TEST SPECIFICATIONS: SAT MATH TEST

A Transparent Blueprint

This section describes the content, format, and distinctive new features of the Math Test in the redesigned SAT, as well as the skills it measures. This section also includes annotated sample questions that help illustrate central aspects of the test.

OVERALL CLAIM FOR THE TEST

The redesigned SAT's Math Test is intended to collect evidence in support of the following claim about student performance:

> Students have fluency with, understanding of, and the ability to apply the mathematical concepts, skills, and practices that are most strongly prerequisite and central to their ability to progress through a range of college courses, career training, and career opportunities.

TEST DESCRIPTION

In keeping with the evidence about essential requirements for college and career readiness described in Section II, the redesigned SAT requires a stronger command of fewer, more important topics. To succeed on the redesigned SAT, students will need to exhibit mathematical practices, such as problem solving and using appropriate tools strategically. The SAT also provides opportunities for richer applied problems.

The redesigned SAT's Math Test has four content areas:

- Heart of Algebra
- Problem Solving and Data Analysis
- Passport to Advanced Math
- Additional Topics in Math

Questions in each content area span the full range of difficulty and address relevant practices, fluency, and conceptual understanding.

Test Summary

The following table summarizes the key content dimensions of the redesigned SAT's Math Test.

SAT MATH TEST CONTENT SPECIFICATIONS		
Time Allotted	80 minutes	
Calculator Portion (38 questions)	55 minutes	
No-Calculator Portion (20 questions)	25 minutes	
	NUMBER	**PERCENTAGE OF TEST**
Total Items	58 questions	100%
Multiple Choice (MC, 4 options)	45 questions	78%
Student-Produced Response (SPR — grid-in)	13 questions	22%
Contribution of Items to Subscores		
Heart of Algebra	19 questions	33%
Analyzing and fluently solving linear equations and systems of linear equations		
Creating linear equations and inequalities to represent relationships between quantities and to solve problems		
Understanding and using the relationship between linear equations and inequalities and their graphs to solve problems		
Problem Solving and Data Analysis	17 questions	29%
Creating and analyzing relationships using ratios, proportional relationships, percentages, and units		
Representing and analyzing quantitative data		
Finding and applying probabilities in context		
Passport to Advanced Math	16 questions	28%
Identifying and creating equivalent algebraic expressions		
Creating, analyzing, and fluently solving quadratic and other nonlinear equations		

SAT MATH TEST CONTENT SPECIFICATIONS		
Creating, using, and graphing exponential, quadratic, and other nonlinear functions		
Additional Topics in Math*	6 questions	10%
Solving problems related to area and volume		
Applying definitions and theorems related to lines, angles, triangles, and circles		
Working with right triangles, the unit circle, and trigonometric functions		
Contribution of Items to Cross-Test Scores		
Analysis in Science	8 questions	14%
Analysis in History/Social Studies	8 questions	14%

*Questions under Additional Topics in Math contribute to the total Math Test score but do not contribute to a subscore within the Math Test.

The test covers all mathematical practices, with an emphasis on problem solving, modeling, using appropriate tools strategically, and looking for and making use of structure to do algebra. The practices emphasized in the redesigned SAT are central to the demands of postsecondary work. Problem solving requires students to make sense of problems and persevere to solve them, a skill highly rated by postsecondary instructors (Conley et al., *Reaching the Goal*, 2011). Modeling stresses applications characteristic of the entire postsecondary curriculum. Students will be asked throughout high school, college, and careers to make choices about which tools to use in solving problems. Finally, structure is fundamental to algebra and to other more advanced mathematics.

As indicated in the test specifications above, the Math Test has two portions. One is a 55-minute portion comprising 38 questions for which students are allowed to use calculators to solve the problems. The other is a 25-minute portion comprising 20 questions for which students are not allowed to use calculators to solve the problems. The blueprint for each of these portions is shown below.

CALCULATOR PORTION

	Number of Questions	% of Test
Total Questions	**38**	**100%**
Multiple Choice (MC)	30	79%
Student-Produced Response (SPR — grid-in)	8	21%
Content Categories	**38**	**100%**
Heart of Algebra	11	29%
Problem Solving and Data Analysis	17	45%
Passport to Advanced Math	7	18%
Additional Topics in Math	3	8%
Time Allocated	55 minutes	

NO-CALCULATOR PORTION

	Number of Questions	% of Test
Total Questions	**20**	**100%**
Multiple Choice (MC)	15	75%
Student-Produced Response (SPR — grid-in)	5	25%
Content Categories	**20**	**100%**
Heart of Algebra	8	40%
Passport to Advanced Math	9	45%
Additional Topics in Math	3	15%
Time Allocated	25 minutes	

Detailed Description of the Content and Skills Measured by the SAT Math Test

The SAT has been redesigned to better align to what research shows students need to know and be able to do in order to be prepared for college and careers. This goal has led to a more focused SAT with a balance across fluency, conceptual understanding, and application. In these and other ways, such as embedding mathematical practices, the redesigned SAT is also a good reflection of college- and career-ready standards.

HEART OF ALGEBRA: LINEAR EQUATIONS AND FUNCTIONS

SAT HEART OF ALGEBRA DOMAIN	
Content Dimension	**Description**
Linear equations in one variable	1. Create and use linear equations in one variable to solve problems in a variety of contexts. 2. Create a linear equation in one variable, and when in context interpret solutions in terms of the context. 3. Solve a linear equation in one variable, making strategic use of algebraic structure. 4. For a linear equation in one variable, a. interpret a constant, variable, factor, or term in a context; b. determine the conditions under which the equation has no solution, a unique solution, or infinitely many solutions. 5. Fluently solve a linear equation in one variable.
Linear functions	Algebraically, a linear function can be defined by a linear expression in one variable or by a linear equation in two variables. In the first case, the variable is the input and the value of the expression is the output. In the second case, one of the variables is designated as the input and determines a unique value of the other variable, which is the output. 1. Create and use linear functions to solve problems in a variety of contexts. 2. Create a linear function to model a relationship between two quantities. 3. For a linear function that represents a context, a. interpret the meaning of an input/output pair, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; b. given an input value, find and/or interpret the output value using the given representation; c. given an output value, find and/or interpret the input value using the given representation, if it exists. 4. Make connections between verbal, tabular, algebraic, and graphical representations of a linear function by a. deriving one representation from the other; b. identifying features of one representation given another representation; c. determining how a graph is affected by a change to its equation. 5. Write the rule for a linear function given two input/output pairs or one input/output pair and the rate of change.
Linear equations in two variables	A linear equation in two variables can be used to represent a constraint or condition on two-variable quantities in situations where neither of the variables is regarded as an input or an output. A linear equation can also be used to represent a straight line in the coordinate plane. 1. Create and use a linear equation in two variables to solve problems in a variety of contexts. 2. Create a linear equation in two variables to model a constraint or condition on two quantities. 3. For a linear equation in two variables that represents a context, a. interpret a solution, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; b. given a value of one quantity in the relationship, find a value of the other, if it exists. 4. Make connections between tabular, algebraic, and graphical representations of a linear equation in two variables by a. deriving one representation from the other; b. identifying features of one representation given the other representation; c. determining how a graph is affected by a change to its equation. 5. Write an equation for a line given two points on the line, one point and the slope of the line, or one point and a parallel or perpendicular line.

SAT HEART OF ALGEBRA DOMAIN	
Content Dimension	**Description**
Systems of two linear equations in two variables	1. Create and use a system of two linear equations in two variables to solve problems in a variety of contexts. 2. Create a system of linear equations in two variables, and when in context interpret solutions in terms of the context. 3. Make connections between tabular, algebraic, and graphical representations of the system by deriving one representation from the other. 4. Solve a system of two linear equations in two variables, making strategic use of algebraic structure. 5. For a system of linear equations in two variables, a. interpret a solution, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; b. determine the conditions under which the system has no solution, a unique solution, or infinitely many solutions. 6. Fluently solve a system of linear equations in two variables.
Linear inequalities in one or two variables	1. Create and use linear inequalities in one or two variables to solve problems in a variety of contexts. 2. Create linear inequalities in one or two variables, and when in context interpret the solutions in terms of the context. 3. For linear inequalities in one or two variables, interpret a constant, variable, factor, or term, including situations where seeing structure provides an advantage. 4. Make connections between tabular, algebraic, and graphical representations of linear inequalities in one or two variables by deriving one from the other. 5. Given a linear inequality or system of linear inequalities, interpret a point in the solution set.

Algebra is the language of much of high school mathematics, and it is also an important prerequisite for advanced mathematics and postsecondary education in many subjects. The redesigned SAT focuses strongly on algebra and recognizes in particular the essentials of the subject that are most essential for success in college and careers. Heart of Algebra will assess students' ability to analyze, fluently solve, and create linear equations and inequalities. Students will also be expected to analyze and fluently solve equations and systems of equations using multiple techniques.

To assess full command of the material, these problems will vary significantly in form and appearance. Problems may be straightforward fluency exercises or may pose challenges of strategy or understanding, such as interpreting the interplay between graphical and algebraic representations or solving as a process of reasoning. Students will be required to demonstrate both procedural skill and a deeper understanding of the concepts that undergird linear equations and functions to successfully exhibit a command of the Heart of Algebra.

Mastering linear equations and functions has clear benefits to students. The ability to use linear equations to model scenarios and to represent unknown quantities is powerful across the curriculum in the postsecondary classroom as well as in the workplace. Further, linear equations and functions remain the bedrock upon which much of advanced mathematics is built. Consider, for example, that derivatives in calculus are used to approximate curves by straight lines and to approximate nonlinear functions by linear ones. Without a strong foundation in the core of algebra, much of this advanced work remains inaccessible.

