data interpretation and analysis. The line of best fit represents the data set as a whole, fitted through the majority of points.	***Investigate patterns of association in bivariate data.*** **8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data including data from Montana American Indian sources on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

KEY TERMS FOR THIS PROGRESSION:

Bivariate data, Scatter-plot, Outlier, Quantitative variable, Slope, *Y*-intercept, Positive association, Negative association, Relative frequency, Histogram, Two-way table

INSTRUCTIONAL STRATEGIES AND RESOURCES:

Cluster: Investigate patterns of association in bivariate data.

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. Students will extend their descriptions and understanding of variation to the graphical displays of bivariate data.

Scatter plots are the most common form of displaying bivariate data in Grade 8. Provide scatter plots and have students practice informally finding the *line of best fit*. Students should create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. Have students use a graphing calculator to determine a linear regression and discuss how this relates to the graph. Students should informally draw a line of best fit for a scatter plot and informally measure the strength of fit. Discussion should include "What does it mean to be above the line, below the line?"

The study of the line of best fit ties directly to the algebraic study of slope and intercept. Students should interpret the slope and intercept of the line of best fit in the context of the data. Then students can make predictions based on the line of best fit.

Instructional Resources/Tools

National Security Agency. *Glued to the Tube or Hooked to the Books?* This lesson provides step-by-step instructions for using the graphing calculator to construct a scatter plot of class data and a line of best fit.

National Council of Teachers of Mathematics, Illuminations:. 2000-2013.

Impact of a Superstar: This lesson uses technology tools to plot data, identify lines of best fit, and detect outliers. Then, students compare the lines of best fit when one element is removed from a data set, and interpret the results.

Exploring Linear Data: In this lesson, students construct scatter plots of bivariate data, interpret individual data points, make conclusions about trends in data, especially linear relationships, and estimate and write equation of lines of best fit.

Ohio Resource Center. 2013. *Lines of Fit:* A video tutorial that shows how to determine a line of best fit for a set of data.

American Statistical Association. *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.*

CONNECTIONS TO OTHER DOMAINS AND/OR CLUSTERS:

8.F.1-5, 8.EE.5, 8.EE.6

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them.
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This Curriculum Organizer was created using the following materials:
ARIZONA - STANDARDS FOR MATHEMATICAL PRACTICE EXPLANATIONS AND EXAMPLES
http://www.azed.gov/standards-practices/mathematics-standards/
DELAWARE – LEARNING PROGRESSIONS
http://www.doe.k12.de.us/infosuites/staff/ci/content_areas/math.shtml
OHIO – INSTRUCTIONAL STRATEGIES AND RESOURCES (FROM MODEL CURRICULUM)
http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=134773

SAMPLE YEAR LONG PLAN

COURSE: 8th Grade Math

Unit	Number Systems	Expressions and Equations - Radicals and Integer Exponents	Expressions and Equations - Proportional Linear Relationships	Expressions and Equations - Systems of Equations	Functions - Linear and Non Linear
(Time)	(10 days)	(20 days)	(30 days)	(20 days)	(30 days)
STANDARDS	8.NS.1 8.NS.2	8.EE.1 8.EE.2 8.EE.3 8.EE.4	8.EE.5 8.EE.6	8.EE.7 a - b 8.EE.8 a - c	8.F.1 8.F.2 8.F.3 8.F.4 8.F.5
Connections to other Domains and/or Clusters:	8.EE.1 8.EE.2 8.EE.3 8.EE.4 8.G.6 8.G.7 8.G.8	8.NS.1 8.NS.2 8.G.6 8.G.7 8.G.8	8.EE.7 8.EE.8	8.EE.5 8.EE.6	8.EE.5-8

8th Grade Math p. 2

Transformations - and Angle Relationships	Geometry - Pythagorean Theorem	Geometry - Volume	Statistics and Probability - Scatter Plots,
(15 days)	(20 days)	(20 days)	(15 days)
8.G.1 a - c	8.G.6	8.G.9	8.SP.1
8.G.2	8.G.7		8.SP.2
8.G.3	8.G.8		8.SP.3
8.G.4			8.SP.4
8.G.5			
8.G.6-8	8.G.1-5	8.G.3-6	8.F.1-5
8.EE.6	8.EE.2		8.EE.5
	8.NS.1-2		8.EE.6

TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: _8th Grade Math______________________ GRADE LEVEL(S): _8th Grade______________________

LAST UNIT	CURRENT UNIT	NEXT UNIT
Transformations and Angle Relationships	**Geometry - Pythagorean Theorem**	Geometry - Volume

UNIT SCHEDULE

Days	Schedule
2	Introduction
2	Prove it is a right Triangle
2	Prove it is an acute or obtuse triangle
3	Find a missing side
4	Real World Applications
2	Special right triangles
3	Distance on a coordinate plane
1	Review
1	Assessment

is about... UNIT MAP

Using the Pythagorean Theorem and the converse of the theorem to solve problems.

- *Introduce the Pythagorean Theorem*
- *Understanding the Pythagorean Theorem*
- *Using the Pythagorean Theorem*
- *Special Triangles*

UNIT SELF TEST

1. Use grid paper to prove the Pythagorean Theorem.
2. Explain why a triangle is or is not a triangle using the Pythagorean Theorem.
3. Use the Pythagorean Theorem to determine whether or not a triangle is obtuse or acute.

MATH STANDARDS	
8.G.6	
8.G.7	
8.G.8	

	4. Use the Pythagorean Theorem to calculate the measurement of a missing side of a right triangle. 5. Use the Pythagorean Theorem to calculate the measurement of a distance/height in a real life situation (tree, flag pole, light stanchion) 6. Use knowledge of special triangles (3-4-5; 45-45-90; 30-60-90) to estimate and calculate the measurement of missing sides of a triangle.		

PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Solving Real-Life Problems: Baseball Jerseys

*Additional student examples and reasoning analysis activities are available in the original document available at the website listed below.

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

For more details, visit: http://map.mathshell.org

Solving A Real-Life Problem: *Baseball Jerseys*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Interpret a situation and represent the variables mathematically.
- Select appropriate mathematical methods to use.
- Explore the effects of systematically varying the constraints.
- Interpret and evaluate the data generated and identify the break-even point, checking it for confirmation.
- Communicate their reasoning clearly.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8.EE Analyze and solve linear equations and pairs of simultaneous linear equations.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson students attempt the *Baseball Jerseys* task individually. You then review their responses and formulate questions for them to consider when thinking about how they could improve their work.
- At the start of the lesson, students think individually about the questions posed.
- Next, students work in small groups to combine their thinking and work together to produce a collaborative solution to the *Baseball Jerseys* task, in the form of a poster.
- In the same small groups, students evaluate and comment on sample responses, identifying the strengths and weaknesses in these responses and comparing them with their own work.
- In a whole-class discussion students compare and evaluate the strategies they have seen and used.
- In a follow-up lesson, students spend ten minutes reflecting on their work and what they have learned.

MATERIALS REQUIRED

- Each individual student will need a copy of the assessment task: *Baseball Jerseys,* some plain paper, a mini-whiteboard, a pen, an eraser, and a copy of the questionnaire *How Did You Work?* Provide calculators if requested.
- Each small group of students will need a large sheet of paper, some felt tipped pens, and copies of *Sample Responses to Discuss*.
- Graph paper should be kept in reserve and used only when requested.
- There is a projector resource to support whole-class discussions.

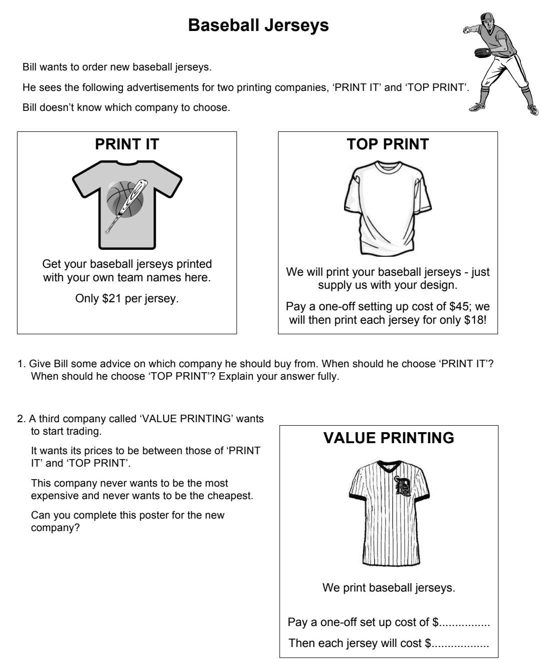
TIME NEEDED

15 minutes before the lesson, a 70-minute lesson and 10 minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task*: Baseball Jerseys* (15 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.



Baseball Jerseys

Bill wants to order new baseball jerseys.

He sees the following advertisements for two printing companies, 'PRINT IT' and 'TOP PRINT'.

Bill doesn't know which company to choose.

1. Give Bill some advice on which company he should buy from. When should he choose 'PRINT IT'? When should he choose 'TOP PRINT'? Explain your answer fully.

2. A third company called 'VALUE PRINTING' wants to start trading.

   It wants its prices to be between those of 'PRINT IT' and 'TOP PRINT'.

   This company never wants to be the most expensive and never wants to be the cheapest.