PROBLEM SOLVING AND DATA ANALYSIS: PROPORTIONAL RELATIONSHIPS, PERCENTAGES, COMPLEX MEASUREMENTS, AND DATA INTERPRETATION AND SYNTHESIS

SAT PROBLEM SOLVING AND DATA ANALYSIS DOMAIN	
Content Dimension	**Description**
Ratios, rates, proportional relationships, and units	Items will require students to solve problems by using a proportional relationship between quantities, calculating or using a ratio or rate, and/or using units, derived units, and unit conversion. 1. Apply proportional relationships, ratios, rates, and units in a wide variety of contexts. Examples include but are not limited to scale drawings and problems in the natural and social sciences. 2. Solve problems involving a. derived units, including those that arise from products (e.g., kilowatt-hours) and quotients (e.g., population per square kilometer); b. unit conversion, including currency exchange and conversion between different measurement systems. 3. Understand and use the fact that when two quantities are in a proportional relationship, if one changes by a scale factor, then the other also changes by the same scale factor.
Percentages	1. Use percentages to solve problems in a variety of contexts. Examples include, but are not limited to, discounts, interest, taxes, tips, and percent increases and decreases for many different quantities. 2. Understand and use the relationship between percent change and growth factor (5% and 1.05, for example); include percentages greater than or equal to 100%.
One-variable data: distributions and measures of center and spread	1. Choose an appropriate graphical representation for a given data set. 2. Interpret information from a given representation of data in context. 3. Analyze and interpret numerical data distributions represented with frequency tables, histograms, dot plots, and boxplots. 4. For quantitative variables, calculate, compare, and interpret mean, median, and range. Interpret (but don't calculate) standard deviation. 5. Compare distributions using measures of center and spread, including distributions with different means and the same standard deviations and ones with the same mean and different standard deviations. 6. Understand and describe the effect of outliers on mean and median. 7. Given an appropriate data set, calculate the mean.
Two-variable data: models and scatterplots	1. Using a model that fits the data in a scatterplot, compare values predicted by the model to values given in the data set. 2. Interpret the slope and intercepts of the line of best fit in context. 3. Given a relationship between two quantities, read and interpret graphs and tables modeling the relationship. 4. Analyze and interpret data represented in a scatterplot or line graph; fit linear, quadratic, and exponential models. 5. Select a graph that represents a context, identify a value on a graph, or interpret information on the graph. 6. For a given function type (linear, quadratic, exponential), choose the function of that type that best fits given data. 7. Compare linear and exponential growth. 8. Estimate the line of best fit for a given scatterplot; use the line to make predictions.

SAT PROBLEM SOLVING AND DATA ANALYSIS DOMAIN	
Content Dimension	**Description**
Probability and conditional probability	Use one- and two-way tables, tree diagrams, area models, and other representations to find relative frequency, probabilities, and conditional probabilities. 1. Compute and interpret probability and conditional probability in simple contexts. 2. Understand formulas for probability and conditional probability in terms of frequency.
Inference from sample statistics and margin of error	1. Use sample mean and sample proportion to estimate population mean and population proportion. Utilize, but do not calculate, margin of error. 2. Interpret margin of error; understand that a larger sample size generally leads to a smaller margin of error.
Evaluating statistical claims: observational studies and experiments	1. With random samples, describe which population the results can be extended to. 2. Given a description of a study with or without random assignment, determine whether there is evidence for a causal relationship. 3. Understand why random assignment provides evidence for a causal relationship. 4. Understand why a result can be extended only to the population from which the sample was selected.

The redesigned SAT's Math Test has responded to the research evidence identifying what is essential for college readiness and success by focusing significantly on problem solving and data analysis: the ability to create a representation of a problem, consider the units involved, attend to the meaning of quantities, and know and use different properties of operations and objects. Problems in this category will require significant quantitative reasoning about ratios, rates, and proportional relationships and will place a premium on understanding and applying unit rate.

Interpreting and synthesizing data are widely applicable skills in postsecondary education and careers. In the redesigned SAT's Math Test, students will be expected to identify quantitative measures of center, the overall pattern, and any striking deviations from the overall pattern and spread in one or two different data sets. This includes recognizing the effects of outliers on the measures of center of a data set. In keeping with the need to stress widely applicable prerequisites, the redesigned SAT emphasizes applying core concepts and methods of statistics, rather than covering broadly a vast range of statistical techniques.

Finally, the redesigned SAT's Math Test emphasizes students' ability to apply math to solve problems in rich and varied contexts and features problems that require the application of problem solving and data analysis to solve problems in science, social studies, and career-related contexts.

PASSPORT TO ADVANCED MATH: ANALYZING ADVANCED EXPRESSIONS

SAT PASSPORT TO ADVANCED MATH DOMAIN	
Content Dimension	**Description**
Equivalent expressions	1. Make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including a. rewriting simple rational expressions; b. rewriting expressions with rational exponents and radicals; c. factoring polynomials. 2. Fluently add, subtract, and multiply polynomials.
Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to a. solve quadratic equations in one variable presented in a wide variety of forms; determine the conditions under which a quadratic equation has no real solutions, one real solution, or two real solutions; b. solve simple rational and radical equations in one variable; c. identify when the procedures used to solve a simple rational or radical equation in one variable lead to an equation with solutions that do not satisfy the original equation (extraneous solutions); d. solve polynomial equations in one variable that are written in factored form; e. solve linear absolute value equations in one variable; f. solve systems of linear and nonlinear equations in two variables, including relating the solutions to the graphs of the equations in the system. 2. Given a nonlinear equation in one variable that represents a context, interpret a solution, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage. 3. Given an equation or formula in two or more variables that represents a context, view it as an equation in a single variable of interest where the other variables are parameters and solve for the variable of interest. 4. Fluently solve quadratic equations in one variable, written as a quadratic expression in standard form equal to zero, where using the quadratic formula or completing the square is the most efficient method for solving the equation.

SAT PASSPORT TO ADVANCED MATH DOMAIN	
Content Dimension	**Description**
Nonlinear functions	1. Create and use quadratic or exponential functions to solve problems in a variety of contexts. 2. For a quadratic or exponential function, a. identify or create an appropriate function to model a relationship between quantities; b. use function notation to represent and interpret input/output pairs in terms of a context and points on the graph; c. for a function that represents a context, interpret the meaning of an input/output pair, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; d. determine the most suitable form of the expression representing the output of the function to display key features of the context, including i. selecting the form of a quadratic that displays the initial value, the zeros, or the extreme value; ii. selecting the form of an exponential that displays the initial value, the end-behavior (for exponential decay), or the doubling or halving time; e. make connections between tabular, algebraic, and graphical representations of the function by i. given one representation, selecting another representation; ii. identifying features of one representation given another representation, including maximum and minimum values of the function; iii. determining how a graph is affected by a change to its equation, including a vertical shift or scaling of the graph. 3. For a factorable or factored polynomial or simple rational function, a. use function notation to represent and interpret input/output pairs in terms of a context and points on the graph; b. understand and use the fact that for the graph of $y = f(x)$, the solutions to $f(x) = 0$ correspond to x-intercepts of the graph and $f(0)$ corresponds to the y-intercept of the graph; interpret these key features in terms of a context; c. identify the graph given an algebraic representation of the function and an algebraic representation given the graph (with or without a context).

As a test that provides an entry point to postsecondary education and careers, the redesigned SAT's Math Test will include topics that are central to the ability of students to progress to later, more advanced mathematics. Problems in Passport to Advanced Math will cover topics that have great relevance and utility for college and career work.

Chief among these topics is the understanding of the structure of expressions and the ability to analyze, manipulate, and rewrite these expressions. This includes an understanding of the key parts of expressions, such as terms, factors, and coefficients, and the ability to interpret complicated expressions made up of these components. Students will be able to show their skill in rewriting expressions, identifying equivalent forms of expressions, and understanding the purpose of different forms.

This category also includes reasoning with more complex equations, including solving quadratic and higher-order equations in one variable and understanding the graphs of quadratic and higher-order functions. Finally, this category includes the ability to interpret and build functions, another skill crucial for success in later mathematics and scientific fields.

ADDITIONAL TOPICS IN MATH

SAT ADDITIONAL TOPICS IN MATH DOMAIN	
Content Dimension	**Description**
Area and volume	1. Solve real-world and mathematical problems about a geometric figure or an object that can be modeled by a geometric figure using given information such as length, area, surface area, or volume. a. Apply knowledge that changing by a scale factor of k changes all lengths by a factor of k, changes all areas by a factor of k^2, and changes all volumes by a factor of k^3. b. Demonstrate procedural fluency by selecting the correct area or volume formula and correctly calculating a specified value.
Lines, angles, and triangles	1. Use concepts and theorems relating to congruence and similarity of triangles to solve problems. 2. Determine which statements may be required to prove certain relationships or to satisfy a given theorem. 3. Apply knowledge that changing by a scale factor of k changes all lengths by a factor of k, but angle measures remain unchanged. 4. Know and directly apply relevant theorems such as a. the vertical angle theorem; b. triangle similarity and congruence criteria; c. triangle angle sum theorem; d. the relationship of angles formed when a transversal cuts parallel lines.
Right triangles and trigonometry	1. Solve problems in a variety of contexts using a. the Pythagorean theorem; b. right triangle trigonometry; c. properties of special right triangles. 2. Use similarity to calculate values of sine, cosine, and tangent. 3. Understand that when given one side length and one acute angle measure in a right triangle, the remaining values can be determined. 4. Solve problems using the relationship between sine and cosine of complementary angles. 5. Fluently apply properties of special right triangles to determine side lengths and calculate trigonometric ratios of 30, 45, and 60 degrees.
Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas, to solve problems. 2. Solve problems using a. radian measure; b. trigonometric ratios in the unit circle. 3. Create an equation to represent a circle in the *xy*-plane. 4. Describe how a. a change to the equation representing a circle in the *xy*-plane affects the graph of the circle; b. a change in the graph of the circle affects the equation of the circle. 5. Understand that the ordered pairs that satisfy an equation of the form $(x - h)^2 + (y - k)^2 = r^2$ form a circle when plotted in the *xy*-plane. 6. Convert between angle measures in degrees and radians. 7. Complete the square in an equation representing a circle to determine properties of the circle when it is graphed in the *xy*-plane, and use the distance formula in problems related to circles.
Complex numbers	1. Apply knowledge and understanding of the complex number system to add, subtract, multiply, and divide with complex numbers and solve problems.

While the overwhelming majority of problems on the redesigned SAT's Math Test fall into the first three domains, the test also addresses additional topics in high school math. In keeping with the approach described in Section II, patterns of selection for these are governed by evidence about their relevance to postsecondary education and work. The additional topics include essential geometric and trigonometric concepts and the Pythagorean theorem, which become powerful methods of analysis and problem solving when connected to other math domains.

SAMPLE QUESTIONS ILLUSTRATING DISTINCTIVE FEATURES OF THE REDESIGNED SAT'S MATH TEST

The following distinctive features of the redesigned SAT's Math Test are illustrated by sample questions that reflect the following:

- An emphasis on mathematical reasoning over reasoning questions disconnected from the mathematics curriculum
- A strong emphasis on both fluency and understanding
- Richer applications, emphasizing career, science, and social studies applications
- Item sets that allow for more than one question about a given scenario
- A no-calculator portion

REASONING ON THE REDESIGNED SAT'S MATH TEST WILL CONNECT MORE DIRECTLY TO ESSENTIAL SKILLS FOR COLLEGE READINESS THAT ARE PART OF A RIGOROUS HIGH SCHOOL CURRICULUM.

To see what this shift means, consider the following question from the current SAT: [1]

Family	Number of Consecutive Nights
Jackson	10
Callan	5
Epstein	8
Liu	6
Benton	8

The table above shows the number of consecutive nights that each of five families stayed at a certain hotel during a 14-night period. If the Liu family's stay did not overlap with the Benton family's stay, which of the 14 nights could be a night on which only one of the five families stayed at the hotel?

A) The 3rd
B) The 5th
C) The 6th
D) The 8th
E) The 10th

This question presents the student with a reasoning puzzle unrelated to the school mathematics curriculum. Being able to solve unfamiliar problems is valuable, but a test based entirely on this idea does not provide as much assurance that students have learned essential math skills and practices — nor does it reward students for their hard work in doing so.

The redesigned SAT's Math Test focuses on applied reasoning skills that are both essential for college readiness and taught in challenging high school math classrooms. This means that the questions will require reasoning and insight as they relate to important curricular skills such as looking for and making use of algebraic structure. In contrast to the question on the left, consider the following sample from the Heart of Algebra category:

EXAMPLE 1: Sample item from the redesigned SAT

If $\frac{1}{2}x + \frac{1}{3}y = 4$, what is the value of $3x + 2y$?

A student may find the solution to this Heart of Algebra problem by noticing the structure of the given equation and seeing that multiplying both sides of the equation $\frac{1}{2}x + \frac{1}{3}y = 4$ by 6 to clear fractions from the equation yields $3x + 2y = 24$.

1 College Board, *Official SAT Practice Test 2013–14* (New York: The College Board, 2013).

A STRONG EMPHASIS ON BOTH FLUENCY AND UNDERSTANDING

In *Adding It Up: Helping Children Learn Mathematics*, the National Research Council (NRC) identified procedural fluency and conceptual understanding as two of the five components of mathematical proficiency. The NRC calls for their inclusion in curricula, instructional materials, and assessments as they define what it means to learn math successfully.[2] As students cannot be ready for college and career without being mathematically proficient, the redesigned SAT assesses fluency with mathematical procedures and conceptual understanding with equal intensity.

The following two sample questions show some of the ways in which fluency and understanding are important on the redesigned SAT.

EXAMPLE 2

$$4x - y = 3y + 7$$
$$x + 8y = 4$$

Based on the system of equations above, what is the value of the product xy?

A) $\frac{3}{2}$

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) $\frac{11}{9}$

Example 2, again from Heart of Algebra, rewards fluency in solving pairs of simultaneous linear equations. Rather than looking for a clever way of back solving the value of the product xy from the system, students can solve the system for the values of x and y, then simply multiply them to get choice C, $\frac{1}{2}$. Note that because the system is not given in standard form, this requires doing some additional algebra, further reinforcing the need for fluency.

2 National Research Council, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: The National Academies Press, 2001).

EXAMPLE 3

The function f is defined by $f(x) = 2x^3 + 3x^2 + cx + 8$, where c is a constant. In the xy-plane, the graph of f intersects the x-axis at the three points $(-4, 0)$, $(\frac{1}{2}, 0)$, and $(p, 0)$. What is the value of c?

A) –18
B) –2
C) 2
D) 10

Example 3, from Passport to Advanced Math, assesses conceptual understanding of polynomials and their graphs. If a student understands these concepts and requires, for example, the point $(-4, 0)$ to lie on the graph, this results in $0 = 2(-4)^3 + 3(-4)^2 + c(-4) + 8$. A student who looks for and makes use of structure will monitor the calculation at this point and recognize an equation that determines the desired value of c, –18. Seeing that he or she is on the right track, the student will then perform the calculations required to solve for c.

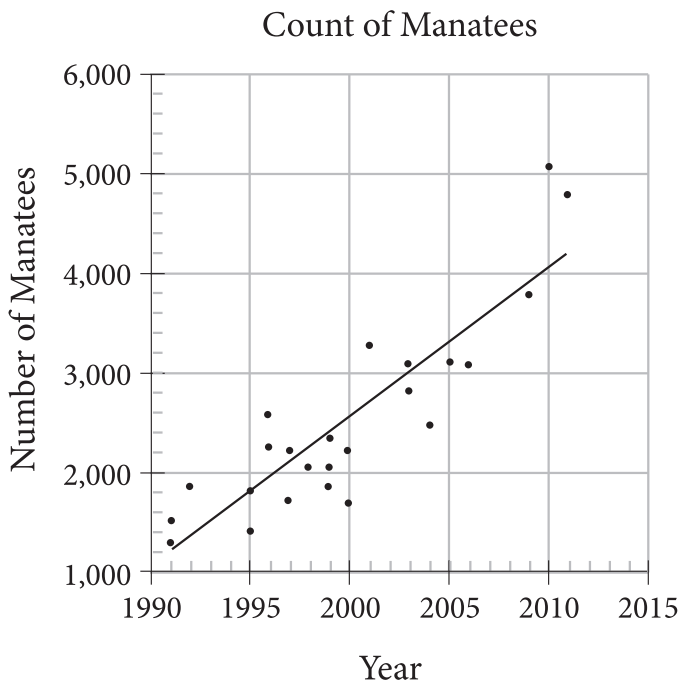
RICHER APPLICATIONS, EMPHASIZING CAREER, SCIENCE, AND SOCIAL STUDIES APPLICATIONS

In response to evidence about essential prerequisites for college and career readiness and success, the redesigned SAT's Math Test requires students to apply their mathematics knowledge, skills, and understandings in challenging, authentic contexts. Students taking the Math Test will encounter a range of disciplines and will be asked to address real-world problems drawn from science, social studies, and careers and demonstrate a capacity for sustained reasoning over the multiple steps required to answer many of the questions on the exam. In these ways, the Math Test also rewards and incentivizes valuable work in the classroom.

Applications on the redesigned SAT's Math Test require students to demonstrate the ability to analyze a situation, determine the essential elements required to solve the problem, represent the problem mathematically, and carry out a solution. These applications often also require linking topics within the mathematics domain (e.g., functions and statistics) and across disciplines (e.g., math and science). Learning to model and problem solve is enhanced when students use the same mathematics (e.g., linear equations) to solve problems in different contexts (e.g., science, social studies, or careers).

Example 4 below is based on real-world methods (aerial observations of wintering spots, or synoptic counts) used by the U.S. Fish and Wildlife Service to count manatees, a type of sea mammal. This type of item is an excellent way of connecting linear functions to statistics. In this item, students are not required to model the line of best fit completely, but they are required to decontextualize the item to understand that they must compute the slope of the line of best fit to get the correct answer, 150.

EXAMPLE 4

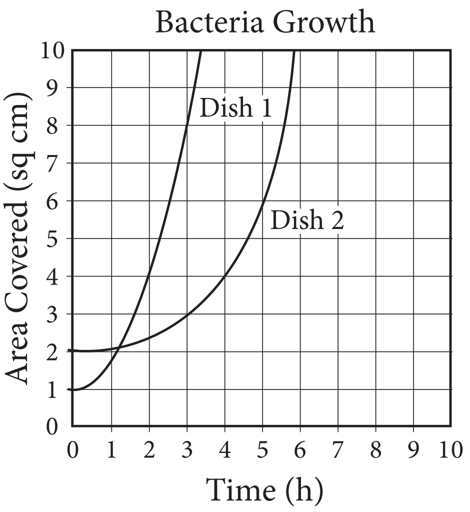


The scatterplot above shows counts of Florida manatees, a type of sea mammal, from 1991 to 2011. Based on the line of best fit to the data shown, which of the following values is closest to the average yearly increase in the number of manatees?

A) 0.75
B) 75
C) 150
D) 750

Example 5 below is another rich application item that uses a science context to make a connection across math domains (functions and statistics) and across subjects (math and science). In this item, students need to synthesize the information given in the graph and the prompt and determine which pieces of information in the graph will help provide them with a correct statement about the data.

EXAMPLE 5



A researcher places two colonies of bacteria into two petri dishes that each have area 10 square centimeters. After the initial placement of the bacteria ($t = 0$), the researcher measures and records the area covered by the bacteria in each dish every ten minutes. The data for each dish were fit by a smooth curve, as shown above, where each curve represents the area of a dish covered by bacteria as a function of time, in hours. Which of the following is a correct statement about the data above?

A) At time $t = 0$, both dishes are 100% covered by bacteria.
B) At time $t = 0$, bacteria covers 10% of Dish 1 and 20% of Dish 2.
C) At time $t = 0$, Dish 2 is covered with 50% more bacteria than Dish 1.
D) For the first hour, the area covered in Dish 2 is increasing at a higher average rate than the area covered in Dish 1.

ITEM SETS THAT ALLOW FOR MORE THAN ONE QUESTION ABOUT A GIVEN SCENARIO

Asking more than one question about a given scenario allows students taking the redesigned SAT to do more sustained thinking and explore situations in greater depth. Students will encounter longer problems like these in their postsecondary work. By including item sets, the redesigned SAT rewards and incentivizes aligned, productive work in classrooms.

Item sets can be used to dig deeper into a student's understanding of a construct or to make connections to other domains. For example, one question from a set may ask about statistics and probability and the next may ask about the function that models the data. In the classroom, item sets manifest the connections between domains and provide opportunities for students to practice and extend their skills of abstraction, analysis, and communication.

Within this subset of questions, students will encounter:

- career-related contexts, which could include scale drawings, estimation, unit rates, percentages, and proportional relationships;
- problem sets that make use of these contexts, allowing multiple questions about a single stimulus;
- real-life scenarios that will likely yield more complex solutions; and
- real-life scenarios that might not be proportional; for instance, students may be asked to demonstrate their proficiency with scaling quantities that aren't proportional or with situations of diminishing returns and accelerated growth.

ITEM SET:
In the classroom, item sets manifest the connections between different domains and provide opportunities for students to practice and extend their skills of abstraction, analysis, and communication. In the redesigned SAT, item sets allow the effective measurement of these skills and inspire productive practice in the classrooms.

EXAMPLE 6

(This is a student-produced response item set. Students grid in their answers, which are machine scored.)

An international bank issues its Traveler credit cards worldwide. When a customer makes a purchase using a Traveler card in a currency different from the customer's home currency, the bank converts the purchase price at the daily foreign exchange rate and then charges a 4% fee on the converted cost.

Sara lives in the United States, but is on vacation in India. She used her Traveler card for a purchase that cost 602 rupees (Indian currency). The bank posted a charge of $9.88 to her account that included the 4% fee.

PART 1

What foreign exchange rate, in Indian rupees per one U.S. dollar, did the bank use for Sara's charge? Round your answer to the nearest whole number.

PART 2

A bank in India sells a prepaid credit card worth 7,500 rupees. Sara can buy the prepaid card using dollars at the daily exchange rate with no fee, but she will lose any money left unspent on the prepaid card. What is the least number of the 7,500 rupees on the prepaid card Sara must spend for the prepaid card to be cheaper than charging all her purchases on the Traveler card? Round your answer to the nearest whole number of rupees.

SOLUTION

PART 1

\$9.88 represents the conversion of 602 rupees plus a 4% fee on the converted cost.