   Can you complete this poster for the new company?

Give each student a copy of the assessment task *Baseball Jerseys* and some plain paper for them to work on. Provide calculators if requested.

> *Read through the questions and try to answer them as carefully as you can. Show all your working, so that I can understand your reasoning.*
>
> *As well as trying to solve the problem, I want you to see if you can present your work in an organized and clear way.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students who sit together often produce similar answers then, when they come to compare their work, they have little to discuss. For this reason, we suggest that, when students do the task individually, you ask them to move to different seats. At the beginning of the formative assessment lesson allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. Research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions, and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students, and write these on the board when you return the work to the students.

Common issues:	**Suggested questions and prompts:**
Student has difficulty getting started	• What do you know? • What do you need to find out? • What calculations could you do with the information you have?
Omits to use all given information For example: The student may not have taken into account the $45 setting up cost for 'Top Print'.	• Write in your own words the information given. • What numbers in the task are fixed? • What can vary? • How could you check your answer?
Students work is unsystematic For example: The student writes 'Print It' 5 and 'Top Print' 10; 10 cost more etc.	• Can you organize the costs of different numbers of jerseys made by the two companies in a systematic way? • What would be sensible values to try? Why? • How might you organize your work?
Makes incorrect assumptions For example: Assumes that Bill should always choose 'Print It' because they don't have a $45 setting up cost like 'Top Print'.	• Will 'Top Print' always be more expensive? • How much will it cost to print 20 shirts with each company?
Students work is poorly presented For example: The student presents the work as a series of unexplained numbers and/or calculations. Or: The student draws a table without headings. Or: The student circles or underlines numbers and it is left to the reader to work out why this is the answer as opposed to any other calculation.	• Would someone unfamiliar with the task easily understand your work? • Have you explained how you arrived at your answer?
Student has difficulties when using graphs or equations For example: The student inaccurately plots lines, does not label axes, or does not explain the purpose of the graph. Or: The student makes a mistake when solving the equation.	• Would someone unfamiliar with the task easily understand your work? • How can you check your answer? • How do your answers help you to solve the problem? • Does your answer seem sensible?
Considers a specific case for comparison For example: The student states that there are nine players in a baseball team and so finds the cost for nine jerseys from each of the two companies (Q1). Or: Completes a set-up fee and cost per jersey for 'Value Printing' based on a specific number of jerseys and does not explore what happens for different numbers of jerseys being bought (Q2).	• What if Bill wanted to buy more/less than nine jerseys? Who should he buy them from? • 'Value Printing' never wants to be the cheapest or the most expensive company. You have shown this is the case for [specific number] jerseys, what would happen if you were buying less/more? Would this still be the case?
Student correctly answers all the questions Student needs an extension task.	• Try to find a different solution for the pricing of 'Value Printing'. Is there a way of describing all possible solutions?

SUGGESTED LESSON OUTLINE

Reviewing individual solutions to the task (10 minutes)

Give each student a mini-whiteboard, a pen, and an eraser and return their work on the *Baseball Jerseys* task. You may want to show the class Slide P-1 of the projector resource.

If you have not added questions to individual pieces of student work, either give each student a printed version of your list of questions with the questions that relate to their work highlighted, or write your list of questions on the board so that students can select questions from the board that are appropriate to their own work.

Recall what we were working on previously. What was the task about?

I have had a look at your work and have some questions I would like you to think about.

On your own, carefully read through the questions I have written. I would like you to use the questions to help you to think about ways of improving your work.

Use your mini-whiteboards to make a note of anything you think will help to improve your work.

If mini-whiteboards are not available, students may want to use the back of their response to the task to jot down their ideas about ways to improve their work. This is an opportunity for students to review their own work before working collaboratively on producing a group solution. Whilst students are reviewing their work, it may be appropriate to ask individual students questions that help them to clarify their thinking.

Collaborative small-group work: preparing joint solutions on posters (20 minutes)

Organize the class into groups of two or three students.

Today you are going to work together in your group to produce a joint solution to the Baseball Jerseys task that is better than your individual work.

Before students have another go at the task, they need to discuss what they have learned from reviewing their individual solutions. This will enable them to decide which of their different approaches is better.

You each have your own individual solution to the task and have been thinking about how you might improve it, using the questions I have posed.

I want you to share your work with your partner(s). Take turns to explain how you did the task and how you think it could be improved.

If explanations are unclear, ask questions until everyone in the group understand the individual solutions.

To check that students understand what they are being asked to do, ask a student to explain it:

Steven, tell me what you are going to do in your groups.

Slide P-2 summarizes these instructions.

Once students have had chance to discuss their work, hand out a sheet of poster paper and some felt tipped pens to each group of students. Provide calculators and graph paper if students request them. Display Slide P-1 of the projector resource.

Having discussed the work you have done individually, in your group agree on the best method for completing the problem and produce a poster that shows a joint solution to the Baseball Jerseys task which is better than your individual work.

Make sure that you answer the task questions clearly and write explanations on your poster. Include any assumptions you have made on your poster.

Again check that students understand what they are being asked to do by asking someone to explain the task to the rest of the class.

While students are working in small groups you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different student approaches

Note any common mistakes. For example, are students consistently using all the given information? Which math do they choose to use? How do they use it? Attend to the students' mathematical decisions. Do they track their progress in their use of their chosen mathematics? Do they notice if they have chosen a strategy that does not seem to be productive? If so, what do they do? You can use this information to focus a whole-class discussion towards the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help them to clarify their thinking, promote further progress and encourage students to develop self-regulation and error detection skills.

You may find that some students find it difficult to keep more than one piece of information in mind at the same time. For example, you may ask them to consider these two questions:

If Bill were to order ten jerseys from 'Print It', how much would they cost?

If Bill were to order ten jerseys from 'Top Print', how much would they cost?

Students who organize their work into a table may struggle to select appropriate column headings or omit column headings for the 'Number of jerseys', 'Cost from Print It' and 'Cost from Top Print'.

How can you make your table clearer to someone who is unfamiliar with the task?

If students are struggling to produce a joint solution to the task, encourage them to identify the strengths and weaknesses of the methods employed in their individual responses. Can any of these methods be improved to produce a group solution that is better than the original individual response? Can they think of any other approaches to try?

What have you done that you both [all] agree on?

What else do you need to find?

Have you used all the information given in the task?

What do you now know that you didn't know before?

Do your calculations make sense?

What assumptions have you made? Do you think they are reasonable?

You may also want to use some of the questions in the *Common issues* table to support your own questioning. The purpose of these questions is to help students to track and review their problem solving strategies. They should be encouraged to give reasons for the choices they have made.

Sharing different approaches (10 minutes)

Once groups have completed their posters, display them at the front of the room. Hold a whole-class discussion on the methods used to produce a group solution. Ask two groups of students to describe the method used and the ways in which this method differs to their initial individual responses. Did the students check their work? If they did, what checking method did they use?

Collaborative analysis of *Sample Responses to Discuss* (20 minutes)

After students have had sufficient time to discuss some different approaches, distribute copies of the *Sample Responses to Discuss* to each group.

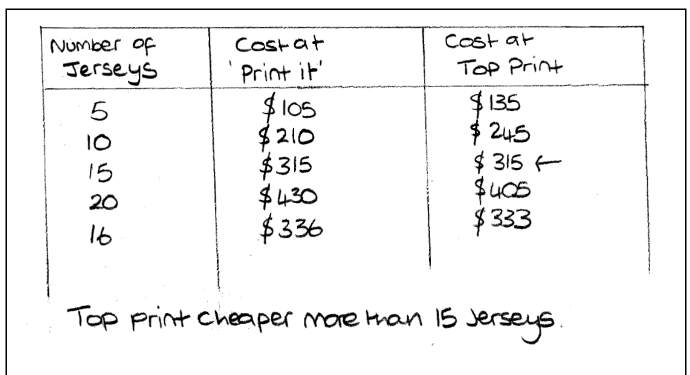
> *In your groups you are now going to look at some student work on the task. Notice in what ways this work is similar to yours, and in which ways it is different.*
>
> *There are some questions for you to answer as you look at the work. You may want to add annotations to the work to make it easier to follow.*

This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy. Students should thoughtfully answer the questions below each piece of sample student work and be encouraged to think carefully about ways in which it could be improved.

It may not be appropriate, or there may not be enough time, for students to analyze all of the four sample responses. If students have been successful, then it may be better to issue sample responses that challenge them to approach the task using a different method. For example, students that have calculated the comparative prices for specific numbers of jerseys could be given Jeremiah's work, which shows the beginnings of an algebraic approach; students that have used an algebraic approach could be given Tanya's work that shows a graphical approach. Students that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

During the small group work, support the students as before. Also, check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample approaches and those the students took in the small group work.