To calculate the original cost of the item in dollars, x:

$$1.04x = 9.88$$
$$x = 9.5$$

Since the original cost is \$9.50, to calculate the exchange rate r, in Indian rupees per one U.S. dollar:

$$9.50 \text{ dollars} \times \frac{r \text{ rupees}}{1 \text{ dollar}} = 602 \text{ rupees}$$
$$r = \frac{602}{9.50}$$
$$\approx 63 \text{ rupees}$$

PART 2

Let d dollars be the cost of the 7,500-rupee prepaid card. This implies that the exchange rate on this particular day is $\frac{d}{7{,}500}$ dollars per rupee. Suppose Sara's total purchases on the prepaid card were r rupees. The value of the r rupees in dollars is $\left(\frac{d}{7{,}500}\right)r$ dollars. If Sara spent the r rupees on the Traveler card instead, she would be charged $(1.04)\left(\frac{d}{7{,}500}\right)r$ dollars. To answer the question about how many rupees Sara must spend in order to make the Traveler card a cheaper option (in dollars) for spending the r rupees, we set up the inequality $1.04\left(\frac{d}{7{,}500}\right)r \geq d$. Rewriting both sides reveals $1.04\left(\frac{r}{7{,}500}\right)d \geq (1)d$, from which we can infer $1.04\left(\frac{r}{7{,}500}\right) \geq 1$. Dividing on both sides by 1.04 and multiplying on both sides by 7,500 finally yields $r \geq 7{,}212$. Hence the least number of rupees Sara must spend for the prepaid card to be cheaper than the Traveler card is 7,212.

Note that Example 6 is not a multiple-choice item. Responses are gridded in by students, which often allows for multiple correct responses and solution processes. Such items allow students to freely apply their critical thinking skills when planning and implementing a solution.

Example 6 is also an item set that includes two student-produced response questions. Student-produced response item set questions on the redesigned SAT measure the complex knowledge and skills that require students to deeply think through the solutions to problems. Set within a range of real-world contexts, these questions require students to make sense of problems and persevere in solving them; make connections between and among the different parts of a stimulus; plan a solution approach, as no scaffolding is provided to suggest a solution strategy; abstract, analyze, and refine an approach as needed; and produce and validate a response. These types of questions require the application of complex cognitive skills.

A NO-CALCULATOR PORTION

The redesigned SAT's Math Test contains two portions: one in which the student may use a calculator and another in which the student may not. The no-calculator portion allows the redesigned SAT to assess fluencies valued by postsecondary instructors and includes conceptual questions for which a calculator will not be helpful. Meanwhile, the calculator portion gives insight into students' capacity to use appropriate tools strategically. The calculator is a tool that students must use (or not use) judiciously.

The calculator portion of the test includes more complex modeling and reasoning questions to allow students to make computations more efficiently. However, this portion will also include questions in which the calculator could be a deterrent to expedience, thus assessing appropriate use of tools. For these types of questions, students who make use of structure or their ability to reason will reach the solution more rapidly than students who get bogged down using a calculator.

EXAMPLE 7

What is one possible solution to the equation $\frac{24}{x+1} - \frac{12}{x-1} = 1$?

Example 7, from the no-calculator portion of the test, requires students to look at the structure of the expression and find a way to rewrite it, again showing the link between fluency and mathematical practices. The student must transform the expression without a calculator, for example by multiplying both sides of the equation by a common denominator as a first step to find the solution. This leads to $x = 5$ and $x = 7$, both of which should be checked in the original equation to ensure that they are not extraneous.

Additional example items showing the distinctive features of the redesigned SAT's Math Test within the four content categories can be found in Appendix B.

Summary

The preceding discussion has presented an overview of the redesigned SAT's Math Test along with a discussion of some of the key features that make the Math Test distinctive both compared to the current SAT's math section and compared to other assessments within the field. As with the Evidence-Based Reading and Writing area of the redesigned SAT, we at the College Board will continue to be guided by research and evidence as we develop the redesigned SAT's Math Test.

Appendix B: Math Sample Questions

The sample questions/tasks in this section are provided to show a number of the key features of the redesigned SAT's Math Test, but do not constitute a full form of the test in terms of the total number of questions/tasks, the range of question difficulty, or examples of all question types and formats.

Sample Questions: Heart of Algebra

SAMPLE 1

Aaron is staying at a hotel that charges \$99.95 per night plus tax for a room. A tax of 8% is applied to the room rate, and an additional one-time untaxed fee of \$5.00 is charged by the hotel. Which of the following represents Aaron's total charge, in dollars, for staying x nights?

A) $(99.95 + 0.08x) + 5$
B) $1.08(99.95x) + 5$
C) $1.08(99.95x + 5)$
D) $1.08(99.95 + 5)x$

CONTENT: Heart of Algebra

KEY: B

CALCULATOR: Permitted

This problem asks students to interpret a situation and formulate a linear expression that represents the situation mathematically. The construction of mathematical models that represent real-world scenarios is a critical skill.

Choice B is correct. The total charge that Aaron will pay is the room rate, the 8% tax on the room rate, and a fixed fee. If Aaron stayed x nights, then the total charge is $(99.95x + 0.08 \times 99.95x) + 5$, which can be rewritten as $1.08(99.95x) + 5$.

Choice A is not the correct answer. The expression includes only a one-night stay in the room and does not accurately account for tax on the room.

Choice C is not the correct answer. The expression includes tax on the fee, and the hotel does not charge tax on the $5.00 fee.

Choice D is not the correct answer. The expression includes tax on the fee and a fee charge for each night.

SAMPLE 2

The gas mileage for Peter's car is 21 miles per gallon when the car travels at an average speed of 50 miles per hour. The car's gas tank has 17 gallons of gas at the beginning of a trip. If Peter's car travels at an average speed of 50 miles per hour, which of the following functions *f* models the number of gallons of gas remaining in the tank *t* hours after the trip begins?

A) $f(t) = 17 - \frac{21}{50t}$

B) $f(t) = 17 - \frac{50t}{21}$

C) $f(t) = \frac{17 - 21t}{50}$

D) $f(t) = \frac{17 - 50t}{21}$

CONTENT: Heart of Algebra

KEY: B

CALCULATOR: Permitted

In this question, students must understand that the number of gallons of gas in the tank is a function of time. The core skill assessed here is the ability to translate from a real-world situation into a mathematical model.

Choice B is correct. Since Peter's car is traveling at an average speed of 50 miles per hour and the car's gas mileage is 21 miles per gallon, the number of gallons of gas used each hour can be found by $\frac{50 \text{ miles}}{1 \text{ hour}} \times \frac{1 \text{ gallon}}{21 \text{ miles}} = \frac{50}{21}$. The car uses $\frac{50}{21}$ gallons of gas per hour, so it uses $\frac{50}{21}t$ gallons of gas in t hours. The car's gas tank has 17 gallons of gas at the beginning of the trip. Therefore, the function that models the number of gallons of gas remaining in the tank t hours after the trip begins is $f(t) = 17 - \frac{50t}{21}$.

Choice A is not the correct answer. The number of gallons of gas used each hour is determined by dividing the average speed by the car's gas mileage.

Choice C is not the correct answer. The number of gallons of gas used each hour is misrepresented as $\frac{21}{50}$. Also, the number of gallons used each hour must be multiplied by time t before it is subtracted from the number of gallons of gas in the tank at the beginning of the trip.

Choice D is not the correct answer. The number of gallons of gas used each hour must be multiplied by time t before it is subtracted from the number of gallons of gas in the tank at the beginning of the trip.

SAMPLE 3

If $-\frac{9}{5} < -3t + 1 < -\frac{7}{4}$, what is one possible value of $9t - 3$?

CONTENT: Heart of Algebra

KEY: Any value greater than $\frac{21}{4}$ and less than $\frac{27}{5}$

CALCULATOR: Permitted

Recognizing the structure of this inequality provides one solution strategy. With this strategy, a student will look at the relationship between $-3t + 1$ and $9t - 3$ and recognize that the latter is -3 multiplied by the former.

Multiplying all parts of the inequality by -3 reverses the inequality signs, resulting in $\frac{27}{5} > 9t - 3 > \frac{21}{4}$, or rather $\frac{21}{4} < 9t - 3 < \frac{27}{5}$ when written with increasing values from left to right. Any value greater than $\frac{21}{4}$ and less than $\frac{27}{5}$ is correct.

SAMPLE 4

$$\frac{5(k+2)-7}{6} = \frac{13-(4-k)}{9}$$

In the equation above, what is the value of k?

A) $\frac{9}{17}$

B) $\frac{9}{13}$

C) $\frac{33}{17}$

D) $\frac{33}{13}$

CONTENT: Heart of Algebra

KEY: B

CALCULATOR: Not Permitted

In this problem, students will demonstrate their fluency in solving equations in one variable.

Choice B is correct. Simplifying the numerators yields $\frac{5k+3}{6} = \frac{9+k}{9}$, and cross-multiplication gives $45k + 27 = 54 + 6k$. Solving for k yields $k = \frac{9}{13}$.

Choice A is not the correct answer. This value may result from not correctly applying the distributive property on the right-hand side, resulting in the expression $13 - 4 - k$ in the numerator. Correctly applying the distributive property yields $13 - (4 - k) = 13 - 4 + k$ in the numerator.

Choice C is not the correct answer. This value may result from not correctly applying the distributive property on the left-hand side, resulting in the expression $5k + 2 - 7$. Correctly applying the distributive property yields $5(k + 2) - 7 = 5k + 3$ in the numerator.

Choice D is not the correct answer. This value may result from not using the appropriate order of operations when simplifying either numerator.

SAMPLE 5

$$4x - y = 3y + 7$$
$$x + 8y = 4$$

Based on the system of equations above, what is the value of the product xy?

A) $-\frac{3}{2}$

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) $\frac{11}{9}$

CONTENT: Heart of Algebra

KEY: C

CALCULATOR: Not Permitted

This question rewards fluency in solving pairs of simultaneous linear equations. Rewriting equations in a way that allows the student to find the values of variables individually is the approach to take here. Students who lack facility in this area of the curriculum may resist just diving in, making success on the item less likely.

Choice C is correct. There are several solution methods possible, but all involve persevering in solving for the two variables and calculating the product. For example, combining like terms in the first equation yields $4x - 4y = 7$ and then multiplying that by 2 gives $8x - 8y = 14$. When this transformed equation is added to the second given equation, the y-terms are eliminated, leaving an equation in just one variable: $9x = 18$, or $x = 2$. Substituting 2 for x in the second equation (one could use either to solve) yields $2 + 8y = 4$, which gives $y = \frac{1}{4}$. Finally, the product xy is $2 \times \frac{1}{4} = \frac{1}{2}$.

Choice A is not the correct answer. Students who select this option have most likely made a calculation error in transforming the second equation (using $-4x - 8y = -16$ instead of $-4x - 32y = -16$) and used it to eliminate the x-terms.

Choice B is not the correct answer. This is the value of y for the solution of the system, but it has not been put back into the system to solve for x to determine the product xy.

Choice D is not the correct answer. Not understanding how to eliminate a variable when solving a system, a student may have added the equations $4x - 4y = 7$ and $x + 8y = 4$ to yield $5x + 4y = 11$. From here, a student may mistakenly simplify the left-hand side of this resulting equation to yield $9xy = 11$ and then proceed to use division by 9 on both sides in order to solve for xy.

SAMPLE 6

If $\frac{1}{2}x + \frac{1}{3}y = 4$, what is the value of $3x + 2y$?

CONTENT: Heart of Algebra

KEY: 24

CALCULATOR: Not Permitted

A student may find the solution to this problem by noticing the structure of the given equation and seeing that multiplying both sides of the equation $\frac{1}{2}x + \frac{1}{3}y = 4$ by 6 to clear fractions from the equation yields $3x + 2y = 24$.

SAMPLE 7

The toll rates for crossing a bridge are \$6.50 for a car and \$10 for a truck. During a two-hour period, a total of 187 cars and trucks crossed the bridge, and the total collected in tolls was \$1,338. Solving which of the following systems of equations yields the number of cars, x, and the number of trucks, y, that crossed the bridge during the two hours?

A) $x + y = 1{,}338$
$6.5x + 10y = 187$

B) $x + y = 187$
$6.5x + 10y = \frac{1{,}338}{2}$

C) $x + y = 187$
$6.5x + 10y = 1{,}338$

D) $x + y = 187$
$6.5x + 10y = 1{,}338 \times 2$

CONTENT: Heart of Algebra

KEY: C

CALCULATOR: Permitted

This question assesses students' ability to create a system of linear equations that represents a real-world situation. Students will have to make sense of the situation presented, choose and define two variables to use, and set up the equations based on the relationships from the information given.

Choice C is correct. If x is the number of cars that crossed the bridge during the two hours and y is the number of trucks that crossed the bridge during the two hours, then $x + y$ represents the total number of cars and trucks that crossed the bridge during the two hours, and $6.5x + 10y$ represents the total amount collected in the two hours. Therefore, the correct system of equations is $x + y = 187$ and $6.5x + 10y = 1{,}338$.

Choice A is not the correct answer. The student may have mismatched the symbolic expressions for total cars and trucks and total tolls collected with the two numerical values given. The expression $x + y$ represents the total number of cars and trucks that crossed the bridge, which is 187.

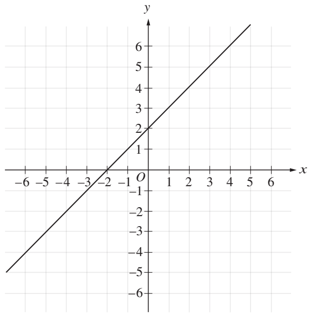
Choice B is not the correct answer. The student may have attempted to use the information that the counts of cars, trucks, and tolls were taken over a period of two hours, but this information is not needed in setting up the correct system of equations. The expression $6.5x + 10y$ represents the total amount of tolls collected, which is \$1,338, not $\frac{\$1,338}{2}$.

Choice D is not the correct answer. The student may have attempted to use the information that the counts of cars, trucks, and tolls were taken over a period of two hours, but this information is not needed in setting up the correct system of equations. The expression $6.5x + 10y$ represents the total amount of tolls collected, which is \$1,338, not \$1,338 × 2.

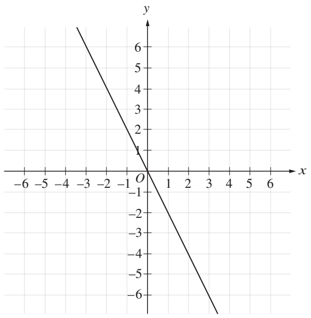
SAMPLE 8

If k is a positive constant different from 1, which of the following could be the graph of $y - x = k(x + y)$ in the xy-plane?

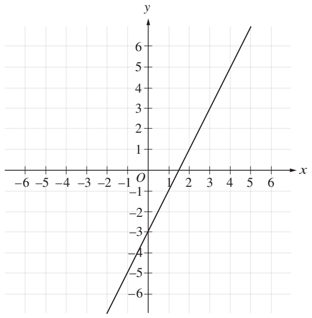
A)



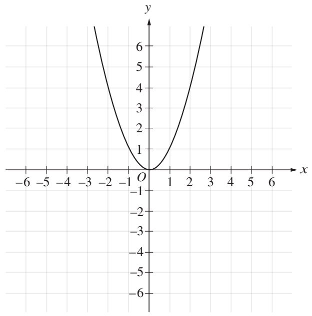
B)



C)



D)



CONTENT: Heart of Algebra

KEY: B

CALCULATOR: Permitted

This problem assesses students' ability to understand the relationship between an equation in two variables and the characteristics of its graph (for example, shape, position, intercepts, extreme points, or symmetry). In addition, it requires the student to transform the given equation into a more suitable form and then make the connection between the obtained equation and the graph.

Choice B is correct. Manipulating the equation to solve for y gives $y = \left(\frac{1+k}{1-k}\right)x$, revealing that the graph of the equation must be a line that passes through the origin. Of the choices given, only the graph shown in choice B satisfies these conditions.

Choice A is not the correct answer. The student may have seen that the term $k(x + y)$ is a multiple of $x + y$ and wrongly concluded that this is the equation of a line with slope 1.

Choice C is not the correct answer. The student may have made incorrect steps when simplifying the equation or may have not seen the advantage that putting the equation in slope-intercept form would give in determining the graph, and thus wrongly concluded the graph has a nonzero y-intercept.

Choice D is not the correct answer. The student may not have seen that the term $k(x + y)$ can be multiplied out and the variables x and y isolated, and wrongly concluded that the graph of the equation cannot be a line.

SAMPLE 9

$$\frac{1}{2}x - \frac{1}{4}y = 5$$
$$ax - 3y = 20$$

In the system of linear equations above, a is a constant. If the system has no solution, what is the value of a?

A) $\frac{1}{2}$
B) 2
C) 6
D) 12

CONTENT: Heart of Algebra

KEY: C

CALCULATOR: Not Permitted

In addition to solving systems of linear equations that have a solution, students should be familiar with systems that have no solution or an infinite number of solutions. Knowing that there are no solutions when two simultaneous equations of the form $ax + by = c$ only differ in their c values will be the first key step in determining the solution to this problem.

Choice C is correct. If the system of equations has no solution, the graphs of the equations in the xy-plane are parallel lines. To be parallel, the lines must have the same slope, and this will be true if the expression $ax - 3y$ is a multiple of the expression $\frac{1}{2}x - \frac{1}{4}y$. Since $-3y = 12\left(-\frac{1}{4}y\right)$, the expression $ax - 3y$ would have to be 12 times the expression $\frac{1}{2}x - \frac{1}{4}y$. This means $ax = 12\left(\frac{1}{2}x\right)$, so $a = 6$. The resulting system is $\frac{1}{2}x - \frac{1}{4}y = 5$ and $6x - 3y = 20$, which is equivalent to $6x - 3y = 60$ and $6x - 3y = 20$, which has no solution.

Choice A is not the correct answer. This may result from the misconception that if each equation in a system has the same x-coefficient, the system cannot have a solution. But if $a = \frac{1}{2}$, subtracting the two equations eliminates x and produces a solution to the system.

Choice B is not the correct answer. This may result from trying to make the second equation in the system a multiple of the first by looking at the ratio of the constants on the right sides, $\frac{20}{5}$, and wrongly concluding that the second equation must be 4 times the first, which would give $a = 4\left(\frac{1}{2}\right)$, or $a = 2$. But the two equations in a system are multiples only if the system has infinitely many solutions, not if the system has no solution.

Choice D is not the correct answer. The student may have found the factor, 12, that multiplies the left side of the first equation to yield the left side of the second, but then neglected to find $a = 12\left(\frac{1}{2}\right)$, or $a = 6$.

SAMPLE 10

When a scientist dives in salt water to a depth of 9 feet below the surface, the pressure due to the atmosphere and surrounding water is 18.7 pounds per square inch. As the scientist descends, the pressure increases linearly. At a depth of 14 feet, the pressure is 20.9 pounds per square inch. If the pressure increases at a constant rate as the scientist's depth below the surface increases, which of the following linear models best describes the pressure p in pounds per square inch at a depth of d feet below the surface?

A) $p = 0.44d + 0.77$
B) $p = 0.44d + 14.74$
C) $p = 2.2d - 1.1$
D) $p = 2.2d - 9.9$

CONTENT: Heart of Algebra

KEY: B

CALCULATOR: Permitted

In approaching this problem, students must determine the relationship between the two variables described within the text: the depth and the pressure.

Choice B is correct. To determine the linear model, one can first determine the rate at which the pressure due to the atmosphere and surrounding water is increasing as the depth of the diver increases. Calculating this gives $\frac{20.9 - 18.7}{14 - 9} = \frac{2.2}{5}$, or 0.44. Then one needs to determine the pressure due to the atmosphere or, in other words, the pressure when the diver is at a depth of 0. Solving the equation $18.7 = 0.44(9) + b$ gives $b = 14.74$. Therefore, the model that can be used to relate the pressure and the depth is $p = 0.44d + 14.74$.

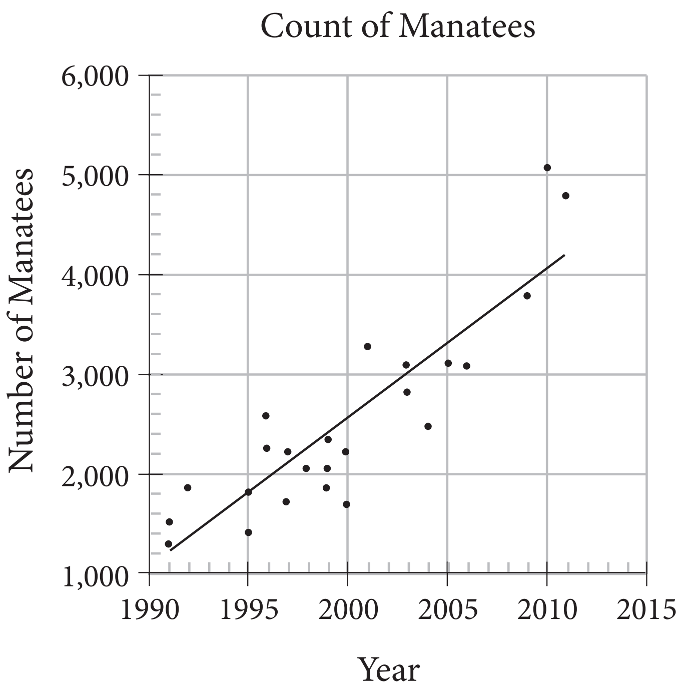
Choice A is not the correct answer. The rate is calculated correctly, but the student may have incorrectly used the ordered pair (18.7, 9) rather than (9, 18.7) to calculate the pressure at a depth of 0 feet.