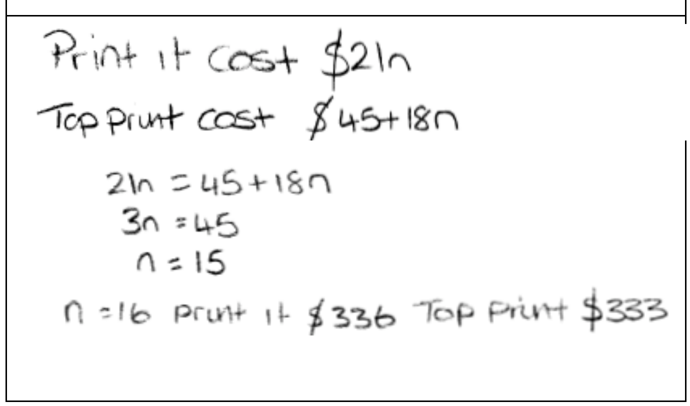
Danny has found an effective way to organize his work using a table, but has made some arithmetical mistakes: 10 jerseys at 'Top Print' should be $225, and 20 jerseys at 'Print It' should be $420. However, his reasoning is correct and these errors do not change his conclusions.



Number of Jerseys	Cost at 'Print it'	Cost at Top Print
5	$105	$135
10	$210	$245
15	$315	$315 ←
20	$430	$405
16	$336	$333

Top print cheaper more than 15 Jerseys.

Jeremiah has tried an algebraic approach and found a correct solution for the break-even point. He checks the costs for more than 15 jerseys by calculating the cost of 16 jerseys from each of the two companies.



Print it cost $21n

Top Print cost $45+18n

21n = 45+18n

3n = 45

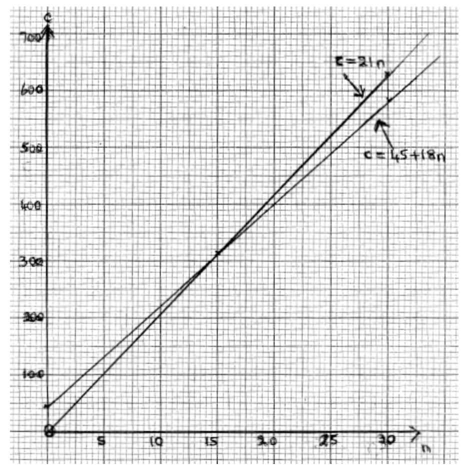
n = 15

n = 16 Print it $336 Top Print $333

Bella has tried a completely different approach. She reasons that once the setting up cost of \$45 has been paid (\$45 ÷ \$3 = 15) ‘Top Print’ is \$3 cheaper **per jersey**.

He would have to buy 15 jerseys to get the \$45 set up cost gone. Then Top Print \$3 cheaper.

Tanya writes equations for the costs of buying jerseys from the two companies: $c = 21n$ and $c = 45 + 18n$. She draws two line graphs representing the two equations but does not use an appropriate scale. She does not explain that the two graphs intersect at the point (15, 315), when the costs for the two companies are equal. The graph shows that for values of n less than 15, the cost for ‘Print It’ is less than ‘Top Print’. For values of n greater than 15, the cost for ‘Print It’ is more than ‘Top Print’.



Whole-class discussion (10 minutes)

Now hold a whole-class discussion to consider the different approaches seen in the sample responses and ask students to compare them with their own work. As students compare the different solution methods, ask them to comment on their strengths and weaknesses.

Did any group use a similar method to Danny/Jeremiah/Bella/Tanya?

What was the same/different about the work?

What is unclear about Danny/Jeremiah/Bella/Tanya’s work?

In what ways could the work be improved?

Did analyzing the responses enable anyone to see errors in their own work?

Of the four sample pieces of work, which do you think has the most complete solution? Which student has adopted the most appropriate approach?

Students should be encouraged to consider the efficiency of different approaches and their appropriateness, as well as the accuracy and completeness of the sample responses. For example, when answering question two of the task a graphical approach provides a sensible strategy, but for question one it may not be considered necessary or advantageous. You may want to demonstrate this by showing students the graph on Slide P-7 of the projector resource. The scales for each axis have been changed, however the accuracy is still an issue.

Does this new scale help to identify the cost of jerseys?

In this case, what are the disadvantages of using a graph?

You may also want to use Slides P-3, P-4, P-5 and P-6 of the projector resource and the questions in the *Common issues* table to support this whole-class discussion.

Follow-up lesson (or possible homework): individual reflection (10 minutes)

Once students have had a chance to discuss the sample responses as a whole-class, distribute the questionnaire *How Did You Work?* Ask students to spend a couple of minutes, working individually, to answer the questions.

Think carefully about your work this lesson and the different methods you have seen and used.

On your own, answer the review questions as carefully as you can.

Some teachers set this as a homework task.

SOLUTIONS

1. For one or two jerseys, the one-off set-up cost means that each jersey is much more expensive from 'Top Print'. As the number of jerseys increases the difference in the costs becomes smaller. For 15 jerseys the costs are equal, and for more than 15 the cost at 'Top Print' becomes less than at 'Print It'.

Tabular method:

Number of jerseys	Cost at 'Print It'	Cost at 'Top Print'
1	$21	$63
2	$42	$81
5	$105	$135
10	$210	$225
15	$315	$315
20	$420	$405
25	$525	$495
16	$336	$333

Algebraic method:

'Top Print' jerseys cost less than 'Print It' jerseys when:

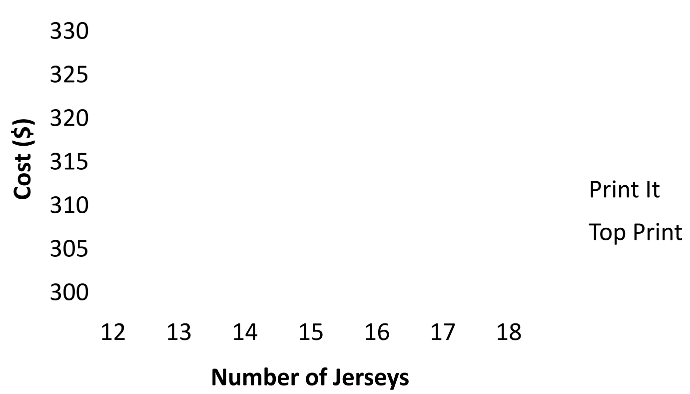
$18n + 45 \leq 21n$ n is the number of jerseys.

$3n \geq 45$

$n \geq 15$

Graphical method:

By plotting the lines C = 18n + 45 and C = 21n where C is the cost of jerseys and n is the number of jerseys it can be seen that the two lines intersect at the point (15, 315). This shows that when buying 15 jerseys it costs the same from both companies.



The advice to Bill is that if he is buying less than 15 jerseys then he should buy from 'Print It' as they are cheaper. If he is buying 15 jerseys then he can choose either company. If he is buying more than 15 jerseys then he should buy from 'Top Print'.

Students may refer to the size of roster or the number of players on the field and assume that Bill will be buying enough jerseys for each player. Any assumptions that are made should be clearly stated and explained.

2. There are multiple solutions to this question and students should be encouraged to extend their work to finding more than one solution. A possible solution would be:

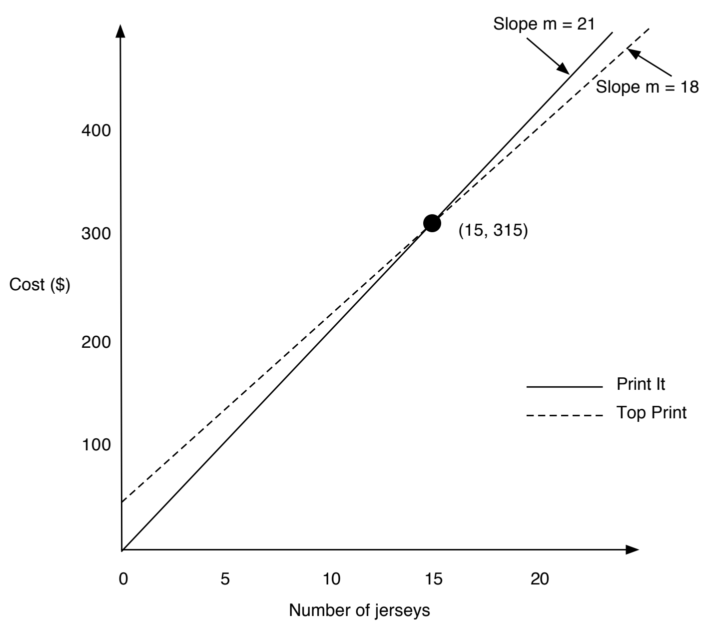
VALUE PRINTING

We print baseball jerseys.

Pay a one-off set up cost of $15

Then each jersey will cost $20

When thinking about this graphically, the line for the third company would need to pass through the point (15, 315) and have a slope between that of the other two lines i.e. $18 < m < 21$:



The third company will need to charge a one-off set-up cost, which is less than $45.

Baseball Jerseys

Bill wants to order new jerseys for his baseball team.

He sees the following advertisements for two printing companies, 'PRINT IT' and 'TOP PRINT'.

Bill doesn't know which company to choose.

PRINT IT	**TOP PRINT**
	
Get your baseball jerseys printed with your own team names here. Only $21 per jersey.	We will print your baseball jerseys - just supply us with your design. Pay a one-off setting up cost of $45; we will then print each jersey for only $18!

1. Give Bill some advice on which company he should buy from. When should he choose 'PRINT IT'? When should he choose 'TOP PRINT'? Explain your answer fully.

2. A third company called 'VALUE PRINTING' wants to start trading.

   It wants its prices to be between those of 'PRINT IT' and 'TOP PRINT'.

   This company never wants to be the most expensive and never wants to be the cheapest.