Choice C is not the correct answer. The rate here is incorrectly calculated by subtracting 20.9 and 18.7 and *not* dividing by 5. The student then uses the coordinate pair $d = 9$ and $p = 18.7$ in conjunction with the incorrect slope of 2.2 to write the equation of the linear model.

Choice D is not the correct answer. The rate here is incorrectly calculated by subtracting 20.9 and 18.7 and *not* dividing by 5. The student then uses the coordinate pair $d = 14$ and $p = 20.9$ in conjunction with the incorrect slope of 2.2 to write the equation of the linear model.

Sample Questions: Problem Solving and Data Analysis

SAMPLE 11



The scatterplot above shows counts of Florida manatees, a type of sea mammal, from 1991 to 2011. Based on the line of best fit to the data shown, which of the following values is closest to the average yearly increase in the number of manatees?

A) 0.75
B) 75
C) 150
D) 750

CONTENT: Problem Solving and Data Analysis

KEY: C

CALCULATOR: Permitted

Modeling with mathematics is a multistep process, and rich applications are instrumental for assessing students' knowledge/skills at every step of the modeling process. In this example, students must interpret the slope of the line of best fit for the scatterplot as the average increase in the number of manatees per year, while taking the scales of the axes into consideration.

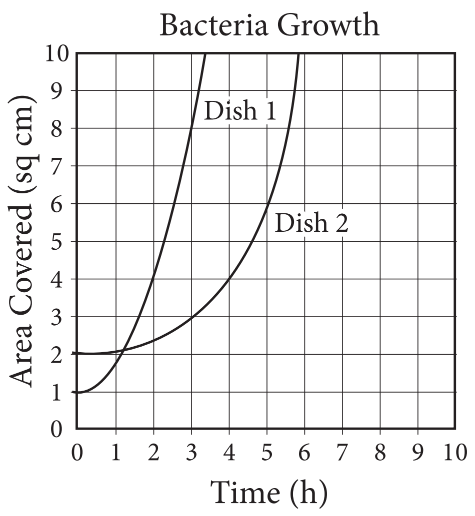
Choice C is correct. The slope of the line of best fit is the value of the average increase in manatees per year. Using approximate values found along the line of best fit (1,200 manatees in 1991 and 4,200 manatees in 2011), the approximate slope can be calculated as $\frac{3{,}000}{20} = 150$.

Choice A is not the correct answer. This value may result from disregarding the actual scale when approximating the slope and interpreting the scale as if each square represents one unit.

Choice B is not the correct answer. This value may result from disregarding the actual scale when approximating the slope, and interpreting the scale as if each square along the x-axis represents one year and each tick mark along the y-axis represents 100 manatees.

Choice D is not the correct answer. This value may result from disregarding the actual scale along the x-axis when approximating the slope and interpreting each square along the x-axis as one year.

SAMPLE 12



A researcher places two colonies of bacteria into two petri dishes that each have area 10 square centimeters. After the initial placement of the bacteria ($t = 0$), the researcher measures and records the area covered by the bacteria in each dish every ten minutes. The data for each dish were fit by a smooth curve, as shown above, where each curve represents the area of a dish covered by bacteria as a function of time, in hours. Which of the following is a correct statement about the data above?

A) At time $t = 0$, both dishes are 100% covered by bacteria.
B) At time $t = 0$, bacteria covers 10% of Dish 1 and 20% of Dish 2.
C) At time $t = 0$, Dish 2 is covered with 50% more bacteria than Dish 1.
D) For the first hour, the area covered in Dish 2 is increasing at a higher average rate than the area covered in Dish 1.

CONTENT: Problem Solving and Data Analysis

KEY: B

CALCULATOR: Permitted

In this question, students need to synthesize all the information given in the graph and the prompt and determine which pieces of information in the graph will help provide them with a description of the colonies when they are first placed in the petri dishes and during the first hour afterward.

Choice B is the correct answer. Each petri dish has area 10 square centimeters, and so at time $t = 0$, Dish 1 is 10% covered $\left(\frac{1}{10}\right)$ and Dish 2 is 20% covered $\left(\frac{2}{10}\right)$. Thus the statement in B is true.

Choice A is not the correct answer. At the end of the observations, both dishes are 100% covered with bacteria, but at time $t = 0$, neither dish is 100% covered.

Choice C is not the correct answer. At time $t = 0$, Dish 1 is covered with 50% less bacteria than is Dish 2, but Dish 2 is covered with 100% more, not 50% more, bacteria than is Dish 1.

Choice D is not the correct answer. After the first hour, it is still true that more of Dish 2 is covered by bacteria than is Dish 1, but for the first hour the area of Dish 1 that is covered has been increasing at a higher average rate (about 0.8 sq cm/hour) than the area of Dish 2 (about 0.1 sq cm/hour).

SAMPLE 13

A typical image taken of the surface of Mars by a camera is 11.2 gigabits in size. A tracking station on Earth can receive data from the spacecraft at a data rate of 3 megabits per second for a maximum of 11 hours each day. If 1 gigabit equals 1,024 megabits, what is the maximum number of typical images that the tracking station could receive from the camera each day?

A) 3
B) 10
C) 56
D) 144

CONTENT: Problem Solving and Data Analysis

KEY: B

CALCULATOR: Permitted

In this problem, students must use the unit rate (data-transmission rate) and the conversion between gigabits and megabits as well as conversions in units of time. Unit analysis is critical to solving the problem correctly, and the problem represents a typical calculation that would be done when working with electronic files and data-transmission rates.
A calculator is recommended in solving this problem.

Choice B is correct. The tracking station can receive 118,800 megabits each day $\left(\frac{3\text{ megabits}}{1\text{ second}} \times \frac{60\text{ seconds}}{1\text{ minute}} \times \frac{60\text{ minutes}}{1\text{ hour}} \times 11\text{ hours}\right)$, which is about 116 gigabits each day $\left(\frac{118{,}800}{1{,}024}\right)$. If each image is 11.2 gigabits, then the number of images that can be received each day is $\frac{116}{11.2} \approx 10.4$. Since the question asks for the maximum number of typical images, rounding the answer down to 10 is appropriate because the tracking station will not receive a completed 11th image in one day.

Choice A is not the correct answer. The student may not have synthesized all of the information. This answer may result from multiplying 3 (rate in megabits per second) by 11 (hours receiving) and dividing by 11.2 (size of image in gigabits), neglecting to convert 3 megabits per second into megabits per hour and to utilize the information about 1 gigabit equaling 1,024 megabits.

Choice C is not the correct answer. The student may not have synthesized all of the information. This answer may result from converting the number of gigabits in an image to megabits (11,470), multiplying by the rate of 3 megabits per second (34,410), and then converting 11 hours into minutes (660) instead of seconds.

Choice D is not the correct answer. The student may not have synthesized all of the information. This answer may result from converting 11 hours into seconds (39,600), then dividing the result by 3 gigabits converted into megabits (3,072), and multiplying by the size of one typical image.

Sample Problem Set

Questions 14 and 15 refer to the following information.

A survey was conducted among a randomly chosen sample of U.S. citizens about U.S. voter participation in the November 2012 presidential election. The table below displays a summary of the survey results.

Reported Voting by Age (in thousands)

	VOTED	DID NOT VOTE	NO RESPONSE	TOTAL
18- to 34-year-olds	30,329	23,211	9,468	63,008
35- to 54-year-olds	47,085	17,721	9,476	74,282
55- to 74-year-olds	43,075	10,092	6,831	59,998
People 75 years old and over	12,459	3,508	1,827	17,794
Total	132,948	54,532	27,602	215,082

SAMPLE 14

According to the table, for which age group did the greatest percentage of people report that they had voted?

A) 18- to 34-year-olds
B) 35- to 54-year-olds
C) 55- to 74-year-olds
D) People 75 years old and over

SAMPLE 15

Of the 18- to 34-year-olds who reported voting, 500 people were selected at random to do a follow-up survey where they were asked which candidate they voted for. There were 287 people in this follow-up survey sample who said they voted for Candidate A, and the other 213 people voted for someone else. Using the data from both the follow-up survey and the initial survey, which of the following is most likely to be an accurate statement?

A) About 123 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.
B) About 76 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.
C) About 36 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.
D) About 17 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

SOLUTION SAMPLE 14

CONTENT: Problem Solving and Data Analysis

KEY: C

CALCULATOR: Permitted

To succeed on these questions, students must conceptualize the context and retrieve relevant information from the table, next manipulating it to form or compare relevant quantities. The first question asks students to select the relevant information from the table to compute the percentage of self-reported voters for each age group and then compare the percentages to identify the largest one, choice C. Of the 55- to 74-year-old group's total population (59,998,000), 43,075,000 reported that they had voted, which represents 71.8% and is the highest percentage of reported voters from among the four age groups.

Choice A is not the correct answer. The question is asking for the age group with the largest percentage of self-reported voters. This answer reflects the age group with the smallest percentage of self-reported voters. This group's percentage of self-reported voters is 48.1%, or $\frac{30,329}{63,008}$, which is less than that of the 55- to 74-year-old group.

Choice B is not the correct answer. The question is asking for the age group with the largest percentage of self-reported voters. This answer reflects the age group with the largest number of self-reported voters, not the largest percentage. This group's percentage of self-reported voters is 63.4%, or $\frac{47{,}085}{74{,}282}$, which is less than that of the 55- to 74-year-old group.

Choice D is not the correct answer. The question is asking for the age group with the largest percentage of self-reported voters. This answer reflects the age group with the smallest number of self-reported voters, not the largest percentage. This group's percentage of self-reported voters is 70.0%, or $\frac{12{,}459}{17{,}794}$, which is less than that of the 55- to 74-year-old group.

SOLUTION SAMPLE 15

CONTENT: Problem Solving and Data Analysis

KEY: D

CALCULATOR: Permitted

The second question asks students to extrapolate from a random sample to estimate the number of 18- to 34-year-olds who voted for Candidate A: this is done by multiplying the fraction of people in the random sample who voted for Candidate A by the total population of voting 18- to 34-year-olds: $\frac{287}{500} \times 30{,}329{,}000 \approx 17$ million, choice D. Students without a clear grasp of the context and its representation in the table might easily arrive at one of the other answers listed.

Choice A is not the correct answer. The student may not have multiplied the fraction of the sample by the correct subgroup of people (18- to 34-year-olds who voted). This answer may result from multiplying the fraction by the entire population, which is an incorrect application of the information.

Choice B is not the correct answer. The student may not have multiplied the fraction of the sample by the correct subgroup of people (18- to 34-year-olds who voted). This answer may result from multiplying the fraction by the total number of people who voted, which is an incorrect application of the information.

Choice C is not the correct answer. The student may not have multiplied the fraction of the sample by the correct subgroup of people (18- to 34-year-olds who voted). This answer may result from multiplying the fraction by the total number of 18- to 34-year-olds, which is an incorrect application of the information.

SAMPLE 16 (STUDENT-PRODUCED RESPONSE)

An international bank issues its Traveler credit cards worldwide. When a customer makes a purchase using a Traveler card in a currency different from the customer's home currency, the bank converts the purchase price at the daily foreign exchange rate and then charges a 4% fee on the converted cost.

Sara lives in the United States, but is on vacation in India. She used her Traveler card for a purchase that cost 602 rupees (Indian currency). The bank posted a charge of $9.88 to her account that included the 4% fee.

PART 1

What foreign exchange rate, in Indian rupees per one U.S. dollar, did the bank use for Sara's charge? Round your answer to the nearest whole number.

PART 2

A bank in India sells a prepaid credit card worth 7,500 rupees. Sara can buy the prepaid card using dollars at the daily exchange rate with no fee, but she will lose any money left unspent on the prepaid card. What is the least number of the 7,500 rupees on the prepaid card Sara must spend for the prepaid card to be cheaper than charging all her purchases on the Traveler card? Round your answer to the nearest whole number of rupees.

CONTENT: Problem Solving and Data Analysis

KEY: 63 (Part 1), 7,212 (Part 2)

CALCULATOR: Permitted

Solution Part 1: $9.88 represents the conversion of 602 rupees plus a 4% fee on the converted cost.

To calculate the original cost of the item in dollars, x:

$$1.04x = 9.88$$
$$x = 9.5$$

Since the original cost is $9.50, to calculate the exchange rate r, in Indian rupees per one U.S. dollar:

$$9.50 \text{ dollars} \times \frac{r \text{ rupees}}{1 \text{ dollar}} = 602 \text{ rupees}$$
$$r = \frac{602}{9.50}$$
$$\approx 63 \text{ rupees}$$

Solution Part 2: Let d dollars be the cost of the 7,500-rupee prepaid card. This implies that the exchange rate on this particular day is $\frac{d}{7{,}500}$ dollars per rupee. Suppose Sara's total purchases on the prepaid card were r rupees. The value of the r rupees in dollars is $\left(\frac{d}{7{,}500}\right)r$ dollars. If Sara spent the r rupees on the Traveler card instead, she would be charged $(1.04)\left(\frac{d}{7{,}500}\right)r$ dollars. To answer the question about how many rupees Sara must spend in order to make the Traveler card a cheaper option (in dollars) for spending the r rupees, we set up the inequality $1.04\left(\frac{d}{7{,}500}\right)r \geq d$. Rewriting both sides reveals $1.04\left(\frac{r}{7{,}500}\right)d \geq (1)d$, from which we can infer $1.04\left(\frac{r}{7{,}500}\right) \geq 1$.

Dividing on both sides by 1.04 and multiplying on both sides by 7,500 finally yields $r \geq 7{,}212$. Hence the least number of rupees Sara must spend for the prepaid card to be cheaper than the Traveler card is 7,212.

Sample Questions: Passport to Advanced Math

SAMPLE 17

If $a^2 + 14a = 51$ and $a > 0$, what is the value of $a + 7$?

CONTENT: Passport to Advanced Math

KEY: 10

CALCULATOR: Not Permitted

There is more than one way to solve this problem. A student can apply standard techniques by rewriting the equation $a^2 + 14a = 51$ as $a^2 + 14a - 51 = 0$ and then factoring. Since the coefficient of a is 14 and the constant term is –51, factoring $a^2 + 14a - 51 = 0$ requires writing 51 as the product of two numbers that differ by 14. This is $51 = (3)(17)$, which gives the factorization $a^2 + 14a - 51 = (a + 17)(a - 3) = 0$. The possible values of a are $a = -17$ and $a = 3$. Since it is given that $a > 0$, it must be true that $a = 3$. Thus, the value of $a + 7$ is $3 + 7 = 10$.

A student could also use the quadratic formula to find the possible values of a:

$$a = \frac{-14 \pm \sqrt{14^2 - 4(1)(-51)}}{2(1)} = \frac{-14 \pm \sqrt{196 - (-204)}}{2} = \frac{-14 \pm \sqrt{400}}{2} = \frac{-14 \pm 20}{2}.$$

The possible values of a are $a = \frac{-14 - 20}{2} = -17$ and $a = \frac{-14 + 20}{2} = 3$. Again, since it is given that $a > 0$, it must be true that $a = 3$. Thus, the value of $a + 7$ is $3 + 7 = 10$.

There is another way to solve this problem that will reward the student who recognizes that adding 49 to both sides of the equation yields $a^2 + 14a + 49 = 51 + 49$, or rather $(a + 7)^2 = 100$, which has a perfect square on each side. Since $a > 0$, the solution $a + 7 = 10$ is evident.

SAMPLE 18

The function f is defined by $f(x) = 2x^3 + 3x^2 + cx + 8$, where c is a constant. In the xy-plane, the graph of f intersects the x-axis at the three points $(-4, 0)$, $\left(\frac{1}{2}, 0\right)$, and $(p, 0)$. What is the value of c ?

A) –18
B) –2
C) 2
D) 10

CONTENT: Passport to Advanced Math

KEY: A

CALCULATOR: Permitted

Students could tackle this problem in many different ways, but the focus is on their understanding of the zeros of a polynomial function and how they are used to construct algebraic representations of polynomials.

Choice A is correct. The given zeros can be used to set up an equation to solve for c. Substituting –4 for x and 0 for y yields $-4c = 72$, or $c = -18$.

Alternatively, since –4, $\frac{1}{2}$, and p are zeros of the polynomial function $f(x) = 2x^3 + 3x^2 + cx + 8$, it follows that $f(x) = (2x - 1)(x + 4)(x - p)$. Were this polynomial multiplied out, the constant term would be $(-1)(4)(-p) = 4p$. (We can see this without performing the full expansion.) Since it is given that this value is 8, it goes that $4p = 8$ or rather, $p = 2$. Substituting 2 for p in the polynomial function yields $f(x) = (2x - 1)(x + 4)(x - 2)$, and after multiplying the factors one finds that the coefficient of the x term, or the value of c, is –18.

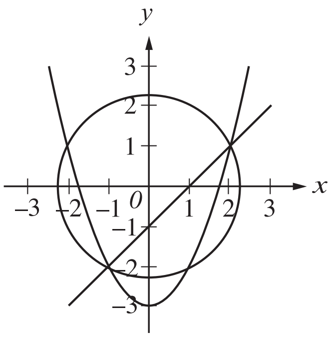
Choice B is not the correct answer. This value is a misunderstood version of the value of p, not c, and the relationship between the zero and the factor (if a is the zero of a polynomial, its corresponding factor is $x - a$) has been confused.

Choice C is not the correct answer. This is the value of p, not c. Using this value as the third factor of the polynomial will reveal that the value of c is –18.

Choice D is not the correct answer. This represents a sign error in the final step in determining the value of c.

SAMPLE 19

$$x^2 + y^2 = 5$$
$$y = x^2 - 3$$
$$x - y = 1$$



A system of three equations and their graphs in the xy-plane are shown above. How many solutions does the system have?

A) One
B) Two
C) Three
D) Four

CONTENT: Passport to Advanced Math

KEY: B

CALCULATOR: Permitted

Students must also demonstrate their conceptual understanding of mathematics. In this problem, the word "solutions" needs to be contextualized within the situation described, and students must recognize that only points where all three graphs have a point in common are solutions to the system.

Choice B is correct. The solutions to the system of equations are the points where the circle, parabola, and line all intersect. These points are (–1, –2) and (2, 1), and these are the only solutions to the system.

Choice A is not the correct answer. This answer may reflect the misconception that a system of equations can have only one solution.

Choice C is not the correct answer. This answer may reflect the misconception that a system of equations has as many solutions as the number of equations in the system.

Choice D is not the correct answer. This answer may reflect the misconception that the solutions of the system are represented by the points where any two of the curves intersect, rather than the correct concept that the solutions are represented only by the points where all three curves intersect.

SAMPLE 20

What is one possible solution to the equation $\frac{24}{x+1} - \frac{12}{x-1} = 1$?

CONTENT: Passport to Advanced Math

KEY: 5 or 7

CALCULATOR: Not Permitted

Students should look for the best solution methods for solving rational equations before they begin. Looking for structure and common denominators will prove very useful at the onset and will help prevent complex computations that do not lead to a solution.

In this problem, multiplying both sides of the equation by the common denominator $(x+1)(x-1)$ yields $24(x-1) - 12(x+1) = (x+1)(x-1)$. Multiplication and simplification then yields $12x - 36 = x^2 - 1$, or $x^2 - 12x + 35 = 0$. Factoring the quadratic gives $(x-5)(x-7) = 0$, so the solutions occur at $x = 5$ and $x = 7$, both of which should be checked in the original equation to ensure that they are not extraneous. In this case, both values are solutions.

SAMPLE 21

$$\frac{1}{x} + \frac{2}{x} = \frac{1}{5}$$

Anise needs to complete a printing job using both of the printers in her office. One of the printers is twice as fast as the other, and together the printers can complete the job in 5 hours. The equation above represents the situation described. Which of the following describes what the expression $\frac{1}{x}$ represents in this equation?

A) The time, in hours, that it takes the slower printer to complete the printing job alone
B) The portion of the job that the slower printer would complete in one hour
C) The portion of the job that the faster printer would complete in two hours
D) The time, in hours, that it takes the slower printer to complete $\frac{1}{5}$ of the printing job

CONTENT: Passport to Advanced Math

KEY: B

CALCULATOR: Not Permitted

This question requires students to interpret/decipher an expression/equation (already constructed for them) that models a real-world situation. This is an important skill to develop and a good practice to foster students' reasoning skills. The students who answer this question correctly are able to interpret the whole expression (or specific parts) in terms of its context.

Choice B is correct. From the description given, $\frac{1}{5}$ is the portion of the job that the two printers, working together, can complete in one hour, and each term in the sum on the left side is the part of this $\frac{1}{5}$ of the job that one of the printers contributes. Since one of the printers is twice as fast as the other, $\frac{2}{x}$ describes the portion of the job that the faster printer is able to complete in one hour and $\frac{1}{x}$ describes the portion of the job that the slower printer is able to complete in one hour.

Choice A is not the correct answer. The student may have not seen that in this context, the rates (that is, the work completed in a fixed time) of the printers can be added to get the combined rate, but the times it takes each printer to complete the job cannot be added to get the time for both printers working together, since the time for printers working together is less than, not greater than, the times for each printer alone. Hence the terms in the sum cannot refer to hours worked. In fact, the time it would take the slower printer to complete the whole job is *x* hours.

Choice C is not the correct answer. The student may have seen that $\frac{1}{x}$ is the smaller term in the sum, wrongly concluded that the smaller term must apply to the faster printer, and then assumed the 2 in the numerator of the second term implies the equation describes work completed in 2 hours. In fact, the portion of the job that the faster printer could complete in 2 hours is $(2)\left(\frac{2}{x}\right) = \frac{4}{x}$.