   Can you complete this poster for the new company?

VALUE PRINTING

We print baseball jerseys.

Pay a one-off set up cost of $................

Then each jersey will cost $.................

How Did You Work?

Mark the boxes and complete the sentences that apply to your work.

1 Our group work was better than my own work ☐

Our group solution was better because

2 We checked our method ☐

We checked our method by

We could check our method by

3 Our method is similar to one of the sample responses ☐ OR Our method is different from **all** the sample responses ☐

Our method is similar to *Add name of sample response*

This is because

Our method is different from all the sample responses because:

4 In our method we assumed that

Baseball Jerseys

PRINT IT	TOP PRINT	VALUE PRINTING
		
Get your baseball jerseys printed with your own team names here. Only $21 per jersey.	We will print your baseball jerseys - just supply us with your design. Pay a one-off setting up cost of $45; we will then print each jersey for only $18!	We print baseball jerseys. Pay a one-off set up cost of $............... Then each jersey will cost $.................

1. Give Bill some advice. When should he choose PRINT IT? When should he choose TOP PRINT?

2. VALUE PRINTING never wants to be the most expensive and never wants to be the cheapest.

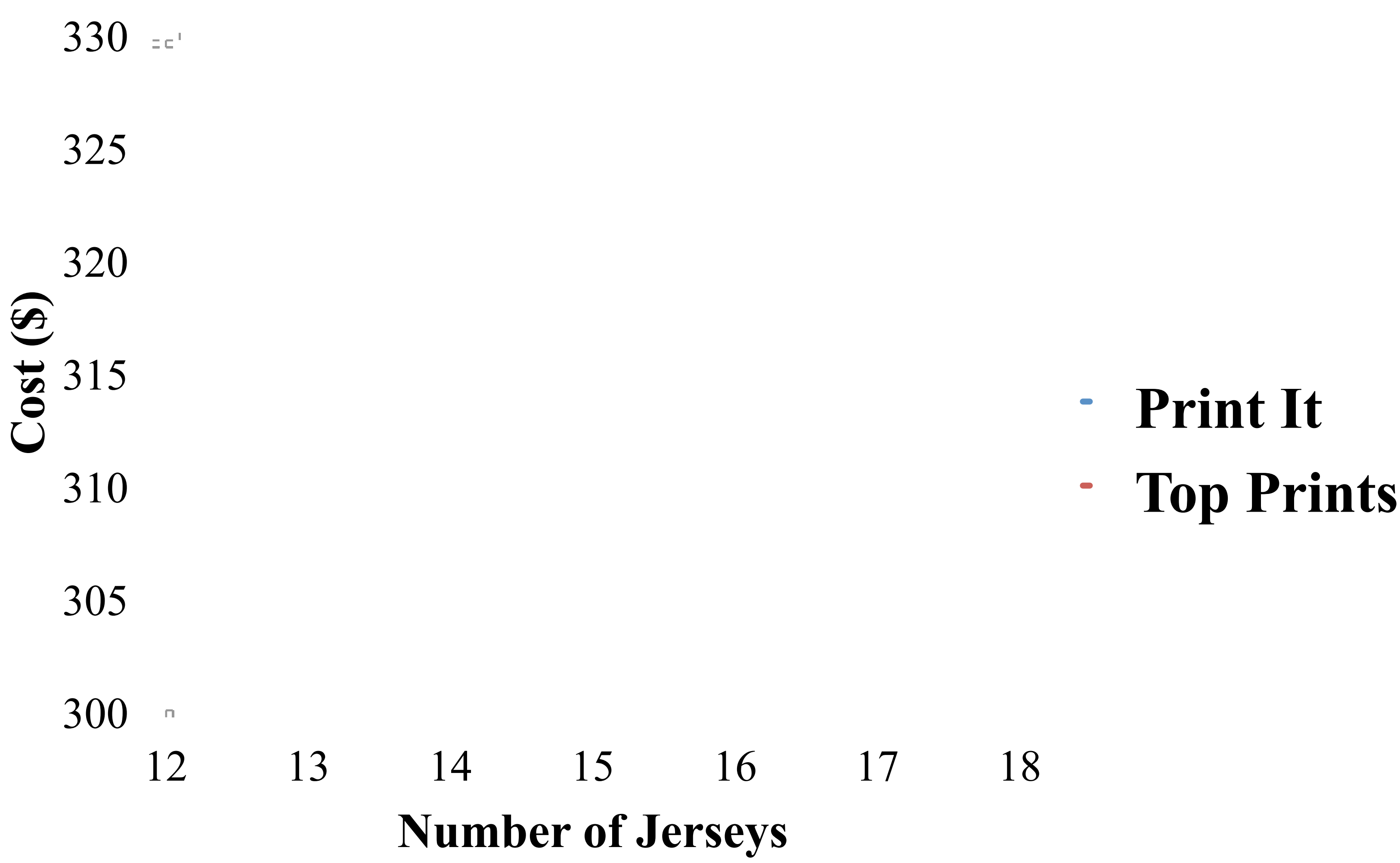
Sharing Individual Solutions

1. Take turns to share your work.

2. Describe how you did the task and how you think it could be improved.

3. If explanations are unclear, ask questions until everyone in the group understands the individual solutions.

Graph Showing Cost of Jerseys



Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions
of these materials in their classrooms, to their students, and to
Judith Mills, Carol Hill, and Alvaro Villanueva who contributed to the design.

This development would not have been possible without the support of
Bill & Melinda Gates Foundation
We are particularly grateful to
Carina Wong, Melissa Chabran, and Jamie McKee

Grade 8 Mathematics Sample ER Item Form Claim 3

MAT.08.ER.3.000EE.E.138 Claim 3

Sample Item ID:	MAT.08.ER.3.000EE.E.138
Grade:	08
Primary Claim:	**Claim 3: Communicating Reasoning** Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	The Number System
Secondary Content Domain(s):	Expressions and Equations
Assessment Target(s):	3 E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is. 3 A: Test propositions or conjectures with specific examples. 1 B: Work with radicals and integer exponents.
Standard(s):	8.EE.2
Mathematical Practice(s):	3
DOK:	3
Item Type:	ER
Score Points:	2
Difficulty:	L
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-specific Attributes (e.g., accessibility issues):	
Notes:	Part of PT set

Ashley and Brandon have different methods for finding square roots.

Ashley's Method

To find the square root of x, find a number so that the product of the number and itself is x. For example, $2 \cdot 2 = 4$, so the square root of 4 is 2.

Brandon's Method

To find the square root of x, multiply x by $\frac{1}{2}$. For example, $4 \cdot \frac{1}{2} = 2$, so the square root of 4 is 2.

Which student's method is **not** correct?

- ☐ Ashley's method
- ☐ Brandon's method

Explain why the method you selected is **not** correct.

Sample Top-Score Response:

Brandon's method is not correct.

Brandon's method works for the square root of 4, but it wouldn't work for the square root of 36. Half of 36 is 18, but the square root of 36 is 6 since 6 times 6 equals 36. Ashley describes the correct way to find the square root of a number.

Scoring Rubric:

Responses to this item will receive 0-2 points, based on the following:

2 points: The student shows a thorough understanding of how to identify correct reasoning regarding square roots. Each part of the response is complete and correct.

1 point: The student shows a partial understanding of how to identify correct reasoning regarding square roots. The student recognizes that Brandon's method is not correct but attempts to explain why Ashley's method is correct instead of showing why Brandon's method is not correct.

0 points: The student shows inconsistent or no understanding of how to identify correct reasoning regarding square roots. Responding only that Brandon's method is not correct is not sufficient to earn any points.

9^{TH}-12^{TH} GRADE

Mathematics

CURRICULUM & STANDARDS

Montana Mathematics K-12 Content Standards and Practices

From the Montana Office of Public Instruction:

GRADE LEVEL STANDARDS & PRACTICES

From the Ravalli County Curriculum Consortium Committee:

After each grade level:

Year Long Plan Samples

Unit Organizer Samples

Lesson Plan Samples

Assessment Sample

Resources

Standards for Mathematical Practice: Grades 9-12 Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
HS.MP.1. Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2. Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3. Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4. Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated ituation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

HS.MP.5. Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6. Attend to precision.	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
HS.MP.7. Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
HS.MP.8. Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Explanations and Examples Grades 9-12
Arizona Department of Education: Standards and Assessment Division

Montana Mathematics Grade 9-12 Content Standards

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Montana Mathematics Grades 9-12 Number and Quantity Content Standards

Numbers and Number Systems

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3, … Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (51/3)3 should be 5(1/3)3 = 51 = 5 and that 51/3 should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities

In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g. acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process might be called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Number and Quantity Overview

The Real Number System
- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

Quantities
- Reason quantitatively and use units to solve problems

The Complex Number System
- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities
- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

The Real Number System — N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.*
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $||c\mathbf{v}|| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2 × 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.