Choice D is not the correct answer. The student may have correctly seen that the value $\frac{1}{5}$ on the right side refers to the portion of the job completed, but not seen that in this context, the rates (that is, the work completed in a fixed time) of the printers can be added to get the combined rate, but the times it takes each printer to complete the job cannot be added to get the time for both printers working together. Hence the terms in the sum cannot refer to hours worked. In fact, the time it takes the slower printer to complete $\frac{1}{5}$ of the job is $\frac{x}{5}$ hours.

SAMPLE 22

$$x^2 + y^2 = 153$$
$$y = -4x$$

If (x, y) is a solution to the system of equations above, what is the value of x^2?

A) –51
B) 3
C) 9
D) 144

CONTENT: Passport to Advanced Math

KEY: C

CALCULATOR: Permitted

In this question, students are asked to manipulate one equation to use in another in order to solve for a given value, making use of structure where appropriate.

Choice C is correct. The second equation gives y in terms of x, so a student can use this to rewrite the first equation in terms of x. Substituting $-4x$ for y in the equation $x^2 + y^2 = 153$ gives $x^2 + (-4x)^2 = 153$. This can be simplified to $x^2 + 16x^2 = 153$, or $17x^2 = 153$. Since the question asks for the value of x^2, not x, dividing both sides of $17x^2 = 153$ by 17 gives the answer: $x^2 = \frac{153}{17} = 9$.

Choice A is not the correct answer. This answer may result from neglecting to square the coefficient -4 in $y = -4x$, which would give $y^2 = -4x^2$. Then the first equation would become $x^2 - 4x^2 = -3x^2 = 153$, which would give –51 as the value of x^2.

Choice B is not the correct answer. This answer may result from finding the value for x, not the value of x^2.

Choice D is not the correct answer. This answer may result from finding the value of y^2, not x^2.

SAMPLE 23

If the expression $\frac{4x^2}{2x-1}$ is written in the equivalent form $\frac{1}{2x-1} + A$, what is A in terms of x?

A) $2x + 1$
B) $2x - 1$
C) $4x^2$
D) $4x^2 - 1$

CONTENT: Passport to Advanced Math

KEY: A

CALCULATOR: Permitted

This question assesses the ability to transform a given expression in a more useful form (from improper to proper rational form). There are multiple approaches to this problem, some longer and more routine, some faster and more insightful.

Choice A is correct. The form of the equation suggests performing long division on $\frac{4x^2}{2x-1}$:

$$\begin{array}{r} 2x+1 \\ 2x-1\overline{)4x^2} \\ \underline{4x^2-2x} \\ 2x \\ \underline{2x-1} \\ 1 \end{array}$$

Since the remainder 1 matches the numerator in $\frac{1}{2x-1}$, it is clear that $A = 2x + 1$.

A short way to find the answer is to use the structure to rewrite the numerator of the expression as $(4x^2 - 1) + 1$, recognizing the term in parentheses as a difference of squares, making the expression equal to $\frac{(2x-1)(2x+1)+1}{2x-1} = 2x + 1 + \frac{1}{2x-1}$. From this, the answer $2x + 1$ is apparent. Another way to find the answer is to isolate A in the form $A = \frac{4x^2}{2x-1} - \frac{1}{2x-1}$ and simplify. As with the first approach, this approach also requires students to recognize $4x^2 - 1$ as a difference of squares that factors.

Choice B is not the correct answer. The student may have made a sign error while subtracting partial quotients in the long division.

Choice C is not the correct answer. The student may misunderstand how to work with fractions and may have tried the incorrect calculation

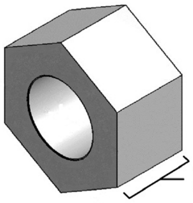
$$\frac{4x^2}{2x-1} = \frac{(1)\left(4x^2\right)}{2x-1} = \frac{1}{2x-1} + 4x^2.$$

Choice D is not the correct answer. The student may misunderstand how to work with fractions and may have tried the incorrect calculation

$$\frac{4x^2}{2x-1} = \frac{1+4x^2-1}{2x-1} = \frac{1}{2x-1} + 4x^2 - 1.$$

Sample Questions: Additional Topics in Math

SAMPLE 24



The figure above shows a metal hex nut with two regular hexagonal faces and a thickness of 1 cm. The length of each side of a hexagonal face is 2 cm. A hole with a diameter of 2 cm is drilled through the nut. The density of the metal is 7.9 grams per cubic cm. What is the mass of this nut, to the nearest gram? (Density is mass divided by volume.)

CONTENT: Additional Topics in Math

KEY: 57

CALCULATOR: Permitted

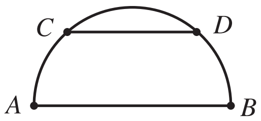
The question above asks students to make connections between physical concepts such as mass and density and essential geometric ideas such as the Pythagorean theorem and volume formulas. There are multiple approaches to solving this problem, but in any of them, the aim is to find the volume of the metal nut and then use the density of the metal to calculate the mass of the nut (57 grams). This is a multistep problem that requires students to devise a multistep strategy and carry out all the algebraic and numerical steps without error.

To solve this problem, students need to find the volume of the hex nut and then use the given fact that density is mass divided by volume.

Finding the volume of the hex nut requires several steps. The first step is to calculate the area of one of the hexagonal faces (without the drilled hole). Each face is a regular hexagon, which can be divided into 6 equilateral triangles with side lengths of 2 cm. Using 30-60-90 triangle properties, the height of each equilateral triangle is $\sqrt{3}$ cm. In turn, the area of one equilateral triangle is $\frac{1}{2}bh = \frac{1}{2}(2)(\sqrt{3}) = \sqrt{3}$ square cm, so the area of the hexagonal face is $6\sqrt{3}$ square cm. The volume of the hexagonal prism is the area of one face multiplied by the height (or thickness in this case), $(6\sqrt{3})(1) = 6\sqrt{3}$ cubic cm. Then, to account for the drilled hole, students need to calculate the volume of a cylinder with diameter 2 cm (or radius 1 cm) and height 1 cm, $V = \pi r^2 h = \pi(1^2)(1) = \pi$ cubic cm, and subtract it from the volume of the hexagonal prism to yield $6\sqrt{3} - \pi$ cubic cm.

Finally, density is mass divided by volume, $7.9 = \frac{\text{mass}}{6\sqrt{3} - \pi}$. Multiplying both sides of the equation by $(6\sqrt{3} - \pi)$ yields the mass of the hex nut as $7.9(6\sqrt{3} - \pi)$ grams. When the values for $\sqrt{3}$ and π are substituted and the result is rounded to the nearest gram, the answer is approximately 57 grams. Note that it is critical for students to attend to the precision of their calculations when solving this problem and not apply any intermediate rounding until the final answer is reached. Here, the use of a calculator provides the ability to attend to precision more effectively and thus is highly encouraged.

SAMPLE 25



The semicircle above has a radius of r inches, and chord $\overline{CD}$ is parallel to the diameter $\overline{AB}$. If the length of $\overline{CD}$ is $\frac{2}{3}$ of the length of $\overline{AB}$, what is the distance between the chord and the diameter in terms of r?

A) $\frac{1}{3}\pi r$

B) $\frac{2}{3}\pi r$

C) $\frac{\sqrt{2}}{2}r$

D) $\frac{\sqrt{5}}{3}r$

CONTENT: Additional Topics in Math

KEY: D

CALCULATOR: Not Permitted

This problem requires students to make use of properties of circles and parallel lines in an abstract setting. Students will have to draw an additional line in order to find the relationship between the distance of the chord from the diameter and the radius of the semicircle. Again, this item provides an opportunity for using different approaches to find the distance required: one can use either the Pythagorean theorem or the trigonometric ratios.

Choice D is correct. This represents the length of the distance between the chord and the diameter, using a radius of the circle to create a triangle, and then the Pythagorean theorem to solve correctly: $r^2 = x^2 + \left(\frac{2}{3}r\right)^2$, where r represents the radius of the circle and x represents the distance between the chord and the diameter.

Choice A is not the correct answer. It does not represent the length of the distance between the chord and the diameter. The student who selects this answer may have tried to use the circumference formula to determine the distance rather than making use of the radius of the circle to create a triangle.

Choice B is not the correct answer. It does not represent the length of the distance between the chord and the diameter. The student who selects this answer may have tried to use the circumference formula to determine the distance rather than making use of the radius of the circle to create a triangle.

Choice C is not the correct answer. It does not represent the length of the distance between the chord and the diameter. The student who selects this answer may have made a triangle within the circle, using a radius to connect the chord and the diameter, but then may have mistaken the triangle for a 45-45-90 triangle and tried to use this relationship to determine the distance.

SAMPLE 26

It is given that $\sin x = a$, where x is the radian measure of an angle and $\frac{\pi}{2} < x < \pi$.

If $\sin w = -a$, which of the following could be the value of w?

A) $\pi - x$
B) $x - \pi$
C) $2\pi + x$
D) $x - 2\pi$

CONTENT: Additional Topics in Math

KEY: B

CALCULATOR: Not Permitted

In problems like this, students must reason how angles x and w are related based on their corresponding sine values and determine the radian measure of angle w, given the parameters of angle x.

Choice B is correct. If an angle with radian measure x such that $\frac{\pi}{2} < x < \pi$ is placed in standard position, its terminal side will fall in Quadrant II, and $\sin x = a$ will be the y-coordinate of the point P where its terminal side intersects the unit circle. If $\sin w = -a$, then when the angle with radian measure w is placed in standard position, its terminal side will intersect the unit circle at a point with y-coordinate equal to $-a$. There are two such points on the unit circle: the reflection of P across the x-axis, which would correspond to an angle with radian measure $-x$ (and also with radian measures $\ldots -6\pi - x, -4\pi - x, -2\pi - x, 2\pi - x, 4\pi - x, 6\pi - x \ldots$); and the reflection of P through the origin, which would correspond to an angle with radian measure $x - \pi$ (and also with radian measures $\ldots x - 5\pi, x - 3\pi, x + \pi, x + 3\pi, x + 5\pi \ldots$). Thus, of the choices given, only $x - \pi$ could be the value of w.

Choice A is not the correct answer. In general $\sin(-x) = -\sin x$ and $\sin(x + \pi) = -\sin x$, so $\sin(\pi - x) = -\sin(-x) = -(-\sin x) = \sin x$. Therefore, $\sin(\pi - x) = a$, not $-a$.

Choice C is not the correct answer. In general, $\sin(2\pi + x) = \sin x$, so $\sin(2\pi + x) = a$, not $-a$.

Choice D is not the correct answer. In general, $\sin(x - 2\pi) = \sin x$, so $\sin(x - 2\pi) = a$, not $-a$.