Montana Mathematics Grades 9-12 Algebra Content Standards

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling
Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Algebra Overview

Seeing Structure in Expressions
- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Functions
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations
- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities
- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Seeing Structure in Expressions — A-SSE

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

Arithmetic with Polynomials and Rational Expressions — A-APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations — A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Reasoning with Equations and Inequalities — A-REI

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

[1]The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Montana Mathematics Grades 9-12 Functions Content Standards

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates
Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematical Practices

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4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
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Functions Overview

Interpreting Functions
- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions
- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models
- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions
- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.★

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x–1)$ *for* $x \neq 1$.
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi – x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g. science, history, and culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Montana Mathematics Grades 9-12 Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

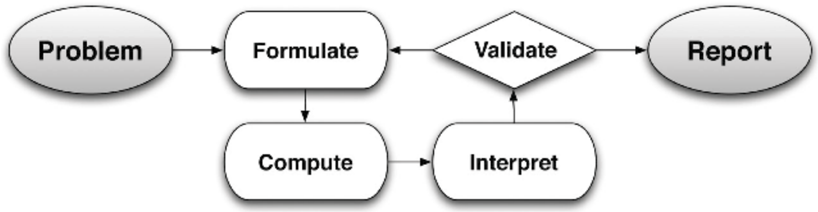
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Model the construction of a Montana American Indian tipi by considering the size of the tipi ring, number of lodge poles, and the angle between the pole and the ground.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO^2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards
Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Montana Mathematics Grades 9-12 Geometry Content Standards

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Various cultural contexts such as American Indian designs in beadwork, star quilts, and tipis provide rich opportunities to apply concepts and skills of geometry.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Mathematical Practices

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Geometry Overview

Congruence
- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry
- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Modeling with Geometry
- Apply geometric concepts in modeling situations

Expressing Geometric Properties with Equations
- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension
- Explain volume formulas and use them to solve problems
- Visualize relationships between two dimensional and three-dimensional objects

Circles
- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Congruence — G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions

12. Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry — G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles — G-C

Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations — G-GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci and directrices.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Geometric Measurement and Dimension — G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry — G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Montana Mathematics Grades 9-12 Statistics and Probability Content Standards

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling
Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data
- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions
- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Conditional Probability and the Rules of Probability
- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions
- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables, and Montana American Indian data sources to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables
5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions — S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability — S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data including information from Montana American Indian data sources when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Notes on Courses and Transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the United States today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Number & Quantity	The Real Number System	Extend the properties of exponents to rational exponents.	N.RN.1		Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 5^(1/3) to be the cube root of 5 because we want [5^(1/3)]^3 = 5^[(1/3) x 3] to hold, so [5^(1/3)]^3 must equal 5
Number & Quantity	The Real Number System	Extend the properties of exponents to rational exponents.	N.RN.2		Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Number & Quantity	The Real Number System	Use properties of rational and irrational numbers.	N.RN.3		Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Number & Quantity	Quantities	Reason quantitatively and use units to solve problems.	N.Q.1		Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Number & Quantity	Quantities	Reason quantitatively and use units to solve problems.	N.Q.2		Define appropriate quantities for the purpose of descriptive modeling.
Number & Quantity	Quantities	Reason quantitatively and use units to solve problems.	N.Q.3		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Number & Quantity	The Complex Number System	Perform arithmetic operations with complex numbers.	N.CN.1		Know there is a complex number i such that i^2 = ?1, and every complex number has the form a + bi with a and b real.
Number & Quantity	The Complex Number System	Perform arithmetic operations with complex numbers.	N.CN.2		Use the relation i^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
Number & Quantity	The Complex Number System	Perform arithmetic operations with complex numbers.	N.CN.3	+	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Number & Quantity	The Complex Number System	Represent complex numbers and their operations on the complex plane.	N.CN.4	+	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
Number & Quantity	The Complex Number System	Represent complex numbers and their operations on the complex plane.	N.CN.5	+	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (-1 ± ?3i)^3 = 8 because (-1 ±?3i) has modulus 2 and argument 120°.
Number & Quantity	The Complex Number System	Represent complex numbers and their operations on the complex plane.	N.CN.6	+	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Number & Quantity	The Complex Number System	Use complex numbers in polynomial identities and equations.	N.CN.7		Solve quadratic equations with real coefficients that have complex solutions.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Number & Quantity	The Complex Number System	Use complex numbers in polynomial identities and equations.	N.CN.8	+	Extend polynomial identities to the complex numbers. For example, rewrite x^2 + 4 as (x + 2i)(x - 2i).
Number & Quantity	The Complex Number System	Use complex numbers in polynomial identities and equations.	N.CN.9	+	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Number & Quantity	Vector & Matrix Quantities	Represent and model with vector quantities.	N.VM.1	+	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
Number & Quantity	Vector & Matrix Quantities	Represent and model with vector quantities.	N.VM.2	+	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
Number & Quantity	Vector & Matrix Quantities	Represent and model with vector quantities.	N.VM.3	+	Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors.
Number & Quantity	Vector & Matrix Quantities	Perform operations on vectors.	N.VM.4	+	**Add and subtract vectors.** *a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.* *b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.* *c. Understand vector subtraction v - w as v + (-w), where (-w) is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction*
Number & Quantity	Vector & Matrix Quantities	Perform operations on vectors.	N.VM.5	+	**Multiply a vector by a scalar.** *a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).* *b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v =? 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).*
Number & Quantity	Vector & Matrix Quantities	Perform operations on matrices & use matrices in applications.	N.VM.6	+	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
Number & Quantity	Vector & Matrix Quantities	Perform operations on matrices & use matrices in applications.	N.VM.7	+	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
Number & Quantity	Vector & Matrix Quantities	Perform operations on matrices & use matrices in applications.	N.VM.8	+	Add, subtract, and multiply matrices of appropriate dimensions.
Number & Quantity	Vector & Matrix Quantities	Perform operations on matrices & use matrices in applications.	N.VM.9	+	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
Number & Quantity	Vector & Matrix Quantities	Perform operations on matrices & use matrices in applications.	N.VM.10	+	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
Number & Quantity	Vector & Matrix Quantities	Perform operations on matrices & use matrices in applications.	N.VM.11	+	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
Number & Quantity	Vector & Matrix Quantities	Perform operations on matrices & use matrices in applications.	N.VM.12	+	Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Algebra	Seeing Structure in Expressions	Interpret the structure of expressions	A.SSE.1		**Interpret expressions that represent a quantity in terms of its context.*** *a. Interpret parts of an expression, such as terms, factors, and coefficients.* *b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.*
Algebra	Seeing Structure in Expressions	Interpret the structure of expressions	A.SSE.2		Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).
Algebra	Seeing Structure in Expressions	Write expressions in equivalent forms to solve problems	A.SSE.3		**Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.** *a. Factor a quadratic expression to reveal the zeros of the function it defines.* *b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.* *c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as [1.15^(1/12)]^(12t) ? 1.012^(12t) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
Algebra	Seeing Structure in Expressions	Write expressions in equivalent forms to solve problems	A.SSE.4		Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
Algebra	Arithmetic with Polynomials and Rational Functions	Perform arithmetic operations on polynomials	A.APR.1		Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Algebra	Arithmetic with Polynomials and Rational Functions	Understand the relationship between zeros and factors of polynomials	A.APR.2		Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).
Algebra	Arithmetic with Polynomials and Rational Functions	Understand the relationship between zeros and factors of polynomials	A.APR.3		Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Algebra	Arithmetic with Polynomials and Rational Functions	Use polynomial identities to solve problems	A.APR.4		Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2 can be used to generate Pythagorean triples.
Algebra	Arithmetic with Polynomials and Rational Functions	Use polynomial identities to solve problems	A.APR.5	+	Know and apply that the Binomial Theorem gives the expansion of (x + y)^n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)
Algebra	Arithmetic with Polynomials and Rational Functions	Rewrite rational expressions	A.APR.6		Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
Algebra	Arithmetic with Polynomials and Rational Functions	Rewrite rational expressions	A.APR.7	+	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Algebra	Creating Equations	Create equations that describe numbers or relationships	A.CED.1		Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
Algebra	Creating Equations	Create equations that describe numbers or relationships	A.CED.2		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
Algebra	Creating Equations	Create equations that describe numbers or relationships	A.CED.3		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
Algebra	Creating Equations	Create equations that describe numbers or relationships	A.CED.4		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.*
Algebra	Reasoning with Equations and Inequalities	Understand solving equations as a process of reasoning and explain the reasoning	A.REI.1		Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Algebra	Reasoning with Equations and Inequalities	Understand solving equations as a process of reasoning and explain the reasoning	A.REI.2		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Algebra	Reasoning with Equations and Inequalities	Solve equations and inequalities in one variable	A.REI.3		Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Algebra	Reasoning with Equations and Inequalities	Solve equations and inequalities in one variable	A.REI.4		**Solve quadratic equations in one variable.** *a. Use the method of completing the square to transforms any quadratic equation in x into an equation of the form (x - p)^2 = q that has the same solutions. Derive the quadratic formula from this form.* *b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.*
Algebra	Reasoning with Equations and Inequalities	Solve systems of equations	A.REI.5		Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Algebra	Reasoning with Equations and Inequalities	Solve systems of equations	A.REI.6		Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Algebra	Reasoning with Equations and Inequalities	Solve systems of equations	A.REI.7		Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle x^2 + y^2 = 3.
Algebra	Reasoning with Equations and Inequalities	Solve systems of equations	A.REI.8	+	Represent a system of linear equations as a single matrix equation in a vector variable.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Algebra	Reasoning with Equations and Inequalities	Solve systems of equations	A.REI.9	+	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
Algebra	Reasoning with Equations and Inequalities	Represent and solve equations and inequalities graphically	A.REI.10		Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Algebra	Reasoning with Equations and Inequalities	Represent and solve equations and inequalities graphically	A.REI.11		Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
Algebra	Reasoning with Equations and Inequalities	Represent and solve equations and inequalities graphically	A.REI.12		Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Functions	Interpreting Functions	Understand the concept of a function and use function notation	F.IF.1		Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
Functions	Interpreting Functions	Understand the concept of a function and use function notation	F.IF.2		Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
Functions	Interpreting Functions	Understand the concept of a function and use function notation	F.IF.3		Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ? 1 (n is greater than or equal to 1).
Functions	Interpreting Functions	Interpret functions that arise in applications in terms of the context	F.IF.4		For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
Functions	Interpreting Functions	Interpret functions that arise in applications in terms of the context	F.IF.5		Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
Functions	Interpreting Functions	Interpret functions that arise in applications in terms of the context	F.IF.6		Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
Functions	Interpreting Functions	Analyze functions using different representations	F.IF.7	only 7d	**Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*** *a. Graph linear and quadratic functions and show intercepts, maxima, and minima.* *b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.* *c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.* *d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. (+)* *e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
Functions	Interpreting Functions	Analyze functions using different representations	F.IF.8		**Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.** *a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.* *b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^(12t), y = (1.2)^(t/10), and classify them as representing exponential growth or decay.*
Functions	Interpreting Functions	Analyze functions using different representations	F.IF.9		Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Functions	Building Functions	Build a function that models a relationship between two quantities	F.BF.1	only 1c	**Write a function that describes a relationship between two quantities.*** *a. Determine an explicit expression, a recursive process, or steps for calculation from a context.* *b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* *c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. (+)*
Functions	Building Functions	Build a function that models a relationship between two quantities	F.BF.2		Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.*
Functions	Building Functions	Build new functions from existing functions	F.BF.3		Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
Functions	Building Functions	Build new functions from existing functions	F.BF.4	only 4b, 4c & 4d	*a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2(x^3) for x > 0 or f(x) = (x+1)/(x-1) for x =? 1 (x not equal to 1).* *b. Verify by composition that one function is the inverse of another. (+)* *c. Read values of an inverse function from a graph or a table, given that the function has an inverse. (+)* *d. Produce an invertible function from a non-invertible function by restricting the domain. (+)*
Functions	Building Functions	Build new functions from existing functions	F.BF.5	+	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Functions	Linear, Quadratic, and Exponential Models	Construct and compare linear and exponential models and solve problems	F.LE.1		**Distinguish between situations that can be modeled with linear functions and with exponential functions.*** *a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.** *b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.** *c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.**
Functions	Linear, Quadratic, and Exponential Models	Construct and compare linear and exponential models and solve problems	F.LE.2		Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
Functions	Linear, Quadratic, and Exponential Models	Construct and compare linear and exponential models and solve problems	F.LE.3		Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*
Functions	Linear, Quadratic, and Exponential Models	Construct and compare linear and exponential models and solve problems	F.LE.4		For exponential models, express as a logarithm the solution to ab^(ct) = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Functions	Linear, Quadratic, and Exponential Models	Interpret expressions for functions in terms of the situation they model	F.LE.5		Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.*
Functions	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle	F.TF.1		Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
Functions	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle	F.TF.2		Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
Functions	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle	F.TF.3	+	Use special triangles to determine geometrically the values of sine, cosine, tangent for (pi)/3, (pi)/4 and (pi)/6, and use the unit circle to express the values of sine, cosine, and tangent for x, [(pi) + x], and [2(pi) - x] in terms of their values for x, where x is any real number.
Functions	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle	F.TF.4	+	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Functions	Trigonometric Functions	Model periodic phenomena with trigonometric functions	F.TF.5		Choose trigonometric functions to model periodic phenomena from a variety of contexts (e.g. science, history, and culture, including those of the Montana American Indian) with specified amplitude, frequency, and midline.*
Functions	Trigonometric Functions	Model periodic phenomena with trigonometric functions	F.TF.6	+	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
Functions	Trigonometric Functions	Model periodic phenomena with trigonometric functions	F.TF.7	+	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*
Functions	Trigonometric Functions	Prove and apply trigonometric identities	F.TF.8		Prove the Pythagorean identity (sin A)^2 + (cos A)^2 = 1 and use it to calculate trigonometric ratios.
Functions	Trigonometric Functions	Prove and apply trigonometric identities	F.TF.9	+	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Geometry	Congruence	Experiment with transformations in the plane	G.CO.1		Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
Geometry	Congruence	Experiment with transformations in the plane	G.CO.2		Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
Geometry	Congruence	Experiment with transformations in the plane	G.CO.3		Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
Geometry	Congruence	Experiment with transformations in the plane	G.CO.4		Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
Geometry	Congruence	Experiment with transformations in the plane	G.CO.5		Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Geometry	Congruence	Understand congruence in terms of rigid motions	G.CO.6		Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
Geometry	Congruence	Understand congruence in terms of rigid motions	G.CO.7		Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
Geometry	Congruence	Understand congruence in terms of rigid motions	G.CO.8		Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Geometry	Congruence	Prove geometric theorems	G.CO.9		Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
Geometry	Congruence	Prove geometric theorems	G.CO.10		Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
Geometry	Congruence	Prove geometric theorems	G.CO.11		Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
Geometry	Congruence	Make geometric constructions	G.CO.12		Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
Geometry	Congruence	Make geometric constructions	G.CO.13		Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Geometry	Similarity, Right Triangles, and Trigonometry	Understand similarity in terms of similarity transformations	G.SRT.1		Verify experimentally the properties of dilations given by a center and a scale factor: -- a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. -- b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Geometry	Similarity, Right Triangles, and Trigonometry	Understand similarity in terms of similarity transformations	G.SRT.2		Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
Geometry	Similarity, Right Triangles, and Trigonometry	Understand similarity in terms of similarity transformations	G.SRT.3		Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
Geometry	Similarity, Right Triangles, and Trigonometry	Prove theorems involving similarity	G.SRT.4		Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
Geometry	Similarity, Right Triangles, and Trigonometry	Prove theorems involving similarity	G.SRT.5		Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Geometry	Similarity, Right Triangles, and Trigonometry	Define trigonometric ratios and solve problems involving right triangles	G.SRT.6		Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Geometry	Similarity, Right Triangles, and Trigonometry	Define trigonometric ratios and solve problems involving right triangles	G.SRT.7		Explain and use the relationship between the sine and cosine of complementary angles.
Geometry	Similarity, Right Triangles, and Trigonometry	Define trigonometric ratios and solve problems involving right triangles	G.SRT.8		Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Geometry	Similarity, Right Triangles, and Trigonometry	Apply trigonometry to general triangles	G.SRT.9	+	Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
Geometry	Similarity, Right Triangles, and Trigonometry	Apply trigonometry to general triangles	G.SRT.10	+	Prove the Laws of Sines and Cosines and use them to solve problems.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Geometry	Similarity, Right Triangles, and Trigonometry	Apply trigonometry to general triangles	G.SRT.11	+	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Geometry	Circles	Understand and apply theorems about circles	G.C.1		Prove that all circles are similar.
Geometry	Circles	Understand and apply theorems about circles	G.C.2		relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Geometry	Circles	Understand and apply theorems about circles	G.C.3		Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Geometry	Circles	Understand and apply theorems about circles	G.C.4	+	Construct a tangent line from a point outside a given circle to the circle.
Geometry	Circles	Find arc lengths and areas of sectors of circles	G.C.5		Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
Geometry	Expressing Geometric Prop's. with Equations	Translate between the geometric description and the equation for a conic section	G.GPE.1		Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Geometry	Expressing Geometric Prop's. with Equations	Translate between the geometric description and the equation for a conic section	G.GPE.2		Derive the equation of a parabola given a focus and directrix.
Geometry	Expressing Geometric Prop's. with Equations	Translate between the geometric description and the equation for a conic section	G.GPE.3	+	Derive the equations of ellipses and hyperbolas given the foci.
Geometry	Expressing Geometric Prop's. with Equations	Use coordinates to prove simple geometric theorems algebraically	G.GPE.4		Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, sqrt3) lies on the circle centered at the origin and containing the point (0, 2)
Geometry	Expressing Geometric Prop's. with Equations	Use coordinates to prove simple geometric theorems algebraically	G.GPE.5		Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
Geometry	Expressing Geometric Prop's. with Equations	Use coordinates to prove simple geometric theorems algebraically	G.GPE.6		Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
Geometry	Expressing Geometric Prop's. with Equations	Use coordinates to prove simple geometric theorems algebraically	G.GPE.7		Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
Geometry	Geometric Measurement and Dimension	Explain volume formulas and use them to solve problems	G.GMD.1		Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
Geometry	Geometric Measurement and Dimension	Explain volume formulas and use them to solve problems	G.GMD.2	+	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Geometry	Geometric Measurement and Dimension	Explain volume formulas and use them to solve problems	G.GMD.3		Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
Geometry	Geometric Measurement and Dimension	Visualize relationships between two-dimensional and three-dimensional objects	G.GMD.4		Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Geometry	Geometric Measurement and Dimension	Apply geometric concepts in modeling situations	G.MG.1		Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder modeling a Montana American Indian tipi as a cone.)★
Geometry	Modeling with Geometry	Apply geometric concepts in modeling situations	G.MG.2		Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
Geometry	Modeling with Geometry	Apply geometric concepts in modeling situations	G.MG.3		Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). *

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Statistics & Probability	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable	S.ID.1		Represent data with plots on the real number line (dot plots, histograms, and box plots).
Statistics & Probability	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable	S.ID.2		Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
Statistics & Probability	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable	S.ID.3		Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
Statistics & Probability	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable	S.ID.4		Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables, and Montana American Indian data sources to estimate areas under the normal curve.
Statistics & Probability	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on two categorical and quantitative variables	S.ID.5		Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
Statistics & Probability	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on two categorical and quantitative variables	S.ID.6		**Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.** *a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* *b. Informally assess the fit of a function by plotting and analyzing residuals.* *c. Fit a linear function for a scatter plot that suggest a linear association.*
Statistics & Probability	Interpreting Categorical and Quantitative Data	Interpret linear models	S.ID.7		Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
Statistics & Probability	Interpreting Categorical and Quantitative Data	Interpret linear models	S.ID.8		Compute (using technology) and interpret the correlation coefficient of a linear fit
Statistics & Probability	Interpreting Categorical and Quantitative Data	Interpret linear models	S.ID.9		Distinguish between correlation and causation

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Statistics & Probability	Making Inferences and Justifying Conclusions	Understand and evaluate random processes underlying statistical experiments	S.IC.1		Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
Statistics & Probability	Making Inferences and Justifying Conclusions	Understand and evaluate random processes underlying statistical experiments	S.IC.2		Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0. 5. Would a result of 5 tails in a row cause you to question the model?
Statistics & Probability	Making Inferences and Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies	S.IC.3		Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
Statistics & Probability	Making Inferences and Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies	S.IC.4		Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
Statistics & Probability	Making Inferences and Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies	S.IC.5		Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
Statistics & Probability	Making Inferences and Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies	S.IC.6		Evaluate reports based on data.
Statistics & Probability	Conditional Probability and the Rules of Probability	Understand independence and conditional probability and use them to interpret data	S.CP.1		Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
Statistics & Probability	Conditional Probability and the Rules of Probability	Understand independence and conditional probability and use them to interpret data	S.CP.2		Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
Statistics & Probability	Conditional Probability and the Rules of Probability	Understand independence and conditional probability and use them to interpret data	S.CP.3		Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
Statistics & Probability	Conditional Probability and the Rules of Probability	Understand independence and conditional probability and use them to interpret data	S.CP.4		Construct and interpret two-way frequency tables of data including information from Montana American Indian data sources when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Statistics & Probability	Conditional Probability and the Rules of Probability	Understand independence and conditional probability and use them to interpret data	S.CP.5		Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
Statistics & Probability	Conditional Probability and the Rules of Probability	Use the rules of probability to compute probabilities of compound events in a uniform probability model	S.CP.6		Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
Statistics & Probability	Conditional Probability and the Rules of Probability	Use the rules of probability to compute probabilities of compound events in a uniform probability model	S.CP.7		Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.
Statistics & Probability	Conditional Probability and the Rules of Probability	Use the rules of probability to compute probabilities of compound events in a uniform probability model	S.CP.8	+	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]*[P(B|A)] =[P(B)]*[P(A|B)], and interpret the answer in terms of the model.
Statistics & Probability	Conditional Probability and the Rules of Probability	Use the rules of probability to compute probabilities of compound events in a uniform probability model	S.CP.9	+	Use permutations and combinations to compute probabilities of compound events and solve problems.
Statistics & Probability	Using Probability to Make Decisions	Calculate expected values and use them to solve problems	S.MD.1	+	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
Statistics & Probability	Using Probability to Make Decisions	Calculate expected values and use them to solve problems	S.MD.2	+	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
Statistics & Probability	Using Probability to Make Decisions	Calculate expected values and use them to solve problems	S.MD.3	+	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
Statistics & Probability	Using Probability to Make Decisions	Calculate expected values and use them to solve problems	S.MD.4	+	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many T V sets would you expect to find in 100 randomly selected households?

Conceptual Category	Domain	Cluster	Code	Exceeds CCR?	Standard
Statistics & Probability	Using Probability to Make Decisions	Use probability to evaluate outcomes of decisions	S.MD.5	+	**Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.** *a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* *b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low- deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
Statistics & Probability	Using Probability to Make Decisions	Use probability to evaluate outcomes of decisions	S.MD.6	+	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
Statistics & Probability	Using Probability to Make Decisions	Use probability to evaluate outcomes of decisions	S.MD.7	+	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

SAMPLE YEAR LONG PLAN

COURSE: Algebra 1

Unit	Intro	Linear equations and inequalities on one variable	Linear equations and inequalities on two variables	Systems
(Time)	(1ST SEMESTER)	(1ST SEMESTER)	(1ST SEMESTER)	(1ST SEMESTER)
STANDARDS	A.SSE.1	A.CED.1	A.CED.2	A.CED.3
		A.CED.4	A.CED.3	A.REI.5
		A.REI.1	A.REI.10	A.REI.6
		A.REI.3	A.REI.12	A.REI.11
			F.IF.5	F.IF.5
			F.IF.6	
			F.IF.7	
			F.BF.1	
			F.BF.2	
			F.LE.1	
			F.LE.2	
			S.ID.8	
			S.ID.9	
			N.Q.1	
			N.Q.2	
			N.Q.3	
			F.LE.5	
				solve by graphing, substitution, linear combos
			scatter plots (correlation, residuals)	
		solving all types (equations and inequalities)		
	Expressions			modeling
			graphing (table, intercepts, slope-intercept)	
	fractions, decimals			
				determining whether a point is a solution
	properties (dist, assoc, comm, id,)	literal equations (rearranging)	modeling	
			zeros (solving, x-int)	
	writing expressions	writing equations	sequences	
	combining like terms		slope (from table, graph, points)	
			write equations	

Key:

- green = major concept
- blue = supporting concept
- yellow = additional concept

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Functions (linear->quadratics)	factoring/ operations with Quadratics	solving quadratics	Statistics
(1ST SEMESTER/2ND SEMESTER)	(2ND SEMESTER)	(2ND SEMESTER)	(2ND SEMESTER)
F.IF.1	A.SSE.2	A.SSE.3	S.ID.1
F.IF.2	A.SSE.3	A.APR.3	S.ID.2
F.IF.3	A.APR.1	A.REI.2	S.ID.3
F.IF.4	F.IF.8	A.REI.4	S.ID.5
F.IF.5		A.REI.10	S.ID.6
		F.IF.5	S.ID.7
		F.IF.7	S.ID.8
		F.IF.8	S.ID.9
		F.BF.3	
		N.Q.1	
		N.Q.2	
		N.Q.3	
		N.CN.1	
		N.CN.2	
		N.CN.7	
	exponent properties		
	operations with poly's		
sequences	factoring/completing the square	modeling	residuals, correlation coefficient
domain/range		solving (zeros, quadratic formula, isolate x)	measures of center,spread, outliers
notations		graphing	graphs (histo, bar, box, stem)
increasing/decreasing		extraneous solutions/complex solutions	
max/min			fitting functions to data (correlation/causation)

SAMPLE YEAR LONG PLAN

COURSE: ALGEBRA II

Key:

green = major concept
blue = supporting concept
yellow = additional concept

Unit	Intro (quadratics)	Rational and Polynomial Expressions	Functions	Exponential Functions
(Time)	(1st Semester)	(1st Semester)	(1st Semester)	(1st/2nd Semester)
STANDARDS	A.SSE.1	N.RN.3	N.RN.1	N.RN.1
		N.CN.1	N.RN.2	N.RN.2
		N.CN.2	A.SSE.1	A.SSE.3C
		N.CN.7	A.SSE.1A	A.SSE.4
		A.SSE.1	A.SSE.1B	A.REI.11
		A.SSE.1AB	A.REI.11	F.IF.3
		A.SSE.2	F.IF4	F.IF.7E
		A.APR.1	F.IF.7B	F.IF.8B
		A.APR.2	F.IF.7C	F.BF.2
		A.APR.3	F.BF.1	F.BF. 4A
		A.APR.4	F.BF.1AB	F.LE.1
		A.APR.6	F.BF.3	F.LE.1ABC
		A.REI.2	F.LE.3	F.LE.2
		A.REI.11		F.LE.3
				F.LE.4
				F.LE.5

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Modelling	Trig Function	Statistics	Probability	Modelling
(throughout year)	(2nd Semester)	(2nd Semester)	(2nd Semester)	(9 DAYS)
	F.IF.7E	S.IC.1	S.CP.1	
	F.TF.1	S.IC.2	S.CP.2	
	F.TF.2	S.IC.3	S.CP.3	
	F.TF.5	S.IC.4	S.CP.4	
	F.TF.8	S.IC.5	S.CP.5	
		S.IC.6	S.CP.6	
		S.ID.4	S.CP.7	

SAMPLE YEAR LONG PLAN

COURSE: H.S. GEOMETRY

Unit	Unit 1- Intro and Constructions	Unit 2- Definitions, Transformations, and Congruence	Unit 3 - Proof of Geometric Theorems	Unit 4- Modeling Project	Unit 5- Similarity
(Time)	(10 days)	(25 days)	(25 days)	(5 days)	(20 days)
STANDARDS	G-CO 12	G-CO 1	G-CO 9	G-MG 1	G-SRT 1
	G-CO 13	G-CO 2	G-CO 10	HS.MP.4	G-SRT 2
		G-CO 3	G-CO 11		G-SRT 3
		G-CO 4			G-SRT 4
		G-CO 5			G-SRT 5
		G-CO 6			
		G-CO 7			
		G-CO 8			
		Involve Modeling Topics Throughout			
		G-MG 1			
		G-MG 2			
		G-MG 3			

Standards Priorities
Major
Supporting
Additional

H.S. Geometry p.2

Unit 6 - Circles and Conics	Unit 7 - Coordinate Geometry	Unit 8 - Trigonometry	Unit 9 - Modeling Project	Unit 10 - Geometric Measurement and
(20 days)	(15 days)	(20 days)	(5 days)	(15 days)
G-C 1	G-GPE 4	G-SRT 6	G-MG 2	G-GMD 1
G-C 2	G-GPE 5	G-SRT 7	G-MG 3	G-GMD 3
G-C 3	G-GPE 6	G-SRT 8	HS.MP.2	G-GMD 4
G-C 4	G-GPE 7	G-SRT 9	HS.MP.5	
G-C 5		G-SRT 10		
G-GPE 1		G-SRT 11		
G-GPE 2				
Involve Modeling Topics Throughout	Involve Modeling Topics Throughout	Involve Modeling Topics Throughout		
G-MG 1	G-MG 1	G-MG 1		
G-MG 2	G-MG 2	G-MG 2		
G-MG 3	G-MG 3	G-MG 3		

Algebra 1 Lesson Plan Scatterplots

Teacher: Ravalli Curriculum Consortium

Date: June 12, 2013

Subject / grade level: Algebra 1

Access to USGS website (or STREAMFLOW APP) to collect stream flow data.

Essential Standards and Clarifying Objectives: F.IF.6 A.CED.1 A.CED.2 F.LE.2 A.REI.10 S.ID.8 S.ID.9

Lesson objective(s): Students will be able to collect data, calculate rate of change, calculate a line of fit to model the date and use the model to make predictions.

Differentiation strategies to meet diverse learner needs:
Intervention - small group work

Extension - collect different data from a variety of sources and situations (time/distance, height/weight, time/population)

ENGAGEMENT

- **Students collect data, plot the data and discuss correlation. Calculate slope as a rate of change and discuss in terms of the context. Calculate a line of fit and use it to make predictions. Discuss the different predictions and see which are reasonable according to the data.**

EXPLORATION

1. **Collect data everyday for 10 days. Plot data and label axes.**
2. **Calculate the slope (using points of your choice).**
3. **Calculate the equation of the line of fit for the points you chose.**
4. **Use your equation to make predictions.**
5. **Discuss results.**

EXPLANATION

Question: ***What does the slope mean in terms of the situation? What does the y-intercept represent in this situation? How do the slopes vary within groups? What is the line of fit? Is it a good fit? Explain your reasoning using complete sentences.***

ELABORATION

- **Discuss the different predictions and see which are reasonable according to the data.**
- **Creatively present your project!**
- **How could this be extended to other data sets?**

EVALUATION

- **Students will be able to precisely calculate slope and a line of fit.**
- **Students are able to accurately articulate their reasoning and process throughout the investigation.**

Algebra 1 **Name:** ______________________________

Scatterplots WS

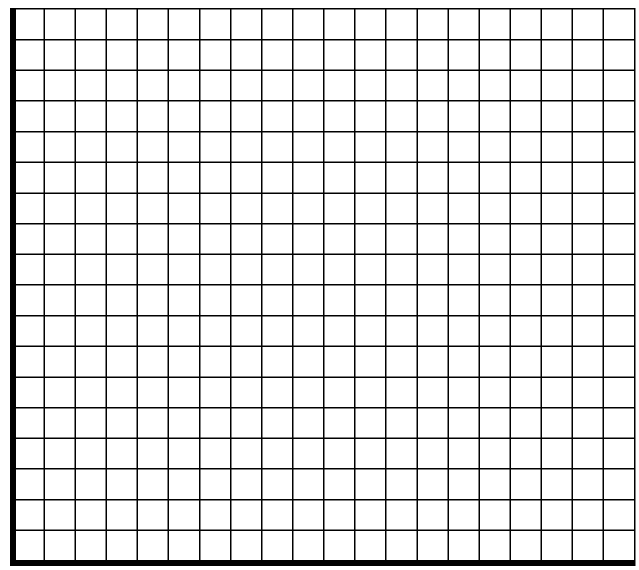
The table below shows the time (days after Nov 29th 2010) and the discharge (In cubic feet per second) of the Blue River below Dillon Reservoir.

Time (days)	0	1	2	3	4	5
CFS	26	28	29	30	33	34

1.) What is the independent variable (x)? ______________________

2.) What is the dependent variable (y)? ___________________

3.) Plot the points on the graph below. Choose an appropriate scale and label each axis. Then draw the line of best fit.



4.) Write the equation of the line of best fit

a.) What are the 2 points you will use?

__________ and __________

b.) Use those 2 points to write the equation of the line. SHOW WORK!

5.) Does your scatterplot have a positive or negative correlation? _______________

6.) What is the slope? ___________
Explain what the slope of this line tells us about the situation. Be specific and use complete sentences.

7.) What are the coordinates of the y-intercept? ____________
Explain what the y-intercept tells us in this situation.

8.) Use your equation to find the discharge (cubic feet per second) of the Blue River 20 days after Nov. 29th 2010.

9.) How many days after Nov. 29th will the discharge of the Blue River be 156 cubic feet per second?

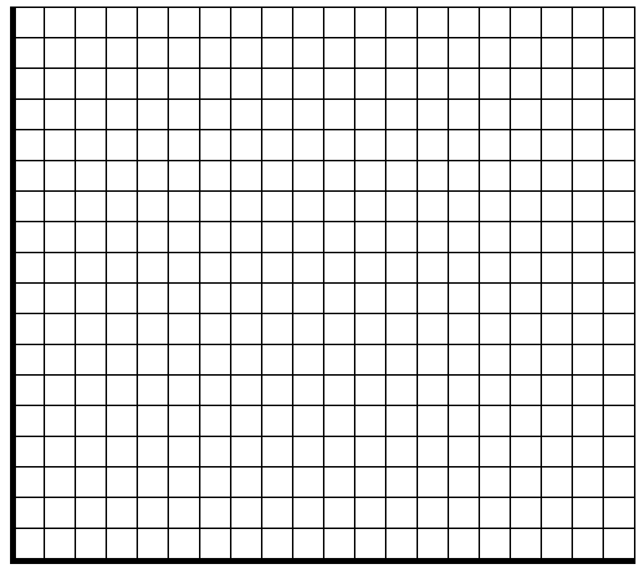
The table below shows the time (days after Dec 1st 2010) and the discharge (In cubic feet per second) of the South Platte at Waterton Canyon.

Time (days)	0	1	2	3	4	5
CFS	156	154	145	140	133	121

1.) What is the independent variable (x)? ________________________

2.) What is the dependent variable (y)? ____________________

3.) Plot the points on the graph below. Choose an appropriate scale and label each axis. Then draw the line of best fit.



4.) Write the equation of the line of best fit

a.) What are the 2 points you will use?

___________ and ___________

b.) Use those 2 points to write the equation of the line. SHOW WORK!

5.) Does your scatterplot have a positive or negative correlation? ________________

6.) What is the slope? ____________
Explain what the slope of this line tells us about the situation. Be specific and use complete sentences.

7.) What are the coordinates of the y-intercept? ______________
Explain what the y-intercept tells us in this situation.

8.) Use your equation to find the discharge (cubic feet per second) of the South Platte 15 days after Dec 1st 2010.

9.) How many days after Dec 1st will the discharge of the South Platte be 0 cubic feet per second? Does this make sense? Why or why not?

APPENDIX

5 E’s Lesson Plan- Blank

Unit Map- Blank

Lesson Plan Template- 5 E's

Teacher:
Date:
Subject area / course / grade level:
Materials:
Practice Standards:
Content Standards:
Lesson Objectives:
ENGAGEMENT • Describe how the teacher will capture students' interest. • What kind of questions should the students ask themselves after the engagement?
EXPLORATION • Describe what hands-on/minds-on activities students will be doing. • List "big idea" conceptual questions the teacher will use to encourage and/or focus students' exploration
EXPLANATION • Student explanations should precede introduction of terms or explanations by the teacher. What questions or techniques will the teacher use to help students connect their exploration to the concept under examination?
ELABORATION • Describe how students will develop a more sophisticated understanding of the concept.
EVALUATION • How will students demonstrate that they have achieved the lesson objective? • This should be embedded throughout the lesson as well as at the end of the lesson

TEACHER: ______________________________ SCHOOL DISTRICT/BUILDING: ______________________________

COURSE: ______________________________ GRADE LEVEL(S): ______________________________

LAST UNIT	**CURRENT UNIT**	NEXT UNIT

UNIT SCHEDULE

is about... UNIT MAP

UNIT SELF TEST QUESTIONS

MATH STANDARDS
